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DETERMINATION OF PRESSURE LOSS DURING FLOW OF VISCO-PLASTIC MIXTURES IN HORIZONTAL PIPELINES IN THE LAMINAR FLOW ZONE

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ABSTRACT

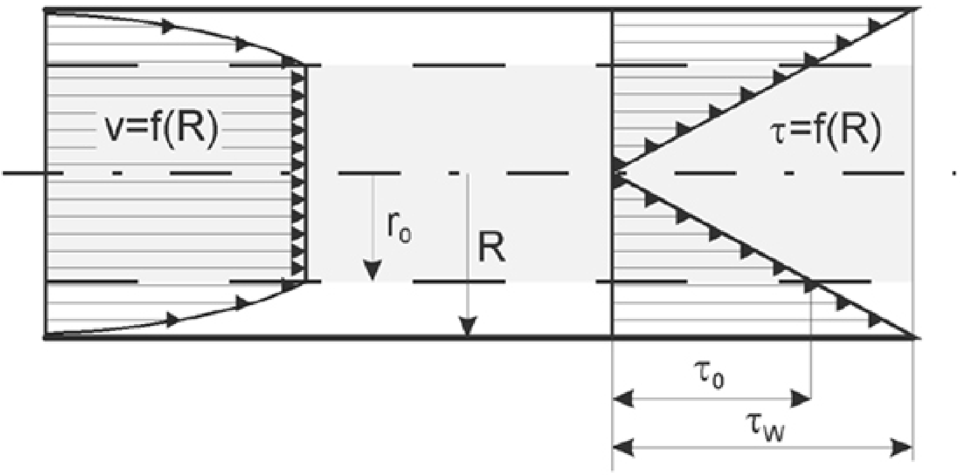
The paper shows a method of determination of pressure loss during flow of the visco-plastic mixtures in pipelines on the basis of dimensionless criterion $\lambda(Re_{gen})$. In order to describe their rheological characters, the 3-parameter generalized Herschel-Bulkley and Vočadlo models were applied. Possible applications of simplified formulas for calculation of the generalized Reynolds number for both considered rheological models were analyzed. The analysis give a range of relative error for assumption that yield stress τ_o in the laminar flow zone has been omitted in calculations.

Key words: rheology, generalized Reynolds number, friction factor, pressure loss.

INTRODUCTION

The pipeline hydro-transport of mixtures is in many cases an alternative solution to the mobile transport (cisterns, water-carts) and it is an optimum both due to the economic and the environmental reasons [6]. Application of the pipeline transport of mixtures with considerable concentration of solid component requires knowledge of hydraulic and rheological characteristic of transported medium. Mixtures usually transported in pipelines are: soil-water mixtures, river and lake aggradate muds, sludges, liquid manure, feed mixtures applied hydraulically, as well as many other liquid organic and mineral materials. Effective transport requires mixtures with the greatest possible concentration of the solid component. However, the maximum concentration should not exceed the thixotropic concentration i.e. the concentration at which mixture loses its rheologically stable behaviour. In most cases, mixtures with considerable concentration of solid parts behave like visco-plastic body showing changeable plastic viscosity. A typical characteristic of these mixtures is an occurrence of yield stress τ_o, exceeding of which results in beginning of a phenomenon of mixture flow in pipeline. A diagram of the cross-sectional average velocity v and the shear stress τ, for flow of the visco-plastic body has been shown in Figure 1 [3]. It is the flow with a characteristic flow core of radius r_o, in which the shear stress τ is lower than the yield stress τ_o ($\tau < \tau_o$). A change in the character of the flow occurs only in the transition or turbulent flow zone i.e. after a critical flow velocity v_{kr} has been exceeded. According to numerous researchers [11], only in this zone the yield stress τ_o can be omitted, and the flow resistance, apart from the impact of rheological characteristics of the mixture, is considerably influenced by a roughness factor of pipe.

Fig. 1. Velocity and shear stress profiles in a cross-section of pipe during the laminar flow of the Bingham plastic



Due to the influence of their internal structure, highly-concentrated visco-plastic mixtures flow in pipelines in the laminar or rarely transitory flow zone. It is also justified by economic reasons.

