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ON THE MODELLING OF MEDIUM THICKNESS PLATES INTERACTING WITH A PERIODIC WINKLER'S SUBSOIL

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ABSTRACT

Plates resting on a subsoil are often met in constructions of the civil engineering. The main aim of the paper is twofold. Firstly, to propose a new averaged model of medium thickness plates resting on a periodic Winkler's 3D subsoil, based on the non-asymptotic tolerance averaging technique, cf. Woźniak and Wierzbicki [19]. The main feature of the model is that it describes the effect of period lengths on the overall behaviour of the plate. Secondly, to apply the non-asymptotic model to analyse free vibrations of a plate strip on a periodic subsoil and show that the aforementioned effect plays a crucial role in dynamic problems.

Key words: medium thickness plate, periodic Winkler's subsoil, effect of period lengths.

INTRODUCTION

Plates interacting with a Winkler's subsoil are applied as important elements of constructions in the civil engineering, e. g. as elements of building foundations. In this paper a special case of these plates is the main object of considerations, i. e. a medium thickness plate (homogeneous and anisotropic) resting on a periodic Winkler's subsoil, cf. Fig. 1. The plates of this kind can be met as constructions under roads, e. g. as concrete plates resting on a weak subsoil, which is reinforced by a system of periodically distributed vertical pillars, made of sand or gravel.

In the plates on a periodic subsoil, a small repeated element, called *a periodicity cell* Δ, may be distinguished. These structures have the properties described by highly oscillating, periodic and also non-continuous functions. An analysis of engineering problems of such plates is too complicated by using exact equations of the plate theory. Hence, different averaged models were proposed, describing certain homogeneous plates with constant homogenized properties instead of real periodic ones. The models based on the method of asymptotic homogenization for periodic solids proposed in Bensoussan et al. [3] should be mentioned. The theory of asymptotically homogenized plates with periodic structures was discussed in a series of papers, e. g. Caillerie

[5], Kohn and Vogelius [10], Matysiak and Nagórko [12], Lewiński [11]. However, in models of this kind *the effect of period lengths* on the overall dynamic plate behaviour is usually neglected. This effect may be taken into account by new non-asymptotic averaged models, based on *the tolerance averaging technique*. This method was proposed and discussed for periodic composites and structures by Woźniak and Wierzbicki [19]. Dynamic problems of different periodic structures were investigated using the tolerance averaging procedure in many papers, e. g. periodic plane structures in Wierzbicki and Woźniak [18], wavy plates in Michalak [13,14], Kirchhoff plates in Jędrysiak [7,8], thin plates with stiffeners in Nagórko and Woźniak [15], Hencky-Bolle plates in Baron [2], where it was shown that the effect of period lengths (called also *the length-scale effect*) cannot be neglected in dynamics of periodic structures.

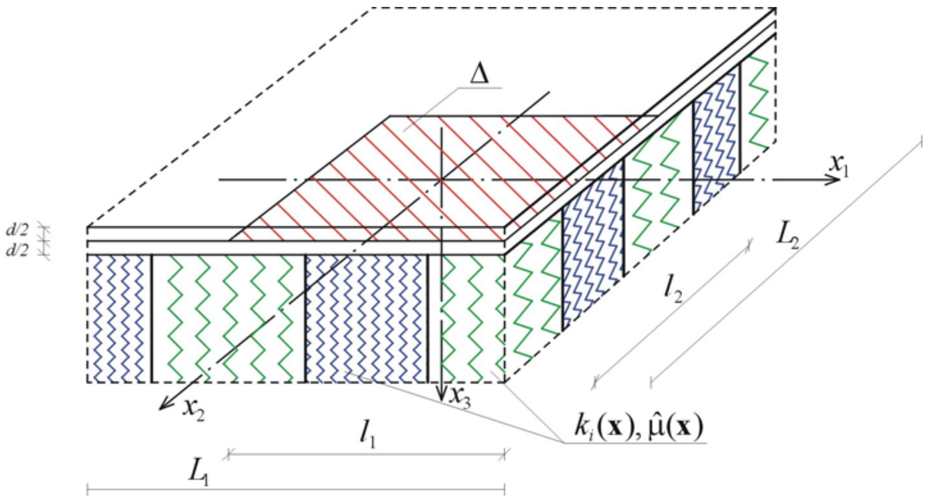
The main purpose of this contribution is to formulate a new non-asymptotic averaged model, describing the above effect on non-stationary problems of medium thickness plates interacting with a periodic three-directional Winkler's subsoil. Moreover, the proposed model is applied as a tool to analyse free vibrations of a plate strip on a periodic subsoil.

Considerations are based on the 2D-Hencky-Bolle plate theory assumptions, cf. Bolle [4], Hencky [6]. Moreover, these assumptions are extended on the effect of Winkler subsoil, cf. Ambartsumyan [1], Szcześniak [16].

GENERAL FORMULATIONS OF THE THEORY OF THE MEDIUM THICKNESS PLATE ON A WINKLER'S SUBSOIL

The orthogonal Cartesian co-ordinate system in the physical space may be denoted by $0x_1x_2x_3$ and the time co-ordinate by t. Throughout the paper subscripts α, β, …(i, j, …) run over 1, 2 (over 1, 2, 3), indices A, B,… (a, b,…) run over 1,…, N (1,…, n). Summation convention holds for all aforementioned indices. Setting $\mathbf{x}\equiv(x_1,x_2)$ and $z\equiv x_3$, it may be assumed that the undeformed plate occupies the region $\Omega:=\{(\mathbf{x},z):-d/2<z<d/2, \mathbf{x}\in\Pi\}$, where Π is the midplane with length dimensions L_1, L_2 along the x_1- and x_2-axis, respectively, and d is the plate thickness. A fragment of the plate example is shown in Fig. 1.

Fig. 1. Part of a medium thickness plate resting on a subsoil with periodic structure



A plate structure under consideration is consisted of an anisotropic and homogeneous medium thickness plate, interacting with a periodic 3D Winkler's subsoil, which rests on a rigid undeformable base, cf. Vlasov and Leontiev [17]. Hence, all material and inertial properties of the plate, e. g. a mass density ρ and elastic modulae a_{ijkl}, are constant. However, the subsoil has heterogeneous periodic structure in planes parallel to the plate midplane, i. e. along the x_1- and x_2-axis directions with periods l_1 and l_2, respectively. Hence, it may be denoted by $\Delta\equiv(-l_1/2, l_1/2)\times(-l_2/2, l_2/2)$ the periodicity basic cell on $0x_1x_2$ plane. It may be assumed that the cell size is described by the parameter $l\equiv(l_1^2+l_2^2)^{½}$, satisfying the condition $d<<l<<L_{\min}$, called *the mesostructure parameter* (where $L_{\min}$ is a minimum characteristic length dimension of the plate in its midplane). Hence, the properties of the subsoil may be described by: a mass density per an unit area $\hat{\mu}=\hat{\mu}(\mathbf{x})$ and Winkler coefficients k_i (i=1,…3) along the x_i-axis directions, which can be periodic functions in $\mathbf{x}=(x_1,x_2)$. In the subsequent considerations it may be assumed $k_3=k(\mathbf{x})$, $k_1=k_2=k_t(\mathbf{x})$. These parameters of the subsoil may be defined following Vlasov and Leontiev

[17]. It is assumed that the plate cannot be torn off from the subsoil. A simplified problem of these structures is analysed by Jędrysiak and Paś [9], where inertia terms of the subsoil are neglected.

