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PLANE ELASTIC CONTACT INVOLVING FRICTION AND BOUNDARY ROUGHNESS

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ABSTRACT

Plane elastic contact for a rigid flat-ended punch and a half-space with rough boundary is studied. Taking into account friction forces it is shown that a partial slip near punch edges can exist. The size of the slip zone is unknown and depends on the contact parameters. The problem is reduced to the system of integral equations, which are being solved numerically.

Key words: contact problem, flat-ended punch, elastic half-space, friction, surface roughness, stick, slip.

INTRODUCTION

When a rigid flat-ended punch is symmetrically pressed by a normal load P into an elastic half-space, the following situations is considered:

a) the contacting surfaces are ideally smooth and frictionless. Then the normal and tangential stresses in the common contact area $|x| \le a$ ($2a$ is the punch width) have the well-known forms [5]

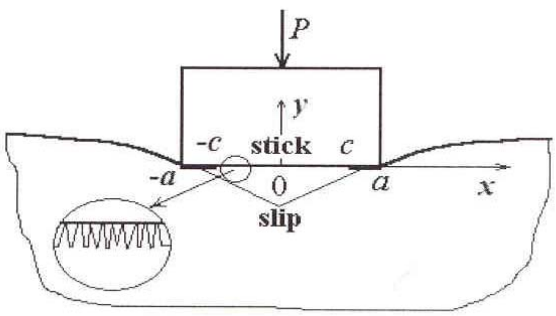
$$p(x) = -\sigma_{yy}(x,0) = \frac{P}{\pi\sqrt{a^2 - x^2}}, \qquad q(x) = \sigma_{xy}(x,0) = 0 \tag{1}$$

b) the friction forces take place in the contact zone. This kind of the contact problems was studied by different approaches in the papers [1,7]. It was shown that for real friction coefficient the contact area is divided into the central stick region ($|x| \le c$) and two lateral zones ($c < |x| \le a$) with the slipping conditions, see Fig. 1. The unknown size of the stick region c can be found from the non-linear equation [7]

$$\frac{K\left(\sqrt{1-(c/a)^2}\right)}{K(c/a)}=\frac{1-2\nu}{2(1-\nu)f} \tag{2}$$

where $K(\cdot)$ is the complete elliptical integral of the first kind, f is the friction coefficient and ν is the Poisson's ratio.

Fig. 1. Geometry of contact



The formulations presented in [1,7] consider, from the geometrical point of view, ideally smooth contacting surfaces. However, real engineering surfaces have the geometrical microstructure, which plays an important role in the contact.

The simultaneous effects due to the friction and the surface roughness in the contact of rigid flat punch and half-space is considered in this paper.

There are many models of the boundary roughness of solids. The well-known Greenwood-Williamson model [2] considers the roughness as elastic asperities with a constant curvature and a Gaussian distribution of heights, which are very densely distributed over the plane surface. The GW model has many modifications and is widely used in the contact problems. However, the GW model can not be applied to formulation of the contact problem involving the friction forces, because it omits tangential forces and horizontal deformation of the rough surfaces. In addition, the GW model was developed for the parabolic contacting bodies and can not be used for the flat-ended geometry.

In this paper, to take into account friction forces in the contact of flat-ended rigid cylinder and elastic rough half space one proposes another model of the boundary roughness. This model is phenomenological and states a modification of the Shtayerman model [6] on the case of tangential forces and deformation. It permits to reduce the considered contact problem to the integral equations which are being solved numerically.

PROBLEM FORMULATION

The contact geometry of the considered problem is shown in Fig. 1 in the Cartesian co-ordinates Oxy. A plane indentation of the rigid flat-ended punch into the elastic half-space is studied. The punch is centrally pressed by the normal load P. It is to assume that the contacting surfaces are rough. Below one proposes a model of the deformation of rough boundary of the half-space, which can be used in the problem under consideration.