Thus, it must be assumed that the hydrotransport of the mixtures with high concentration, but lower than the thixotropic concentration, will take place in the laminar flow zone with velocities exceeding the threshold velocity v_{gr}. This assumption results in a necessity of including the yield stress value in calculations regarding flow of visco-plastic mixtures in pipelines.

In author's opinion, it allows the thesis that correct dimensioning of pump and pipe installations for the visco-plastic mixtures requires application of a type $\lambda(Re_{gen})$ dimensionless criterion based on the generalized three-parameter rheological model e.g. Herschel-Bulkley or Vočadlo model. The main aim of this paper is to prove the expressed earlier thesis.

RHEOLOGICAL CHARACTERIZATION OF VISCO-PLASTIC BODY

For determination of rheological behaviour of visco-plastic mixtures, the models including the yield stress τ_o are applied. Apart from numerous existing models that meet this condition, only the two-parameter Bingham model and the three-parameter generalized Herschel-Bulkley and Vočadlo models find practical application in determination of the pipeline hydrotransport conditions of visco-plastic mixtures in the laminar flow zone [1,10, 5].

Bingham model

$$\begin{aligned} &\tau = \tau_0 + \eta_p \cdot G \quad \text{for } \tau > \tau_0 \\ &G = 0 \qquad\qquad\quad \text{for } \tau < \tau_0 \end{aligned} \tag{1}$$

Herschel-Bulkley model

$$\begin{aligned} &\tau = \tau_0 + k \cdot G^n \quad \text{for } \tau > \tau_0 \\ &G = 0 \qquad\qquad\quad \text{for } \tau < \tau_0 \end{aligned} \tag{2}$$

Vočadlo model

$$\begin{aligned} &\tau = \left(\tau_0^{1/n} + KG\right)^n \quad \text{for } \tau > \tau_0 \\ &G = 0 \qquad\qquad\qquad\quad \text{for } \tau < \tau_0 \end{aligned} \tag{3}$$

The generalized three-parameter models (2), (3) include simpler two- and one-parameter models.

In order to determine the rheological parameters of mixture, the viscometric research ought to be carried out with the use of rotary or pipe viscometers. For the economic and practical reasons, the rotary viscometers find the widest application. Necessary viscometric tests can be carried out even with a small-volume sample allowing its complete and accurate rheological characterization. Moreover, the viscometric research, as distinct from the pipeline investigations, makes possible the direct determination of the yield stress.

DETERMINATION OF RHEOLOGICAL PARAMETERS OF MIXTURE

The viscometric research with the use of a rotary viscometer as well as determination of the pseudo-curves of flow of the visco-plastic mixtures ought to be carried out in accordance with the methodology provided by the author [8].

Determination of the rheological parameters on the basis of a prior determined true flow curve can be performed in accordance with the methodology described by Czaban [1].

Particular attention ought to be paid to determination of the yield stress τ_o, which, in the author's opinion, requires closer elaboration. This parameter determines conditions of flow of visco-plastic mixtures in pipelines. According to many authors [11, 10, 6], the yield stress should be treated as a physical parameter, permanent to various models describing the rheological behaviour of the visco-plastic mixtures. The value of yield stress can be directly measured in the course of viscometric research.

The visco-plastic mixtures in the motionless state have a spatial structure whose flexibility is large enough to resist any shear stress τ lower than the yield stress τ_o. If the yield stress τ_o is exceeded, the spatial structure is damaged and the system acquires the characteristics of pseudo-plastic or Newtonian fluid subjected to τ-τ_o stress. The spatial structure of mixture will start to reconstruct as soon as shear stress τ drops below the yield stress τ_o.

One of the most common methods of direct determination of the yield stress during tests with the rotary viscometer is lowering of a deformation pseudo-velocity G_p to zero ($G_p \rightarrow 0$). Practically, the value of yield stress τ_o is obtained after a drive of a rotary viscometer had been turned off as the direct reading from a scale of the device. For more recent viscometers e.g. those manufactured by Haake, it is possible to determine the yield stress automatically i.e. at the beginning of measurement the applied shear stress is measured up to the moment when the rotary cylinder starts to move. The beginning of shearing in a gap between the internal and external cylinder of the rotary viscometer ($G_p \neq 0$) is recognized as exceeding the yield stress τ_o and the momentary value of shear stress is registered as τ_o value.