If denoted by u_i, ε_{ij}, σ_{ij} displacements, strains, stresses, respectively; by $\bar{u}_i$, $\bar{\varepsilon}_{ij}$ virtual displacements and virtual strains; by p^+, p^- loadings (in the z-axis direction) on the bottom and upper surfaces of the plate, respectively; by q_i loadings on these surfaces along the x_i-axis directions, describing the effect of the subsoil. These loadings are defined as:

$$\begin{aligned} q_\alpha(\mathbf{x},d/2,t) &= k_t(\mathbf{x})u_\alpha(\mathbf{x},d/2,t)+\hat{\mu}(\mathbf{x})\ddot{u}_\alpha(\mathbf{x},d/2,t), \qquad \alpha=1,2 \\ q_3(\mathbf{x},d/2,t) &= k(\mathbf{x})u_3(\mathbf{x},d/2,t)+\hat{\mu}(\mathbf{x})\ddot{u}_3(\mathbf{x},d/2,t) \end{aligned}$$

It may be assumed that the horizontal planes (z=const) are planes of elastic symmetry. This means $a_{3\alpha\beta\gamma}=0$, $a_{333\gamma}=0$ and $a_{\alpha\beta\gamma\delta}$, $a_{\alpha\beta33}$, a_{3333} are non-zero terms of the elastic modulae tensor. If denoted $c_{\alpha\beta\gamma\delta}\equiv a_{\alpha\beta\gamma\delta}-a_{\alpha\beta33}a_{\gamma\delta33}(a_{3333})^{-1}$, $b_{\alpha\beta}\equiv\psi a_{\alpha3\beta3}$, where ψ is a shear coefficient.

The considerations are carried out in the framework of the 2D-Hencky-Bolle theory. Hence, the well known assumptions may be recalled:

- *the kinematic constraints*

$$u_\alpha(\mathbf{x},z,t)=z\phi_\alpha(\mathbf{x},t), \qquad u_3(\mathbf{x},z,t)=u(\mathbf{x},t) \tag{2.1}$$

where $u(\mathbf{x},t)$ are deflections of points of the midplane, $\phi_\alpha(\mathbf{x},t)$ are independent rotations; for virtual displacements similar constraints may be obtained:

$$\bar{u}_\alpha(\mathbf{x},z)=z\bar{\phi}_\alpha(\mathbf{x}), \qquad \bar{u}_3(\mathbf{x},z)=\bar{u}(\mathbf{x}) \tag{2.2}$$

- *the strain-displacement relations*

$$\varepsilon_{ij}=u_{(i,j)} \tag{2.3}$$

- *the stress-strain relations* (under the plane stress assumption s_{33}=0)

$$\sigma_{\alpha\beta}=c_{\alpha\beta\gamma\delta}\varepsilon_{\gamma\delta}, \qquad \sigma_{\alpha3}=2b_{\alpha\beta}\varepsilon_{\beta3} \tag{2.4}$$

- *the virtual work principle*

$$\begin{aligned} &\int_\Pi\int_{-d/2}^{d/2}\rho(\mathbf{x},z)\ddot{u}_j(\mathbf{x},z,t)\bar{u}_i(\mathbf{x},z)\delta_{ij}dzda+\int_\Pi\int_{-d/2}^{d/2}[\sigma_{\alpha\beta}(\mathbf{x},z,t)\bar{\varepsilon}_{\alpha\beta}(\mathbf{x},z)+2\sigma_{\alpha3}(\mathbf{x},z,t)\bar{\varepsilon}_{\alpha3}(\mathbf{x},z)]dzda= \\ &=\int_\Pi[p^-(\mathbf{x},t)\bar{u}_3(\mathbf{x},-\tfrac{1}{2}d)+p^+(\mathbf{x},t)\bar{u}_3(\mathbf{x},\tfrac{1}{2}d)]da-\int_\Pi q_j(\mathbf{x},\tfrac{1}{2}d,t)\bar{u}_i(\mathbf{x},\tfrac{1}{2}d)\delta_{ij}da \end{aligned} \tag{2.5}$$

with $da=dx_1dx_2$, which has to hold for arbitrary virtual displacements $\bar{u}_i$ defined as $\bar{u}_\alpha=z\bar{\phi}_\alpha$, $\bar{u}_3=\bar{u}$, where $\bar{\phi}_\alpha$, $\bar{u}$ are sufficiently regular, independent functions.

Equations (2.1)-(2.5) are the basis for the derivation of the well known differential equations for generalized displacements ϕ_α, u of the medium thickness plate. For periodic structures under consideration obtained governing equations have highly oscillating, periodic, functional and, in general, non-continuous coefficients. Because the direct application of these equations to special problems is difficult, the equations are approximated by the ones with constant coefficients. However, employing the asymptotic homogenization methods, usually model equations may be obtained, which neglects the effect of the period lengths on the overall dynamic behaviour of medium thickness plates on a periodic subsoil. To take this effect into account, the tolerance averaging technique will be applied.

At the end of this section quantities, averaged over thickness:

$$\mu\equiv\int_{-d/2}^{d/2}\rho dz, \quad \vartheta\equiv\int_{-d/2}^{d/2}\rho z^2dz, \quad d_{\alpha\beta\gamma\delta}\equiv\int_{-d/2}^{d/2}c_{\alpha\beta\gamma\delta}z^2dz, \quad f_{\alpha\beta}\equiv\int_{-d/2}^{d/2}\psi b_{\alpha\beta}dz$$

describing plate properties as a mass density per an unit area, a rotational inertia and bending stiffnesses, respectively; which will be employed in the subsequent considerations, may be introduced.

TOLERANCE AVERAGING APPROACH

Introductory concepts.
The governing equations with constant coefficients, which describe the effect of period lengths, will be derived by using *the tolerance averaging method* (proposed by Woźniak and Wierzbicki [19] for periodic composites and structures) to the equations of motion of medium thickness plates on a periodic Winkler's subsoil. In the modelling procedure additional concepts introduced in this monography, i. e. *an averaging operator*, *a tolerance system*, *a slowly varying function*, *a periodic-like function*, *an oscillating function*, will be used. Following the aforesaid book some of them will be recalled.

Define by $\Delta(\mathbf{x})\equiv\mathbf{x}+\Delta$ a periodicity cell at $\mathbf{x}\in\Pi_\Delta$, $\Pi_\Delta=\{\mathbf{x}: \mathbf{x}\in\Pi,\ \Delta(\mathbf{x})\subset\Pi\}$. The *averaging operator* for periodic structures under consideration is given by

$$<\varphi>=<\varphi>(\mathbf{x})\equiv(l_1 l_2)^{-1}\int_{\Delta(\mathbf{x})}\varphi(\mathbf{y})d\mathbf{y},\quad \mathbf{x}\in\Pi_\Delta,\ \mathbf{y}\in\Delta(\mathbf{x}) \tag{3.1}$$

for an arbitrary integrable function φ defined on the midplane Π. If φ is a periodic function its averaged value calculated from (3.1) is constant.