According to this model the vertical u_y and tangential u_x displacements of the rough surface of the half-space under the punch consist of two parts

$$u_y(x,0)=u_y^{(a)}(x)+u_y^{(b)}(x,0), \quad |x|\le a \tag{3}$$

$$u_x(x,0)=u_x^{(a)}(x)+u_x^{(b)}(x,0), \quad |x|\le a \tag{4}$$

where $u_x^{(a)}(x)$ and $u_y^{(a)}(x)$ describe displacements due to the deformation of asperities, and $u_x^{(b)}(x,0)$ and $u_y^{(b)}(x,0)$ are bulk displacements. For the first ones a phenomenological model was used, according to which

$$u_y^{(a)}(x) = \alpha p(x), \quad |x| \le a \tag{5}$$

$$u_x^{(a)}(x) = \beta q(x), \quad |x| \le a \tag{6}$$

where α, β are called the roughness parameters. These parameters can be obtained from experimental data or using the method proposed in [4].

The bulk displacements are caused by elastic deformation of the half-space, which is subjected to the action of the tractions $p(x)$ and $q(x)$ [3]

$$u_y^{(b)}(x,0) = -\frac{2(1-\nu^2)}{\pi E}\int_{-a}^{a} p(\xi)\ln|\xi - x| d\xi + \frac{(1-2\nu)(1+\nu)}{2E}\int_{-a}^{a} q(\xi)\,\mathrm{sgn}(\xi - x) d\xi \tag{7}$$

$$u_x^{(b)}(x,0) = -\frac{2(1-\nu^2)}{\pi E}\int_{-a}^{a} q(\xi)\ln|\xi - x| d\xi - \frac{(1-2\nu)(1+\nu)}{2E}\int_{-a}^{a} p(\xi)\,\mathrm{sgn}(\xi - x) d\xi \tag{8}$$

where E is the Young's modulus of the half-space material.

As one can see the model proposed here states the extension of the Shtayerman's one on the action of the friction forces and involves the tangential deformation of the roughness due to these forces. It depends on two parameters α, β which should be determined experimentally. Notice that the first one describes the stiffness of the asperities in the normal direction and second one – in the tangential direction.

The similar model of the boundary roughness was used in [4], where the Cattaneo-Mindlin problem for the rough half-space was studied.

PROBLEM OF NORMAL CONTACT

For further analysis the following assumption were made: the tangential traction $q(x)$ has no effect on the normal displacements. It is known [3], that this effect, generally, is small, what means that the second term in the right side of the presentation (7) can be omitted. This assumption makes possible to consider separately the problem of determination of the normal pressure $p(x)$, and, after this, the tangential problem can be studied.
Satisfying with help of formulae (3), (5), (7) the boundary condition of the normal problem

$$u_y(x,0) = \delta, \qquad |x| \le a \tag{9}$$

where δ is the rigid punch approach, and using the above assumption one arrives at the integral equation of the Fredholm type of the second kind for $p(x)$

$$\alpha p(x) - \frac{2(1-\nu^2)}{\pi E}\int_{-a}^{a} p(\xi)\ln|\xi - x| d\xi = \delta, \quad |x| \le a \tag{10}$$

This equation must be considered with the equilibrium condition

$$\int_{-a}^{a} p(x)dx = P \tag{11}$$

Introducing dimensionless variables, functions and parameters

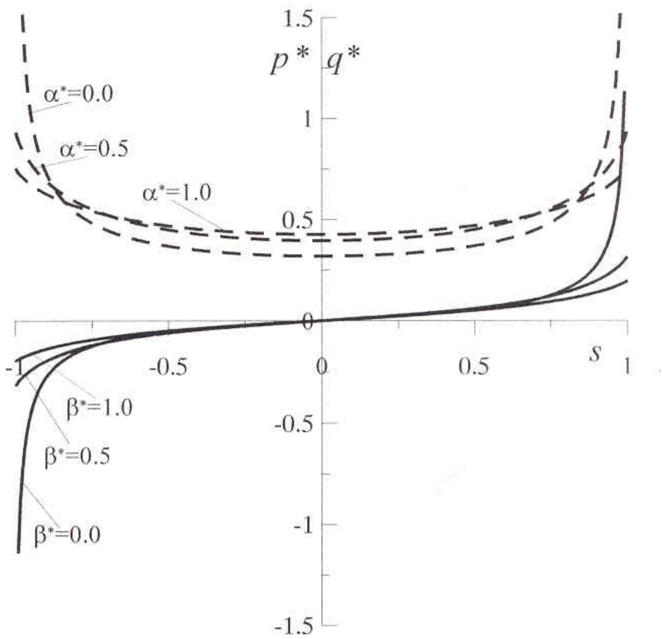
$$s = \frac{x}{a}, \qquad \eta = \frac{\xi}{a}, \qquad p^*(s) = \frac{ap(x)}{P}, \qquad \alpha^* = \frac{\alpha E}{2(1-\nu^2)a}, \quad \delta^* = \frac{\delta E}{2(1-\nu^2)P} \tag{12}$$

the system of integral equations (10), (11) can be rewritten in the normalized form

$$\alpha^* p^*(s) - \frac{1}{\pi}\int_{-1}^{1} p^*(\eta)\ln|\eta - s|d\eta = \delta^*, \quad |s| \le 1 \tag{14}$$