HYDRAULIC CHARACTERIZATION OF THE FLOW

Rate of flow in a pipeline of diameter D for viscoplastic mixtures, according to [1, 5, 6, 4] equals:

- for Herschel-Bulkley model

$$Q = \frac{\pi D^3}{8}\left(\frac{\tau_w}{k}\right)^{\frac{1}{n}} \frac{n}{n+1}\left(1-\frac{\tau_0}{\tau_w}\right)^{\frac{n+1}{n}} \left\{1-\frac{2n}{3n+1}\left(1-\frac{\tau_0}{\tau_w}\right)\left[1+\frac{n}{2n+1}\cdot\frac{\tau_0}{\tau_w}\right]\right\} \tag{4}$$

- for Vočadlo model

$$Q = \frac{\pi D^3}{8K}\tau_w^{\frac{1}{n}}\left[\frac{n}{3n+1}-\frac{1}{3}\left(\frac{\tau_0}{\tau_w}\right)^{\frac{1}{n}}+\frac{1}{3(3n+1)}\left(\frac{\tau_0}{\tau_w}\right)^{\frac{3n+1}{n}}\right] \tag{5}$$

Assumption n = 1 for equations (4, 5) will lead to the Reiner-Buckingham formula for flow described with 2-parameter Bingham model. Average velocity of flow in pipeline equals:

- for Herschel-Bulkley model

$$V = \frac{D}{2}\left(\frac{\tau_w}{k}\right)^{\frac{1}{n}} \frac{n}{n+1}\left(1-\frac{\tau_0}{\tau_w}\right)^{\frac{n+1}{n}} \left\{1-\frac{2n}{3n+1}\left(1-\frac{\tau_0}{\tau_w}\right)\left[1+\frac{n}{2n+1}\cdot\frac{\tau_0}{\tau_w}\right]\right\} \tag{6}$$

- for Vočadlo model

$$V = \frac{D}{2K}\tau_w^{\frac{1}{n}}\left[\frac{n}{3n+1}-\frac{1}{3}\left(\frac{\tau_0}{\tau_w}\right)^{\frac{1}{n}}+\frac{1}{3(3n+1)}\left(\frac{\tau_0}{\tau_w}\right)^{\frac{3n+1}{n}}\right] \tag{7}$$

By transformation of equations (6), (7) the author derived formula allowing calculation of tangential stresses on the wall of a pipe:

- for Herschel-Bulkley model

$$\tau_w = \frac{(2v)^n k}{D^n\left|\frac{n}{n+1}\left(1-\frac{\tau_0}{\tau_w}\right)^{\frac{n+1}{n}}\left\{1-\frac{2n}{3n+1}\left(1-\frac{\tau_0}{\tau_w}\right)\left[1+\frac{n}{2n+1}\cdot\frac{\tau_0}{\tau_w}\right]\right\}\right|^n} \tag{8}$$

Vočadlo model

$$\tau_w = \frac{(2vK)^n}{D^n\left[\frac{n}{3n+1}-\frac{1}{3}\left(\frac{\tau_0}{\tau_w}\right)^{1/n}+\frac{1}{3(3n+1)}\left(\frac{\tau_0}{\tau_w}\right)^{\frac{3n+1}{n}}\right]^n} \tag{9}$$

DETERMINATION OF HYDRAULICAL LOSS IN PIPELINE

Govier, Aziz [2] give the equation of flow for visco-plastic fluid and suggest its solution, for evaluation of pressure loss, by means of the trial-and-error method. Friction factor λ , in the opinion of the authors, is functionally related to the Reynolds number Re for the pseudo-plastic model, Y parameter resulting from yield stress τ_0 and the structural number n. This is similar to known solution of λ(Re, He) type for Bingham's model (Hedstrom 1952, Govier 1959), which generates in the laminar flow zone a family of curves λ(Re), successively shifted depending on the Hedstrom number He. Determination of the friction factor λ is more complicated in this method.