The functions φ, Ψ defined on the midplane Π and a periodic function f may be introduced. If approximation

$$<f\Psi>(\mathbf{x})\cong<f>\Psi(\mathbf{x}),\qquad \mathbf{x}\in\Pi_\Delta \tag{3.2}$$

holds for every f (with the required accuracy, dependent on f) then Ψ is called *a slowly varying function*. If for every $\mathbf{x}\in\Pi_\Delta$ there exists a periodic function $\varphi_\mathbf{x}$ such that approximation

$$<f\varphi>(\mathbf{x})\cong<f\varphi_\mathbf{x}>(\mathbf{x}),\qquad \mathbf{x}\in\Pi_\Delta \tag{3.3}$$

holds as above, then φ is referred to as *a periodic-like function* and $\varphi_\mathbf{x}$ is termed a periodic approximation of φ at $\mathbf{x}$.

The aforementioned formulae are known as the tolerance averaging approximations, cf. Woźniak and Wierzbicki [19], where it was shown that these formulae are dependent on Δ and a certain tolerance system T. It is possible to write $\Psi\in SV(T)$ for a function Ψ, which is slowly varying together with its derivatives and $\varphi\in PL(T)$ for a periodic-like function φ.

A periodic-like function φ with the condition $<\mu\varphi>(\mathbf{x})\cong 0$ for every $\mathbf{x}\in\Pi_\Delta$, where $\mu(\cdot)$ is a positive value periodic function, is called *an oscillating function*. The set of oscillating functions with the weight μ is denoted by $PL^\mu(T)$. For constant values functions μ the condition takes the form $\mu<\varphi>(\mathbf{x})=<\varphi>(\mathbf{x})\cong 0$ for every $\mathbf{x}\in\Pi_\Delta$.

In the modelling procedure the above concepts, defined by Woźniak and Wierzbicki [19], and lemmas and assertions, formulated and proved in this work, are used.

Modelling assumptions.
In the tolerance averaging technique the additional assumption called *the Conformability Assumption* (*CA*), cf. Woźniak and Wierzbicki [19], is formulated. It states that the generalized displacements – the rotations ϕ_α and the deflection u – of the medium thickness plate on a periodic subsoil have to conform to this periodic structure, i. e. they can be represented by periodic-like functions:

$$\phi_\alpha(\cdot,t),\ u(\cdot,t)\in PL(T)$$

Similarly, the virtual displacements $\bar{\phi}_\alpha, \bar{u}$ are periodic-like functions, i. e.:

$$\bar{\phi}_\alpha(\cdot), \bar{u}(\cdot)\in PL(T)$$

These conditions may be violated only near the plate boundary.

Outline of the modelling procedure.
The modelling procedure of the tolerance averaging can be divided into three steps.

1) The generalized displacements of the plate (the rotations ϕ_α and the deflection u) can be decomposed in the form

$$\phi_\alpha(\cdot,t) = \varphi_\alpha(\cdot,t) + \varphi^*_\alpha(\cdot,t), \qquad u(\cdot,t) = w(\cdot,t) + w^*(\cdot,t) \tag{3.4}$$

where φ_α is the averaged part of the rotations defined by $\varphi_\alpha(\cdot,t) \equiv <\phi_\alpha>(\cdot,t)$; w is the averaged part of the deflection defined by $w(\cdot,t) \equiv <u>(\cdot,t)$; $\varphi^*_\alpha(\cdot,t) \in PL^1(T)$ are *fluctuations of the rotations* and hold the condition $<\varphi^*_\alpha(\cdot,t)> = 0$; $w^*(\cdot,t) \in PL^1(T)$ is *a fluctuation of the deflection* and satisfies the condition $<w^*(\cdot,t)>=0$. Because $\phi_\alpha(\cdot,t)$, $u(\cdot,t) \in PL(T)$ it is clear that $\varphi_\alpha(\cdot,t)$, $w(\cdot,t) \in SV(T)$. Hence, the functions $\varphi_\alpha(\cdot,t)$ and $w(\cdot,t)$ are called *the macrorotations* and *the macrodeflection*, respectively. Moreover, it may be assumed that the virtual displacements of the plate $\overline{\phi}_\alpha, \overline{u}$ can be decomposed in the form similar to (3.4), i. e.:

$$\overline{\phi}_\alpha(\cdot) = \overline{\varphi}_\alpha(\cdot) + \overline{\varphi}^*_\alpha(\cdot), \qquad \overline{u}(\cdot) = \overline{w}(\cdot) + \overline{w}^*(\cdot) \tag{3.5}$$

in which components satisfy similar conditions to those for components of the generalized displacements (3.4).
2) It is assumed that the fluctuations of the displacements in (3.4) and (3.5) can be approximated by the truncated series (cf. Woźniak and Wierzbicki, [19]) in the following form, respectively:

$$\varphi^*_\alpha(\mathbf{x},t) = h^a(\mathbf{x})\Phi^a_\alpha(\mathbf{x},t), \qquad w^*(\mathbf{x},t) = g^A(\mathbf{x})V^A(\mathbf{x},t) \tag{3.6}$$

and

$$\overline{\varphi}^*_\alpha(\mathbf{x}) = h^a(\mathbf{x})\overline{\Phi}^a_\alpha(\mathbf{x}), \qquad \overline{w}^*(\mathbf{x}) = g^A(\mathbf{x})\overline{V}^A(\mathbf{x}) \tag{3.7}$$

where $a=1,\dots,n$, $A=1,\dots,N$. Functions g^A, h^a are known ones called *mode-shape functions*, obtained from periodic problems for the periodicity cell. These functions g^A or h^a stand the system of N or n linear-independent periodic functions, such that $<g^A>=0$ and $g^A(\cdot), lg^A_{,\alpha}(\cdot) \in O(l)$ or $<h^a>=0$ and $h^a(\cdot), lh^a_{,\alpha}(\cdot) \in O(l)$. These functions approximate the expected form of the oscillating part of free vibration modes of the periodic structure of the plate, cf. Woźniak and Wierzbicki (2000). However, slowly varying functions $\Phi^a_\alpha(\cdot,t), V^A(\cdot,t) \in SV(T)$ are new kinematic unknowns and $\overline{\Phi}^a_\alpha(\cdot), \overline{V}^A(\cdot) \in SV(T)$ are new virtual displacements. The new unknowns $\Phi^a_\alpha(\cdot,t)$ and $V^A(\cdot,t)$ will be called *the fluctuation variables* for the rotations and for the deflection, respectively.

3) Substituting (3.4) and (3.5) into (2.1)-(2.5), using (3.6) and (3.7) and also the tolerance averaging approximations (3.2) and (3.3), after some manipulations the equations for the macrodeflection w, the macrorotations φ_α and fluctuation variables Φ^a_α and V^A may be obtained. These equations are presented in the subsequent section.