The equations (13), (14) were solved numerically for some values of the dimensionless roughness parameter α^*. The effects of this parameter on the normal pressure distribution are presented in Fig. 2 by discontinuous lines. For $\alpha^*=0$ we obtain the Sadowsky's solution (1) which is unbounded for $s\to\pm 1$. But in the problem involving boundary roughness the contact pressure at the punch edges, as one can see, is bounded.

Fig. 2. Distribution of the normal pressure for some values of parameter α^* (discontinuous lines). Distribution of the tangential traction for some values of parameter β^* and for $\alpha^*=0.5$, $\nu=0.3$ (continuous lines)



PUNCH PERFECTLY CONNECTED WITH HALF-SPACE

First one considers the case when the friction forces are sufficiently great to prevent slip in the whole contact area. Then the boundary conditions of the tangential problem are as follows

$$u_x(x,0) = 0, \qquad |x| \le a \tag{15}$$

$$q(x) < fp(x), \qquad |x| \le a \tag{16}$$

Satisfying the condition (15) with help of presentation (4), (6), (8) one arrives at the following integral equation for unknown function $q(x)$

$$\beta q(x) - \frac{2(1-\nu^2)}{\pi E}\int_{-a}^{a} q(\xi)\ln|\xi - x|d\xi = \frac{(1-2\nu)(1+\nu)}{2E}\int_{-a}^{a} p(\xi)\,\mathrm{sgn}(\xi - x)d\xi, \quad |x| \le a \tag{17}$$

Here the contact pressure $p(x)$ is assumed to be known from the previous Section.

This equation must be solved together with the equilibrium condition

$$\int_{-a}^{a} q(x)dx = 0 \tag{18}$$

The equations (17), (18) were solved numerically in normalized terms (12) and additionally

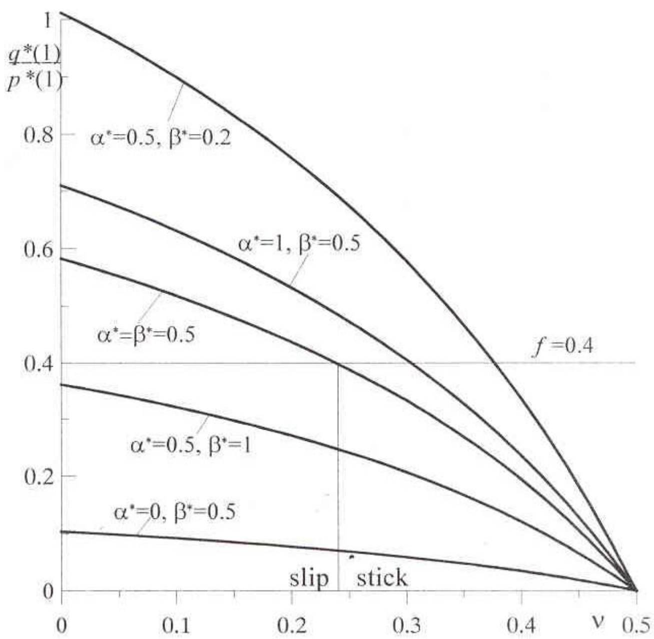
$$q^*(s) = \frac{aq(x)}{P}, \quad \beta^* = \frac{\beta E}{2(1-\nu^2)a} \tag{19}$$

The distributions of the frictional traction for some values of the parameters β^* and for $\alpha^*=0.5$, $\nu=0.3$ are shown is Fig. 2 by continuous lines. One can see that in the problem involving boundary roughness the friction forces are bounded at the punch edges. Upon increasing the parameter β^*, these forces decrease.

For purposes of studying a possible slip between the contacting surfaces the ratio of tangential to normal tractions should be investigated. In the classical formulation [1,7] this ratio is a function of the Poisson's module ν and the coordinate x, and tends to infinity at the punch edges. This means that the condition (16) can not be satisfied in this point for physically accepted values of the friction coefficient $f<\infty$. As a consequence, a slip is expected near the contact area edge.