Application of the dimensionless criterion of $\lambda(Re_{gen})$ type solves this problem, but also requires determination of the generalized Reynolds number for a rheological model assumed. The generalized Reynolds number can be calculated on basis of the equations (15), (16):

$$Re_{gen} = \frac{8\rho v^2}{\tau_w} \tag{10}$$

The author, in accordance with the papers [5, 4] and the equations (8), (9), derived for the first time the formula for calculation of the generalized Reynolds number for the Herschel-Bulkley and Vočadlo models:

- for the Herschel-Bulkley model

$$Re_H = \frac{8v^{(2-n)}D^n\rho}{2^n k}\left|\frac{n}{n+1}\left(1-\frac{\tau_0}{\tau_w}\right)^{\frac{n+1}{n}}\left\{1-\frac{2n}{3n+1}\left(1-\frac{\tau_0}{\tau_w}\right)\left[1+\frac{n}{2n+1}\cdot\frac{\tau_0}{\tau_w}\right]\right\}\right|^n \tag{11}$$

- for the Vočadlo model

$$Re_v = \frac{8v^{(2-n)}D^n\rho}{(2K)^n}\left[\frac{n}{3n+1}-\frac{1}{3}\left(\frac{\tau_0}{\tau_w}\right)^{\frac{1}{n}}+\frac{1}{3(3n+1)}\left(\frac{\tau_0}{\tau_w}\right)^{\frac{3n+1}{n}}\right]^n \tag{12}$$

The usage of equations (11), (12) for the generalized Reynolds number requires prior determination of shearing stress at the wall of pipe τ_w, what is not possible during rotary viscometer rheological tests. This type of viscometer only enables establishing of parameters of the rheological model assumed for approximation of flow curve, e.g. τ_o, k for the Herschel- Bulkley model. The author, to avoid the above-mentioned difficulty, has introduced equations for calculation of shearing stress τ_w for the analyzed models of Herschel-Bulkley and Vočadlo, for given rheological characteristics of mixture and flow parameters in the pipe of diameter D; equation (8), (9).

Assuming $\tau_w > \tau_o$, a simple iteration would lead directly to determination of the ultimate τ_w value that results from compatibility of formulas (8) or (9) in dependence on the given rheological model.

Many researches like Dodge, Metzner, Hornig [9] trivialize influence of the yield stress on the fluid rheological behaviour in the laminar flow zone and simplify calculation of the Reynolds number to the following equations:

- for the Herschel-Bulkley model

$$Re_H = \frac{8v^{(2-n)}D^n\rho}{k\left(6+\frac{2}{n}\right)^n} \tag{13}$$

- for the Vočadlo model

$$Re_v = \frac{8v^{(2-n)}D^n\rho}{K^n\left(6+\frac{2}{n}\right)^n} \tag{14}$$

Such simplification is equivalent to change of the visco-plastic body model to the pseudoplastic body model and is admissible only in the transitory or turbulent flow zone. The simplified procedure used earlier was also a result of absence of the formula allowing the determination of the Reynolds number for considered three-parameter models. In the laminar flow zone the friction factor λ can be determined from the Poiseuille equation, similarly as for the Newtonian fluid:

$$\lambda = \frac{64}{Re_{gen}} \tag{15}$$

Value of pressure loss Δp can be then determined basing on the Darcy-Weisbach equation (11) for the flow of the visco-plastic mixtures in horizontal pipelines:

$$\Delta p = \lambda \frac{L}{D}\frac{v^2}{2}\rho \tag{16}$$

DISCUSSION AND CONCLUSIONS

To assess a possibility of application of the dimensionless criterion $\lambda(Re_{gen})$, theoretic calculation of the relative error of the generalized Reynolds number determination has been performed. In this calculation following formula was used, involving equations that take into consideration the yield stress τ_o or not:

$$\delta = \left(\frac{Re_{gen(13,14)} - Re_{gen(11,12)}}{Re_{gen(13,14)}}\right) \cdot 100\% \tag{17}$$

Detailed equations for determination of the relative error δ for the analyzed rheological models, after introducing of equations (11) or (12) and (13) or (14), take following forms:

Herschel-Bulkley model

$$\delta = \left[1 - \frac{\left[\left(1 - \frac{\tau_0}{\tau_w}\right)^{\frac{n+1}{n}} \left\{1 - \frac{2n}{3n+1}\left(1 - \frac{\tau_0}{\tau_w}\right)\left[1 + \frac{n}{2n+1} \cdot \frac{\tau_0}{\tau_w}\right]\right\}\right]^n}{\left(1 - \frac{2n}{3n+1}\right)^n}\right] \cdot 100\% \tag{18}$$

Vočadlo model

$$\delta = \left[1 - \frac{\left[\frac{n}{3n+1} - \frac{1}{3}\left(\frac{\tau_0}{\tau_w}\right)^{\frac{1}{n}} + \frac{1}{3(3n+1)}\left(\frac{\tau_0}{\tau_w}\right)^{\frac{3n+1}{n}}\right]^n}{\left(\frac{n}{3n+1}\right)^n}\right] \cdot 100\% \tag{19}$$

Variability of the relative error δ determined on the basis of equations (18), (19) for the analyzed rheological models is shown in Figures 2, 3. Analyzing the diagrams of dependencies $\delta = f(t_0/t_w, n)$, a clear difference of their runs may be observed depending on the applied rheological model. For Herschel-Bulkley model (fig.2.), the relative error value depends to small extent on the structural number n and remains, for $\tau_o/\tau_w < 0.5$ (appropriate for practical applications – Reiner, Fiłatow [9]), in the range 10-65%. It means that the evaluation of yield stress τ_o while performing rheological assessment of visco-plastic mixtures with use of the Herschel-Bulkley model is very important. Excluding this factor in the laminar flow zone, as shown on the diagram, causes serious error in determination of the generalized Reynolds number and may lead to faulty estimation of flow resistance in pipeline.
For Vočadlo model (fig. 3), the analyzed relative error δ depends clearly on the structural number n. This parameter controls value of the relative error of determination of the Reynolds number Re_v. For $\tau_o/\tau_w < 0.5$ (reccomended for practical application) and resulting values of the structural number n=0.1-0.5, value of the relative error δ vary from

0 to 20%. Only for n>0.8, the relative error δ reaches the values similar to those for the Herschel-Bulkley model. Therefore, it may be considered for approximation of flow curves with the generalized Vočadlo model, that neglect of the yield stress τ_o for values of the structural number n lower than 0.5 would produce error of determination of the Reynolds number Re_v lower than 20%. For values of the structural number n exceeding 0.5, the complete formula (5) describing generalized Reynolds number Re_v with allowance for yield stress τ_o ought to be rigorously applied.

Fig. 2. Relative error $\delta = f(\tau_0/\tau_w, n)$ Re_H determination

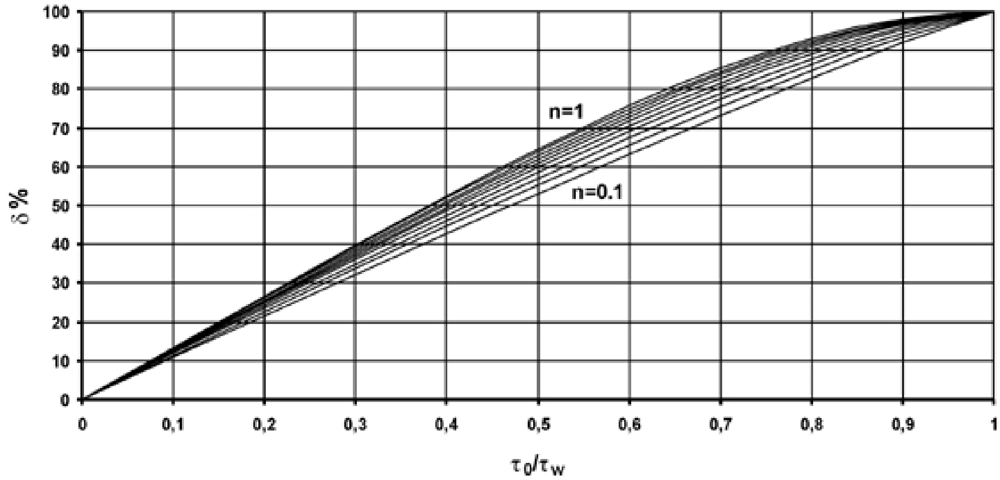
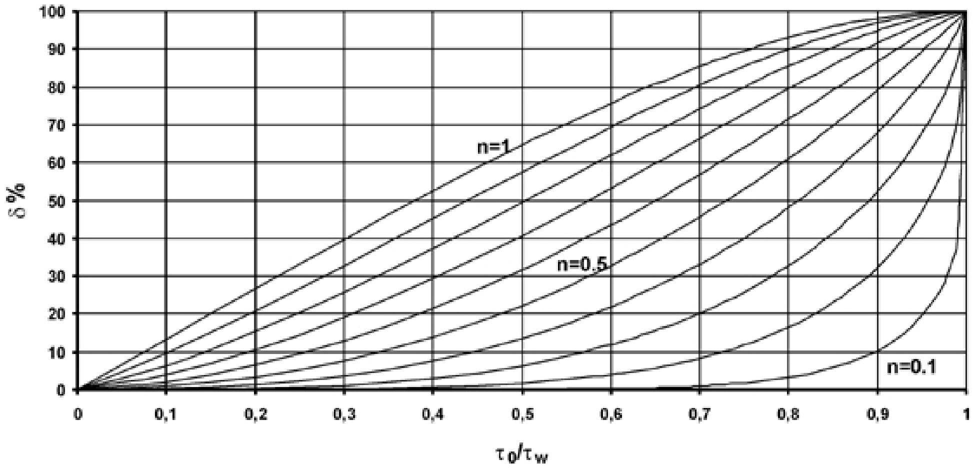