GOVERNING EQUATIONS

Using the procedure of the tolerance averaging and denoting

$$\begin{gathered}
B_{\alpha\beta\gamma\delta} \equiv \tfrac{d^3}{12}\tfrac{E}{1-\nu^2}[(1-\nu)\delta_{\alpha\gamma}\delta_{\beta\delta} + \nu\delta_{\alpha\beta}\delta_{\gamma\delta}], \quad D_{\alpha\beta} \equiv \psi d \tfrac{E}{1+\nu}\delta_{\alpha\beta}, \quad \mu \equiv \rho d, \quad \vartheta \equiv \rho\tfrac{d^3}{12} \\
P \equiv <p>, \quad P^A \equiv l^{-1} <pg^A> \\
G^{AB} \equiv l^{-2} <g^A g^B>, \quad G^{AB}_{\alpha\beta} \equiv <g^A_{,\alpha} g^B_{,\beta}> \\
H^{ab} \equiv l^{-2} <h^a h^b>, \quad H^{ab}_{\alpha\beta} \equiv <h^a_{,\alpha} h^b_{,\beta}>, \quad F^{Aa}_\alpha \equiv <g^A_{,\alpha} h^a> \\
m \equiv <\mu>, \quad m^{AB} \equiv l^{-2} <\mu g^A g^B>, \quad \tilde{m}^a \equiv l^{-1} <\mu h^a>, \quad \tilde{m}^{ab} \equiv l^{-2} <\mu h^a h^b> \\
K \equiv <k>, \quad K^A \equiv l^{-1} <kg^A>, \quad K^{AB} \equiv l^{-2} <kg^A g^B> \\
K_t \equiv <k_t>, \quad K_t^{\ a} \equiv l^{-1} <k_t h^a>, \quad K_t^{\ ab} \equiv l^{-2} <k_t h^a h^b>
\end{gathered} \tag{4.1}$$

the governing equations of the tolerance model of medium thickness plates resting on a periodic subsoil are derived:

$$
\begin{aligned}
&B_{\alpha\beta\gamma\delta}\varphi_{\gamma,\beta\delta}-D_{\alpha\beta}(\varphi_{\beta}+w_{,\beta})-\vartheta\ddot{\varphi}_{\alpha}-\tfrac{1}{4}d^{2}(K_{t}\varphi_{\alpha}+m\ddot{\varphi}_{\alpha}+lK_{t}{}^{a}\Phi^{a}_{\alpha}+l\widetilde{m}^{a}\ddot{\Phi}^{a}_{\alpha})=0\\
&D_{\alpha\beta}(\varphi_{\beta,\alpha}+w_{,\alpha\beta})-Kw-(\mu+m)\ddot{w}-lK^{A}V^{A}-lm^{A}\ddot{V}^{A}=-P\\
&l^{2}\vartheta H^{ab}\ddot{\Phi}^{b}_{\alpha}+B_{\alpha\beta\gamma\delta}H^{ab}_{\beta\delta}\Phi^{b}_{\gamma}+l^{2}D_{\alpha\beta}H^{ab}\Phi^{b}_{\beta}+lD_{\alpha\beta}F^{Aa}_{\beta}V^{A}+\\
&+\tfrac{1}{4}d^{2}(lK_{t}{}^{a}\varphi_{\alpha}+l\widetilde{m}^{a}\ddot{\varphi}_{\alpha}+l^{2}K_{t}{}^{ab}\Phi^{b}_{\alpha}+l^{2}\widetilde{m}^{ab}\ddot{\Phi}^{b}_{\alpha})=0\\
&lK^{A}w+lm^{A}\ddot{w}+lD_{\alpha\beta}F^{Aa}_{\alpha}\Phi^{a}_{\beta}+(D_{\alpha\beta}G^{AB}_{\alpha\beta}+l^{2}K^{AB})V^{B}+l^{2}(\mu G^{AB}+m^{AB})\ddot{V}^{B}=lP^{A}
\end{aligned}
\tag{4.2}
$$

where w, φ_α, V^A, Φ^a_α are basic unknowns, being slowly varying functions.

Summarizing, *the tolerance model* is defined by:

1° Equations (4.2) for N+1 and 2(n+1) unknowns, w, V^A, A=1,…, N, and $\varphi_\alpha, \Phi^a_\alpha$;

2° Applicability conditions of the model, i. e. equations (4.2) have physical sense for the unknowns $w(\cdot,t)$, $V^A(\cdot,t)$ and $\varphi_\alpha(\cdot,t), \Phi^a_\alpha(\cdot,t)$, being slowly varying functions for every t;

3° The model equations (4.2) describe the effect of period lengths by terms involving the mesostructure parameter l;

4° The plate deflection may be approximated by means of the formula

$$u(\mathbf{x},t)\approx w(\mathbf{x},t)+g^{A}(\mathbf{x})V^{A}(\mathbf{x},t)$$

and the plate rotations by

$$\phi_\alpha(\mathbf{x},t)\approx \varphi_\alpha(\mathbf{x},t)+h^{a}(\mathbf{x})\Phi^{a}_{\alpha}(\mathbf{x},t)$$

where the approximation „≈" is related to the assumption that the fluctuations of the deflection and the rotations are defined in the form of the truncated series $g^A(\cdot)V^A(\cdot,t)$, A=1,…, N, and $h^a(\cdot)\Phi^a_\alpha(\cdot,t)$, a=1,…, n.

It may be noticed that to obtain the above equations, one must previously derive the mode-shape functions g^A, A=1,…,N, and h^a, a=1,…, n, for every periodic plate under consideration as solutions to certain periodic problems for the periodicity cell. However, the investigations are usually restricted to approximate forms of these solutions and to small numbers N, n of mode shapes, what is sufficient from the computational point of view, cf. Jędrysiak [7]. In the subsequent sections N=n=1 is assumed, i. e. only one mode-shape function $g\equiv g^1$ and $h\equiv h^1$.

In order to evaluate obtained results *the homogenized model of medium thickness plates resting on a periodic subsoil* may be introduced. It is governed by equations in the form:

$$
\begin{aligned}
&B_{\alpha\beta\gamma\delta}\varphi_{\gamma,\beta\delta}-D_{\alpha\beta}(\varphi_{\beta}+w_{,\beta})-\vartheta\ddot{\varphi}_{\alpha}-\tfrac{1}{4}d^{2}(K_{t}\varphi_{\alpha}+m\ddot{\varphi}_{\alpha})=0\\
&D_{\alpha\beta}(\varphi_{\beta,\alpha}+w_{,\alpha\beta})-Kw-(\mu+m)\ddot{w}=-P
\end{aligned}
\tag{4.3}
$$

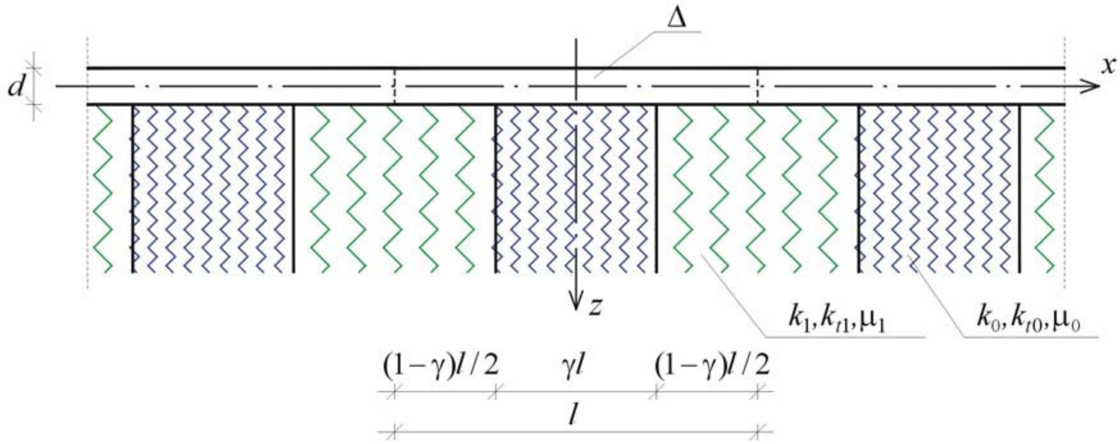
where the effect of period lengths is not taken into account. Coefficients in the above equations are defined by formulae (4.1). Equations (4.3) are obtained from equations (4.2) after neglecting terms with the mesostructure parameter l.