Quite different situation is observed in the contact problem involving the boundary roughness. The ratio $q(x)/p(x)$ is now a function of ν, x, as well as of the roughness parameters α^* and β^*. This ratio at the punch edges is shown in Fig. 3 versus the Poisson's ratio for some sets of α^* and β^*. One can see that it is bounded, and there are the values of parameters ν, α^*, β^*, for which the stick condition (16) is satisfied for the given friction coefficient f. For example, if $f=0.4$ and roughness parameters $\alpha^*=\beta^*=0.5$ there is the range of the Poisson's ratio $0.24<\nu<0.5$ (see Fig. 3), for which the condition (16) is satisfied and the contacting surfaces remain stuck within the whole contact region. If $0<\nu<0.24$ this condition is disturbed at $x=\pm a$ and a slip must occur near the punch edges. Such formulation of the contact will be considered in the next Section. The possibility of an expected slip for other sets of parameters f, α^*, β^* can be read from Fig.3. Generally, a slip is bigger for smaller β^* when α^* is fixed.

Fig. 3. Ratio $q^*(1)/p^*(1)$ as function of the Poisson's ratio ν for some sets of parameters α^* and β^*



PARTIAL SLIP CONTACT PROBLEM

Let it consider the problem, in which the contacting surfaces are stuck in the central part of the contact area, while near the punch edges a slip takes place. The problem is naturally symmetric according to the *x*-variable. The boundary conditions of this problem are as follows

$$u_x(x,0) = 0, \qquad |x| \le c \tag{20}$$

$$q(x) < fp(x), \qquad |x| \le c \tag{21}$$

$$q(x) = fp(x), \qquad c < |x| \le a \tag{22}$$

$$u_x(x,0) > 0, \qquad c < |x| \le a \tag{23}$$

One can notice that the stick area size c is unknown.

Satisfying the boundary condition (20) with help of the formulae (4), (6), (8) the integral equation for the tangential traction may be obtained

$$\beta q(x) - \frac{2(1-\nu^2)}{\pi E}\int_{-a}^{a} q(\xi)\ln|\xi - x|d\xi = \frac{(1-2\nu)(1+\nu)}{2E}\int_{-a}^{a} p(\xi)\,\mathrm{sgn}(\xi - x)d\xi, \quad |x| \le c \tag{24}$$

where the contact pressure $p(x)$ as assumed to be known from the solution of the normal problem.

The integral equation (24) can not be solved in the present form because it is written within the stick area while the unknown function $q(x)$ is defined on the whole contact region. For purposes to obviate this difficulty the distribution of tangential traction is presented in the form

$$q(x) = fp(x)\,\mathrm{sgn}(x) + \begin{cases} q_0(x), & |x| \le c \\ 0, & c < |x| \le a \end{cases} \tag{25}$$

where $q_0(x)$ is the new unknown called the corrective traction. One can notice that the presentation (25) satisfies the boundary conditions (21), (22) if $q_0(x)<0$.

Substituting (25) into (24), the integral equation for the corrective traction can be derived

$$\begin{aligned} \beta q_0(x) - \frac{2(1-\nu^2)}{\pi E}\int_{-c}^{c} q_0(\xi)\ln|\xi - x|d\xi &= \frac{(1-2\nu)(1+\nu)}{2E}\int_{-a}^{a} p(\xi)\,\mathrm{sgn}(\xi - x)d\xi \\ - f\beta p(x)\,\mathrm{sgn}(x) + -\frac{2(1-\nu^2)f}{\pi E}\int_{-a}^{a} p(\xi)\,\mathrm{sgn}(\xi)\ln|\xi - x|d\xi, &\qquad |x| \le c \end{aligned} \tag{26}$$