Fig. 3. Relative error $\delta = f(\tau_0/\tau_w, n)$ Re_v determination



The author has analyzed results of tests of liquid cattle manure flow in a pipeline of diameter D=0,125 m and estimated its rheological characteristic with application of the Herschel-Bulkley and the Vočadlo model. Basic determined parameters of the mixture are: ρ=1029 kg m^{-1}, τ_0=0,5 Pa, k=0,824 Pa s^n, K=0,699 $Pa^{1/n}$ s, n=0.540 [7]. The author has presented measured and calculated values of (λ) and measured values of the friction factor (λ_H, λ_V), estimated values of calculation of relative error of the generalized Reynolds number. The author performed the calculation of the relative error in accordance with the equations (18) and (19), taking into account determined earlier value of the yield stress τ_0 and, as a second case, omitting the yield stress in calculations. Results of the calculations can be found in Table 1.

For low velocities of flow ranging from 0.3-0.4 m s^{-1} the values of the friction factor λ_H, λ_V are overcalculated for both analyzed rheological models, referred to the measured values. For velocities above 0.4 m s^{-1} there is high conformity of theoretically calculated and measured values of friction factor.

Estimation of the relative error δ of the Reynolds number determination with or without considering the yield stress τ_0 is fully conforming the theoretical calculations.

On the basis of theoretical analysis and own research the author came to following conclusions:

1. Determination of the rheological properties of visco-plastic mixtures requires application of three-parameter generalized rheological model considering the yield stress, for instance the Herschel-Bulkley or the Vočadlo model.

Table 1. Calculated and measured values of the friction factor λ and the relative error

Lp	v [m·s^{-1}]	Q [m^3·s^{-1}]	ΔP [Pa]	τ_w [Pa]	τ_0/τ_w	λ	λ_H	λ_V	$\delta_{H(18)}$ [%]	$\delta_{V(19)}$ [%]
1	0.18	0.0021	1601.78	2.82	0.18	0.68	1.08	0.87	22.13	3.59
2	0.22	0.0026	1848.20	3.25	0.15	0.51	0.76	0.63	19.21	2.75
3	0.26	0.0031	2217.84	3.91	0.13	0.43	0.57	0.48	16.03	1.96
4	0.32	0.0037	2341.06	4.12	0.12	0.32	0.43	0.37	15.19	1.77
5	0.38	0.0044	2710.70	4.77	0.10	0.26	0.33	0.29	13.13	1.35
6	0.45	0.0053	3203.55	5.64	0.09	0.21	0.24	0.22	11.12	0.99
7	0.53	0.0062	3696.41	6.51	0.08	0.18	0.19	0.17	9.64	0.76
8	0.62	0.0073	4312.48	7.59	0.07	0.15	0.15	0.14	8.27	0.57
9	0.75	0.0088	4682.12	8.24	0.06	0.11	0.11	0.10	7.62	0.49
10	0.91	0.0107	5051.76	8.90	0.06	0.08	0.08	0.08	7.06	0.42