AN APPLICATION TO ANALYSE FREE VIBRATIONS OF A PLATE STRIP

Free vibration frequencies within the tolerance model.

As an example of the application of the proposed model, free vibrations of a plate strip, with the span L along the x_1-axis, resting on a periodic subsoil will be analysed, cf. Fig. 2. Hence, loadings, p=0 were neglected. It may be denoted $x\equiv x_1$ and assumed that the plate strip is simply supported on the opposite edges x=0, L and is made of an isotropic material with constant properties - the Young's modulus E, the Poisson's ratio ν, the mass density ρ.

Fig. 2. Part of a plate strip on a subsoil with periodic structure



However, the subsoil is made of periodic, piece-wise constant material. Hence, the Winkler coefficients k, k_t are given by

$$k(x), k_t(x) = \begin{cases} k_0, k_{t0} & \text{if} \quad x \in [-\frac{1}{2}\gamma l, \frac{1}{2}\gamma l], \\ k_1, k_{t1} & \text{if} \quad x \in [-\frac{1}{2}l, -\frac{1}{2}\gamma l) \cup (\frac{1}{2}\gamma l, \frac{1}{2}l] \end{cases} \tag{5.1}$$

and the mass density of the subsoil is assumed as

$$\hat{\mu}(x) = \begin{cases} \mu_0 & \text{if} \quad x \in [-\frac{1}{2}\gamma l, \frac{1}{2}\gamma l], \\ \mu_1 & \text{if} \quad x \in [-\frac{1}{2}l, -\frac{1}{2}\gamma l) \cup (\frac{1}{2}\gamma l, \frac{1}{2}l] \end{cases} \tag{5.2}$$

where $\gamma \in [0,1]$, cf. Fig. 2. For the considered periodicity cell, the mode-shape functions are assumed in the form

$$g(x) = h(x) = l\cos(2\pi x/l) \tag{5.3}$$

It mat be denoted $B_1 \equiv B_{1111}, B_2 \equiv B_{2121}, D \equiv D_{11} = D_{22}, H \equiv H^{11}, H_1 \equiv H_{11}^{11}, G \equiv G^{11}, G_1 \equiv G_{11}^{11}$ and $V \equiv V^1$, $\Phi_\alpha \equiv \Phi_\alpha^1$. It can be shown that for the mode-shape functions (5.3) $H = G, H_1 \equiv G_1$, $m^1 = \tilde{m}^1, m^{11} = \tilde{m}^{11}$ have been obtained. Hence, the equations of free vibrations in the framework of the tolerance model (4.2) take the form of the system of four differential equations for w, V, φ_1, Φ_1:

$$\begin{aligned} &(\mu + m)\ddot{w} + Kw - D(w_{,11} + \varphi_{1,1}) + l(K^1 V + m^1 \ddot{V}) = 0 \\ &w_{,1} + (D + \tfrac{1}{4}d^2 K_t)\varphi_1 - B_1 \varphi_{1,11} + (\vartheta + \tfrac{1}{4}d^2 m)\ddot{\varphi}_1 + \tfrac{1}{4}d^2 l[(K_t^1 \Phi_1 + m^1 \ddot{\Phi}_1)] = 0 \\ &l(m^1 \ddot{w} + K^1 w) + (DH_1 + l^2 K^{11})V + l^2(\mu H + m^{11})\ddot{V} = 0 \\ &\tfrac{1}{4}d^2 l(m^1 \ddot{\varphi}_1 + K_t^1 \varphi_1) + l^2(\vartheta H + \tfrac{1}{4}d^2 m^{11})\ddot{\Phi}_1 + [l^2(DH + \tfrac{1}{4}d^2 K_t^{11}) + B_1 H_1]\Phi_1 = 0 \end{aligned} \tag{5.4}$$

and the independent system of two differential equations for φ_2, Φ_2:

$$\begin{aligned} &(D + \tfrac{1}{4}d^2 K_t)\varphi_2 - B_2 \varphi_{2,11} + (\vartheta + \tfrac{1}{4}d^2 m)\ddot{\varphi}_2 + \tfrac{1}{4}d^2 l(K_t^1 \Phi_2 + m^1 \ddot{\Phi}_2) = 0 \\ &\tfrac{1}{4}d^2 l(m^1 \ddot{\varphi}_2 + K_t^1 \varphi_2) + l^2(\vartheta H + \tfrac{1}{4}d^2 m^{11})\ddot{\Phi}_2 + [l^2(DH + \tfrac{1}{4}d^2 K_t^{11}) + B_2 H_1]\Phi_2 = 0 \end{aligned} \tag{5.5}$$

Assume solutions to equations (5.4) and (5.5) in the form satisfying boundary conditions of the simply supported plate strip:

$$\begin{aligned} &w(x,t) = A_0 \sin\alpha x \cos\omega t, && V(x,t) = A_1 \sin\alpha x \cos\omega t \\ &\varphi_1(x,t) = B_0 \cos\alpha x \cos\omega t, && \Phi_1(x,t) = B_1 \cos\alpha x \cos\omega t \\ &\varphi_2(x,t) = C_0 \cos\alpha x \cos\omega t, && \Phi_2(x,t) = C_1 \cos\alpha x \cos\omega t \end{aligned} \tag{5.6}$$

where α is a wave number, $A_0, A_1, B_0, B_1, C_0, C_1$ are amplitudes.