The equilibrium condition (18) with help of presentation (25) can be transformed to the equation

$$\int_{-c}^{c} q_0(x)dx = 0 \tag{27}$$

In the normalized variables and functions (12), (19) and

$$q_0^*(s) = \frac{aq_0(x)}{fP}, \quad c^* = \frac{c}{a} \tag{28}$$

the equations (26), (27) read

$$\beta^* q_0^*(s) - \frac{1}{\pi}\int_{-c^*}^{c^*} q_0^*(\eta)\ln|\eta - s|d\eta = \frac{1-2\nu}{4(1-\nu)f}\int_{-1}^{1} p^*(\eta)\,\mathrm{sgn}(\eta - s)d\eta -$$
$$- \beta^* p^*(s)\,\mathrm{sgn}(s) + \frac{1}{\pi}\int_{-1}^{1} p^*(\eta)\,\mathrm{sgn}(\eta)\ln|\eta - s|d\eta, \qquad |s| \le c^* \tag{29}$$

$$\int_{-c^*}^{c^*} q_0^*(s)ds = 0 \tag{30}$$

The integral equations (29), (30) were solved numerically. Unknown normalized size of the stick zone c^* was calculated iteratively satisfying the physical conditions

$$q_0^*(\pm c^*) = 0 \tag{31}$$

which, according to the formula (25), provides the continuity of the tangential traction within the whole contact area.

After calculation of the corrective traction, the distribution of friction forces can be obtained from the presentation (25).

Input data for the calculations are: the Poisson's ratio ν, the roughness parameters α^* and β^*, and the friction coefficient f. For calculation one takes ν=0.3.

Figures 4a and 4b show the distributions of tangential traction for two sets of the roughness parameters and some values of the friction coefficient. One can observe the decreasing of this traction upon decreasing the friction coefficient. Simultaneously the stick zone shrinks. If the friction coefficient increases, the distributions of the shear traction tend to those obtained in the case of the perfectly connected punch.

Fig. 4a. Distributions of tangential traction for $\alpha^* = \beta^*$=0.5 and some values of the friction coefficient f

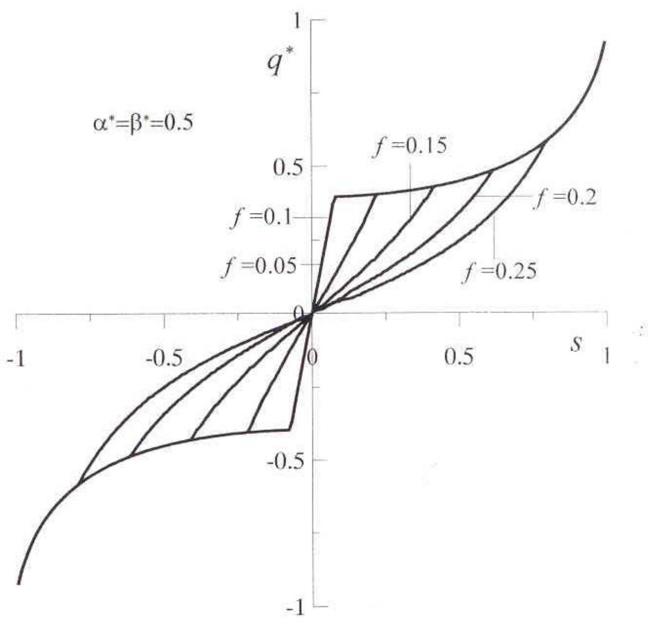
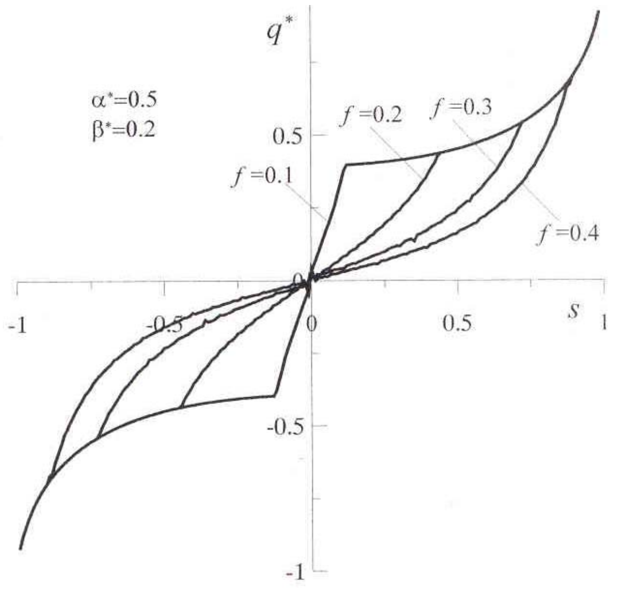


Fig. 4b. Distributions of tangential traction for α^*=0.5, β^*=0.2 and some values of the friction coefficient *f*



The dimensionless stick region size c^* as functions of the friction coefficient *f* is presented in Fig. 5a and Fig. 5b for the same sets of the roughness parameters. Increasing of the stick zone size with the friction coefficient is observed. At some values of the friction coefficient one obtains c^*=1 what means that the slip zone vanishes to zero and the contacting surfaces are in the stick conditions.