2. Determination of pressure loss in the laminar flow zone on the basis of the dimensionless criterion $\lambda(Re_{gen})$ requires prior determination of rheological characteristic of mixture and application of the full generalized Reynolds number for the considered rheological model. The author for the first time derived the formula for calculation of the generalized Reynolds number for the Herschel-Bulkley and Vočadlo models.

3. The author on basis of his own research affirmatively verified the possibility of application of the generalized criterion of $\lambda(Re_{gen})$ type to prediction of pressure loss in the laminar flow zone in pipeline.

4. Omitting the value of the yield stress τ_0 in the formula for calculation of the Reynolds number leads to significant errors, that depend in detail on the rheological and hydraulic properties of mixture.

SUMMARY

Application of the dimensionless criterion $\lambda(Re_{gen})$ for assessment and generalization of research results regarding hydraulic loss in pipelines requires conducting of the viscometric research in order to determine rheological characteristics of the mixture examined. For visco-plastic mixtures, the models including yield stress τ_o should be used for description of their rheological behaviour. In this case, the author recommends application of the three-parameter generalized Herschel-Bulkley and Vočadlo models, which include more simple two- and one-parameter models. However, a necessary condition for application of the $\lambda(Re_{gen})$ criterion is correct determination of the generalized Reynolds number taking into account rheological characteristics of tested mixture. The value of yield stress τ_o determines character of flow of the visco-plastic mixtures in the laminar flow zone. Correct determination of the friction factor λ requires appropriate evaluation of the yield stress τ_o value and including in calculations the full generalized Reynolds number Re_{gen}. After determination of the friction factor λ, one may then evaluate pressure loss during the laminar flow of the visco-plastic mixtures in pipelines from the Darcy-Weisbach equation (16). The analysis carried out by the author showed that reccomended for practical application Herschel-Bulkley (2) and Vočadlo (3) models respond to changes of the rheological parameters τ_o and n in different ways. The Herschel-Bulkley model (2) formed by adding the τ_o parameter to the De Waele-Ostwald model of pseudo-plastic body requires accurate and direct determination of the yield stress. Neglect of the yield stress τ_o in the calculations of the generalized Reynolds number Re_H leads, as it has been proved by the author, to serious error in estimation of the friction factor λ for each value of the structural number n. The Vočadlo model (3) slightly depends on the yield stress τ_o but shows great dependence on the structural number n.

In the author's opinion, selection of one of the two proposed rheological models is directly associated with the possibility of exact determination of the yield stress τ_o. In case of direct τ_o measurement with the rotary viscometer, both Herschel-Bulkley (2) and Vočadlo (3) models can be applied. The yield stress τ_o can not be determined directly by measurements with the pipe viscometer. However, τ_o can be calculated on the basis of approximation of the flow curve τ(G). This indirect, unprecise determination of the τ_o parameter justifies, according to the author, application of the Vočadlo model (3). In this case, application of the Herschel-Bulkley model (2) is too uncertain.

NOTATION

D [m] – diameter of pipe
g [$\mathrm{m \cdot s^{-2}}$] – acceleration due to gravity
G [$\mathrm{s^{-1}}$] – absolute value of the shear rate
K [$\mathrm{Pa^{1/n}s}$] – consistency index for Vočadlo model
k [$\mathrm{Pa \cdot s^{n}}$] – consistency index for Herschel-Bulkley model
L [m] – length of pipe
Re_{gen} – generalized Reynolds number
R [m] – radius of pipe
v [$\mathrm{m \cdot s^{-1}}$] – cross-sectional average velocity
λ – friction factor (measured values)
τ [Pa] – shear stresses
τ_0 [Pa] – yield stress
τ_w [Pa] – shearing stress at the wall of pipe
ρ [$\mathrm{kg \cdot m^{-3}}$] – density
Δp [Pa] – pressure loss

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