Now, the solutions $(5.6)_{1,2,3,4}$ may be substituted for (5.4) and denoted:

$$\tilde{a}_2 \equiv [(\tfrac{1}{4}d^2m+\vartheta)(\tfrac{1}{4}d^2m^{11}+\vartheta H)-(\tfrac{1}{4}d^2m^1)^2][(\mu+m)(\mu H+m^{11})-(m^1)^2]$$

$$\begin{aligned}
\tilde{b}_1 \equiv\ & (\mu+m)DH_1[(\tfrac{1}{4}d^2m+\vartheta)(\tfrac{1}{4}d^2m^{11}+\vartheta H)-(\tfrac{1}{4}d^2m^1)^2]+\\
& +(\tfrac{1}{4}d^2m+\vartheta)B_1H_1[(\mu+m)(\mu H+m^{11})-(m^1)^2]
\end{aligned}$$

$$\begin{aligned}
\tilde{b}_2 \equiv\ & [(\tfrac{1}{4}d^2m+\vartheta)(\tfrac{1}{4}d^2m^{11}+\vartheta H)-(\tfrac{1}{4}d^2m^1)^2][(\mu+m)K^{11}+(D\alpha^2+K)(\mu H+m^{11})-2K^1m^1]+\\
& +[(\mu+m)(\mu H+m^{11})-(m^1)^2][(\tfrac{1}{4}d^2m+\vartheta)(DH+\tfrac{1}{4}d^2K_t^{11})+\\
& \qquad +(B_1\alpha^2+D+\tfrac{1}{4}d^2K_t)(\tfrac{1}{4}d^2m^{11}+\vartheta H)-\tfrac{1}{8}d^4K_t^1m^1]
\end{aligned}$$

$$\tilde{c}_0 \equiv (\mu+m)B_1D(H_1)^2(\tfrac{1}{4}d^2m+\vartheta)$$

$$\begin{aligned}
\tilde{c}_1 \equiv\ & (D\alpha^2+K)DH_1[(\tfrac{1}{4}d^2m+\vartheta)(\tfrac{1}{4}d^2m^{11}+\vartheta H)-(\tfrac{1}{4}d^2m^1)^2]+\\
& +B_1H_1(B_1\alpha^2+D+\tfrac{1}{4}d^2K_t)[(\mu+m)(\mu H+m^{11})-(m^1)^2]+\\
& +(\mu+m)DH_1[(\tfrac{1}{4}d^2m+\vartheta)(DH+\tfrac{1}{4}d^2K_t^{11})+\\
& \qquad +(B_1\alpha^2+D+\tfrac{1}{4}d^2K_t)(\tfrac{1}{4}d^2m^{11}+\vartheta H)-\tfrac{1}{8}d^4K_t^1m^1]+\\
& +(\tfrac{1}{4}d^2m+\vartheta)B_1H_1[(\mu+m)K^{11}+(D\alpha^2+K)(\mu H+m^{11})-2K^1m^1]
\end{aligned}$$

$$\begin{aligned}
\tilde{c}_2 \equiv\ & [(\tfrac{1}{4}d^2m+\vartheta)(\tfrac{1}{4}d^2m^{11}+\vartheta H)-(\tfrac{1}{4}d^2m^1)^2][(D\alpha^2+K)K^{11}-(K^1)^2]+\\
& +[(\mu+m)(\mu H+m^{11})-(m^1)^2][(B_1\alpha^2+D+\tfrac{1}{4}d^2K_t)(DH+\tfrac{1}{4}d^2K_t^{11})-(\tfrac{1}{4}d^2K_t^1)^2]+\\
& +[(\mu+m)K^{11}+(D\alpha^2+K)(\mu H+m^{11})-2K^1m^1][(\tfrac{1}{4}d^2m+\vartheta)(DH+\tfrac{1}{4}d^2K_t^{11})+\\
& \qquad +(B_1\alpha^2+D+\tfrac{1}{4}d^2K_t)(\tfrac{1}{4}d^2m^{11}+\vartheta H)-\tfrac{1}{8}d^4K_t^1m^1]-\\
& -(\mu H+m^{11})(\tfrac{1}{4}d^2m^{11}+\vartheta H)(D\alpha)^2
\end{aligned}$$

$$\tilde{d}_0 \equiv B_1D(H_1)^2[(\mu+m)(B_1\alpha^2+D+\tfrac{1}{4}d^2K_t)+(\tfrac{1}{4}d^2m+\vartheta)(D\alpha^2+K)]$$

$$\begin{aligned}
\tilde{d}_1 \equiv\ & (\mu+m)DH_1[(B_1\alpha^2+D+\tfrac{1}{4}d^2K_t)(DH+\tfrac{1}{4}d^2K_t^{11})-(\tfrac{1}{4}d^2K_t^1)^2]+\\
& +(\tfrac{1}{4}d^2m+\vartheta)B_1H_1[(D\alpha^2+K)K^{11}-(K^1)^2]+\\
& +(D\alpha^2+K)DH_1[(\tfrac{1}{4}d^2m+\vartheta)(DH+\tfrac{1}{4}d^2K_t^{11})+\\
& \qquad +(B_1\alpha^2+D+\tfrac{1}{4}d^2K_t)(\tfrac{1}{4}d^2m^{11}+\vartheta H)-\tfrac{1}{8}d^4K_t^1m^1]+\\
& +(B_1\alpha^2+D+\tfrac{1}{4}d^2K_t)B_1H_1[(\mu+m)K^{11}+(D\alpha^2+K)(\mu H+m^{11})-2K^1m^1]-\\
& -(D\alpha)^2[DH_1(\tfrac{1}{4}d^2m^{11}+\vartheta H)+B_1H_1(\mu H+m^{11})]
\end{aligned}$$

$$\begin{aligned}
\tilde{d}_2 \equiv\ & [(D\alpha^2+K)K^{11}-(K^1)^2][(\tfrac{1}{4}d^2m+\vartheta)(DH+\tfrac{1}{4}d^2K_t^{11})+\\
& \qquad +(B_1\alpha^2+D+\tfrac{1}{4}d^2K_t)(\tfrac{1}{4}d^2m^{11}+\vartheta H)-\tfrac{1}{8}d^4K_t^1m^1]+\\
& +[(B_1\alpha^2+D+\tfrac{1}{4}d^2K_t)(DH+\tfrac{1}{4}d^2K_t^{11})-(\tfrac{1}{4}d^2K_t^1)^2][(\mu+m)K^{11}+\\
& \qquad +(D\alpha^2+K)(\mu H+m^{11})-2K^1m^1]-\\
& -(D\alpha)^2[(\mu H+m^{11})(DH+\tfrac{1}{4}d^2K_t^{11})+(\tfrac{1}{4}d^2m^{11}+\vartheta H)K^{11}]
\end{aligned}$$

$$\tilde{e}_0 \equiv B_1D(H_1)^2[(D\alpha^2+K)(B_1\alpha^2+D+\tfrac{1}{4}d^2K_t)-(D\alpha)^2]$$

$$\begin{aligned}
\tilde{e}_1 \equiv\ & (D\alpha^2+K)DH_1[(B_1\alpha^2+D+\tfrac{1}{4}d^2K_t)(DH+\tfrac{1}{4}d^2K_t^{11})-(\tfrac{1}{4}d^2K_t^1)^2]+\\
& +(B_1\alpha^2+D+\tfrac{1}{4}d^2K_t)B_1H_1[(D\alpha^2+K)K^{11}-(K^1)^2]-\\
& -(D\alpha)^2[DH_1(DH+\tfrac{1}{4}d^2K_t^{11})+B_1H_1K^{11}]
\end{aligned}$$

$$\begin{aligned}
\tilde{e}_2 \equiv\ & [(B_1\alpha^2+D+\tfrac{1}{4}d^2K_t)(DH+\tfrac{1}{4}d^2K_t^{11})-(\tfrac{1}{4}d^2K_t^1)^2][(D\alpha^2+K)K^{11}-(K^1)^2]-\\
& -(D\alpha)^2(DH+\tfrac{1}{4}d^2K_t^{11})K^{11}
\end{aligned}$$