Fig. 5a. Dimensionless stick area size as function of the friction coefficient *f* for α^*= β^*=0.5 and some values of the Poisson's ratio *ν*.

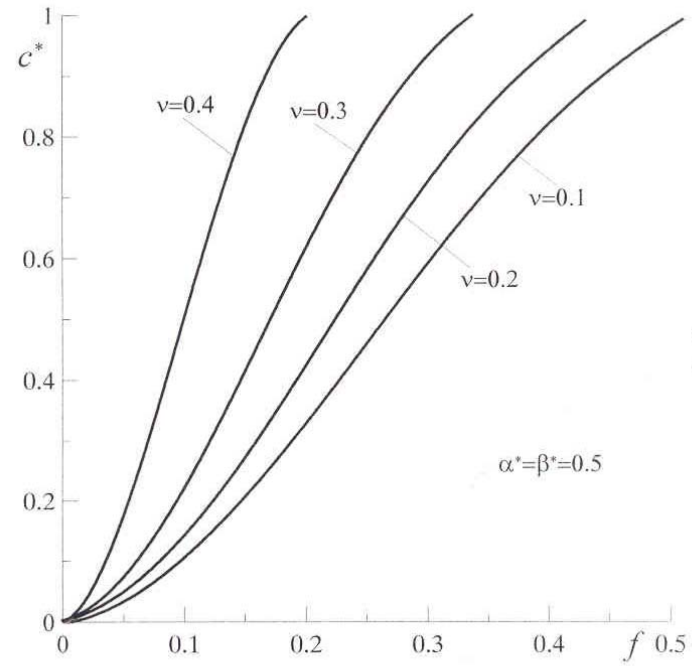
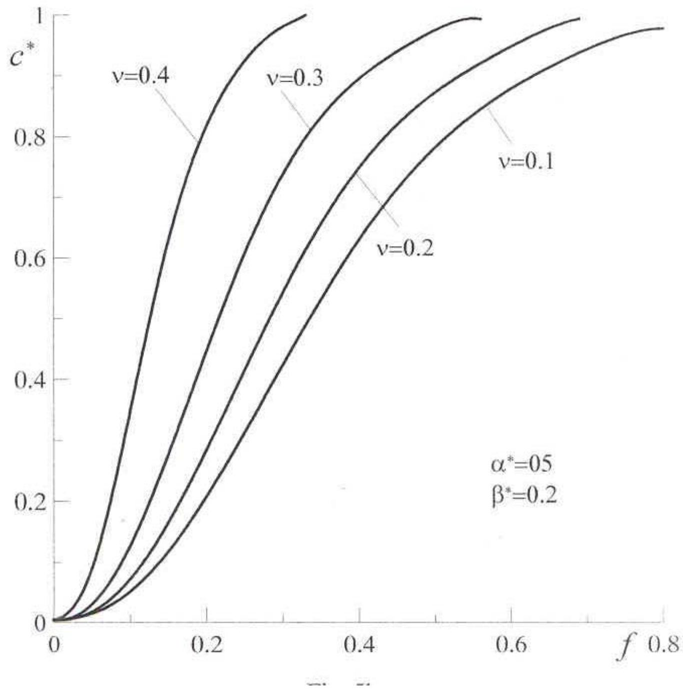


Fig. 5b. Dimensionless stick area size as function of the friction coefficient *f* for α^{*}=0.5, β^{*}=0.2 and some values of the Poisson's ratio *ν*



CONCLUSION

The plane contact of the rigid flat-ended punch and elastic half-space is studied. The simultaneous effects due to the friction under the punch and the roughness of the half-space surface are investigated. A new model of the surface roughness is developed. This model permits to solve considered contact problems by the boundary integral equation method. Derived integral equations are being solved numerically.

New phenomena due to the surface roughness are observed. In contrast to the corresponding classical contact, in the problem under consideration the contact with perfectly connected surfaces is physically possible for the finite values of the friction coefficient.

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