After some calculations that were obtained at the characteristic equation within *the tolerance model* for vibrations of w, V, φ_1, Φ_1 was arrived at:

$$\tilde{a}_2l^4\omega^8-(\tilde{b}_2l^2+\tilde{b}_1)l^2\omega^6+(\tilde{c}_2l^4+\tilde{c}_1l^2+\tilde{c}_0)\omega^4-(\tilde{d}_2l^4+\tilde{d}_1l^2+\tilde{d}_0)\omega^2+\tilde{e}_2l^4+\tilde{e}_1l^2+\tilde{e}_0=0 \quad (5.7)$$

Denoting

$$\tilde{a}\equiv\tilde{a}_2l^4,\quad \tilde{b}\equiv(\tilde{b}_2l^2+\tilde{b}_1)l^2,\quad \tilde{c}\equiv\tilde{c}_2l^4+\tilde{c}_1l^2+\tilde{c}_0,\quad \tilde{d}\equiv\tilde{d}_2l^4+\tilde{d}_1l^2+\tilde{d}_0,\quad \tilde{e}\equiv\tilde{e}_2l^4+\tilde{e}_1l^2+\tilde{e}_0$$

and also

$$\tilde{\Delta} \equiv 12\tilde{a}\tilde{e} - 3\tilde{b}\tilde{d} + \tilde{c}^2, \qquad \tilde{\Gamma} \equiv 9[3\tilde{a}\tilde{d}^2 + 3\tilde{b}^2\tilde{e} - (\tilde{b}\tilde{d} + 8\tilde{a}\tilde{e})\tilde{c}] + 2\tilde{c}^3$$

$$\tilde{\Phi} \equiv \frac{\tilde{b}^2}{4\tilde{a}^2} - \frac{2\tilde{c}}{3\tilde{a}}, \qquad \tilde{\Psi} \equiv 8\tilde{d} - \frac{4\tilde{b}\tilde{c}}{\tilde{a}} + \frac{\tilde{b}^3}{\tilde{a}^2}$$

$$\tilde{\Lambda} \equiv \sqrt[3]{\tilde{\Gamma} + \sqrt{\tilde{\Gamma}^2 - 4\tilde{\Delta}^3}}, \qquad \tilde{\Xi} \equiv \tilde{\Phi} + \frac{\sqrt[3]{2}}{3\tilde{a}}\frac{\tilde{\Delta}}{\tilde{\Lambda}} + \frac{1}{3\sqrt[3]{2}\tilde{a}}\tilde{\Lambda}$$

from equation (5.7) the following formulae of free vibration frequencies are derived:

$$\begin{aligned}
\omega_{-1}^{\;2} &= \frac{\tilde{b}}{4\tilde{a}} - \frac{1}{2}\sqrt{\tilde{\Xi}} - \frac{1}{2}\sqrt{3\tilde{\Phi} - \tilde{\Xi} - \frac{\tilde{\Psi}}{4\tilde{a}\sqrt{\tilde{\Xi}}}}, \qquad & \omega_{+1}^{\;2} &= \frac{\tilde{b}}{4\tilde{a}} - \frac{1}{2}\sqrt{\tilde{\Xi}} + \frac{1}{2}\sqrt{3\tilde{\Phi} - \tilde{\Xi} - \frac{\tilde{\Psi}}{4\tilde{a}\sqrt{\tilde{\Xi}}}} \\
\omega_{-2}^{\;2} &= \frac{\tilde{b}}{4\tilde{a}} + \frac{1}{2}\sqrt{\tilde{\Xi}} - \frac{1}{2}\sqrt{3\tilde{\Phi} - \tilde{\Xi} + \frac{\tilde{\Psi}}{4\tilde{a}\sqrt{\tilde{\Xi}}}}, \qquad & \omega_{+2}^{\;2} &= \frac{\tilde{b}}{4\tilde{a}} + \frac{1}{2}\sqrt{\tilde{\Xi}} + \frac{1}{2}\sqrt{3\tilde{\Phi} - \tilde{\Xi} + \frac{\tilde{\Psi}}{4\tilde{a}\sqrt{\tilde{\Xi}}}}
\end{aligned} \tag{5.8}$$

where ω_{-1}, ω_{-2} are the lower free vibration frequencies for w, φ_1; ω_{+1}, ω_{+2} are the higher free vibration frequencies for V, Φ_1. The higher frequencies are related to a periodic structure of the plate strip on a periodic subsoil.

Substituting $(5.6)_{5,6}$ for (5.5) and denoting:

$$\begin{aligned}
\bar{c}_1 &\equiv (\tfrac{1}{4}d^2m + \vartheta)(\tfrac{1}{4}d^2m^{11} + \vartheta H) - (\tfrac{1}{4}d^2m^1)^2 \\
\bar{d}_0 &\equiv (\tfrac{1}{4}d^2m + \vartheta)B_2H_1 \\
\bar{d}_1 &\equiv (\tfrac{1}{4}d^2m + \vartheta)(DH + \tfrac{1}{4}d^2K_t^{\;11}) + (B_2\alpha^2 + D + \tfrac{1}{4}d^2K_t)(\tfrac{1}{4}d^2m^{11} + \vartheta H) - \tfrac{1}{8}d^4K_t^{\;1}m^1 \\
\bar{e}_0 &\equiv B_2H_1(B_2\alpha^2 + D + \tfrac{1}{4}d^2K_t) \\
\bar{e}_1 &\equiv (B_2\alpha^2 + D + \tfrac{1}{4}d^2K_t)(DH + \tfrac{1}{4}d^2K_t^{\;11}) - (\tfrac{1}{4}d^2K_t^{\;1})^2
\end{aligned}$$

the characteristic equation within *the tolerance model* for vibrations of φ_2, Φ_2 has got the form

$$\bar{c}_1l^2\omega^4 - (\bar{d}_1l^2 + \bar{d}_0)\omega^2 + \bar{e}_1l^2 + \bar{e}_0 = 0 \tag{5.9}$$

Denoting

$$\bar{c} \equiv \bar{c}_1l^2, \quad \bar{d} \equiv \bar{d}_1l^2 + \bar{d}_0, \quad \bar{e} \equiv \bar{e}_1l^2 + \bar{e}_0$$

from equation (5.9) the following formulae of free vibration frequencies are derived:

$$\omega_{-}^{\;2} = \frac{\bar{d} - \sqrt{\bar{d}^2 - 4\bar{c}\bar{e}}}{2\bar{c}}, \qquad \omega_{+}^{\;2} = \frac{\bar{d} + \sqrt{\bar{d}^2 - 4\bar{c}\bar{e}}}{2\bar{c}}$$

where ω_- is the lower free vibration frequency for φ_2; ω_+ is the higher free vibration frequency for Φ_2, which is related to a periodic structure of the plate strip on a periodic subsoil.

In the subsequent sections the analysis will be restricted only to the free vibration frequencies for w, V, φ_1, Φ_1, which are determined by the formulae (5.8).

Free vibration frequencies within the homogenized model.
For free vibrations of the plate strip within the homogenized model, equations (4.3) take the form of the system of two equations for the macrodeflection w and the macrorotation φ_1:

$$\begin{aligned}
&(\mu + m)\ddot{w} + Kw - D(w_{,11} + \varphi_{1,1}) = 0 \\
&-D(w_{,1} + \varphi_1) + B_1\varphi_{1,11} - \vartheta\ddot{\varphi}_1 - \tfrac{1}{4}d^2(K_t\varphi_1 + m\ddot{\varphi}_1) = 0
\end{aligned} \tag{5.10}$$

and the independent equation for the macrorotation φ_2

$$B_2\varphi_{2,11} - D\varphi_2 - \vartheta\ddot{\varphi}_2 - \tfrac{1}{4}d^2(K_t\varphi_2 + m\ddot{\varphi}_2) = 0 \tag{5.11}$$

If substitute solutions $(5.6)_{1,3}$ for equations (5.10) and denote:

$$\hat{c} \equiv (\mu + m)(\tfrac{1}{4}d^2 m + \vartheta)$$
$$\hat{d} \equiv (\tfrac{1}{4}d^2 m + \vartheta)(D\alpha^2 + K) + (\mu + m)(B_1\alpha^2 + D + \tfrac{1}{4}d^2 K_t)$$
$$\hat{e} \equiv (B_1\alpha^2 + D + \tfrac{1}{4}d^2 K_t)(D\alpha^2 + K) - (D\alpha)^2$$

the characteristic equation within *the homogenized model* for vibrations of w, φ_1 has the form

$$\hat{c}\omega^4 - \hat{d}\omega^2 + \hat{e} = 0$$

From the above equation the following formulae of free vibration frequencies may be derived:

$$\omega_1{}^2 = \frac{\hat{d} - \sqrt{\hat{d}^2 - 4\hat{c}\hat{e}}}{2\hat{c}}, \qquad \omega_2{}^2 = \frac{\hat{d} + \sqrt{\hat{d}^2 - 4\hat{c}\hat{e}}}{2\hat{c}} \tag{5.12}$$

where ω_1, ω_2 are the lower free vibration frequencies for w and φ_1.
Now, the solution $(5.6)_5$ may be substituted for (5.11) and denoted:

$$\breve{d} \equiv \tfrac{1}{4}d^2 m + \vartheta, \qquad \breve{e} \equiv B_2\alpha^2 + D + \tfrac{1}{4}d^2 K_t$$

Hence, the characteristic equation within *the homogenized model* for vibrations of φ_2 has the following form

$$-\breve{d}\omega^2 + \breve{e} = 0$$

From the above equation one free vibration frequency of *the homogenized model* is derived

$$\omega^2 = \frac{\breve{e}}{\breve{d}}$$

which is *the lower frequency* for φ_2.

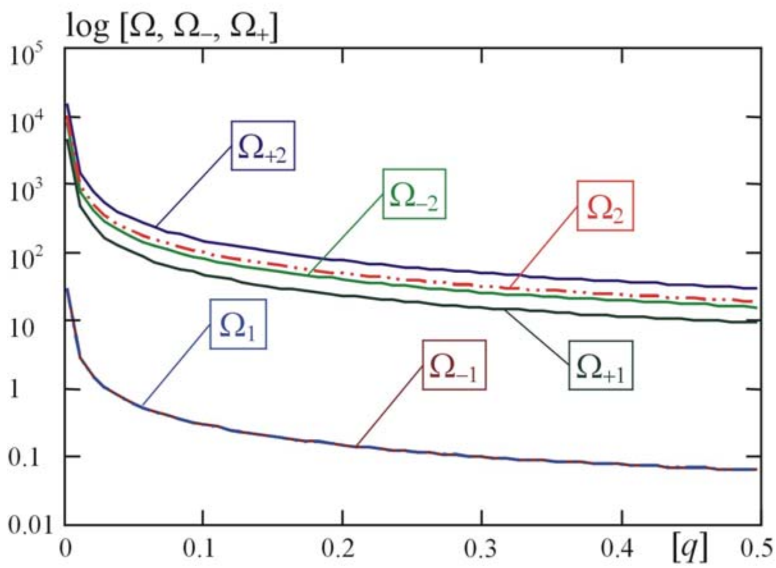
The following considerations will be restricted only to the free vibration frequencies for w, φ_1, which are determined by formulae (5.12).

Calculation results.
Introduce dimensionless parameters:

$$\Omega_{-1,-2} \equiv \omega_{-1,-2}\chi\alpha^{-1}, \qquad \Omega_{+1,+2} \equiv \omega_{+1,+2}\chi\alpha^{-1}, \qquad \Omega_{1,2} \equiv \omega_{1,2}\chi\alpha^{-1}, \qquad q \equiv \alpha l$$

where $\Omega_{-1,-2}, \Omega_{+1,+2}, \Omega_{1,2}$ are *frequency parameters*, which describe the free vibration frequencies determined by $(5.8)_{1,3}$, $(5.8)_{2,4}$, $(5.12)_{1,2}$, respectively; q is *the dimensionless wave number*; $\chi^2 \equiv \rho E^{-1}$.

Fig. 3. Diagrams of dispersion curves of relations Ω-*q*



Results of calculations are presented in Fig. 3 as diagrams of relations between the frequency parameters Ω and the dimensionless wave number q ($\Omega - q$). Calculations are made for the Poisson's ratio $\nu = 1/6$, for the shear coefficient $\psi = 5/6$, and for parameters: $q \in (0{,}0.5]$, d/l=0.2, $\gamma = 0.5, k_0 d / E = 10^{-4}, k_0 / k_{t0} = 5, k_1 / k_0 = \mu_1 / \mu_0 = 0, \mu / \mu_0 = 1,$ with the parameter γ describing a length of a cell part with the Winkler coefficients k_0, k_{t0} and the mass density μ_0, cf. (5.1), (5.2) and Fig. 2.

CONCLUSIONS

The *tolerance averaging technique*, proposed for periodic composites and structures by Woźniak and Wierzbicki [19], and applied in this paper to medium thickness plates resting on a periodic Winkler's subsoil, leads to the governing equations with constant coefficients of a new averaged model. This model describes the effect of period lengths on vibrations of plates of this kind and is called *the tolerance model*.
After the analyses of new obtained results some final remarks were formulated.

1. The obtained *tolerance model* is governed by equations, which involve terms dependent explicitly on *the mesostructure parameter l* (describing the size of the periodicity cell). Hence, this model makes possible to investigate certain phenomena, caused by the internal periodic structure of the system of the plate and the subsoil, in dynamic problems.
2. One such phenomenon is manifested in additional higher free vibration frequencies, which cannot be obtained in the framework of known homogenized models.
3. The results obtained in the presented example make possible to formulate some comments, cf. Fig. 3.

- The values of the first lower frequency calculated within the tolerance model are nearly identical with values of this frequency by the homogenized model.
- The values of the second lower frequency by the tolerance model are smaller than values of this frequency by the homogenized model.
- The values of the first higher frequency by the tolerance model, related to the fluctuation of the plate deflection, are smaller than values of the second lower frequency related to the macrorotation (i. e. the averaged rotation).
- The values of the second higher frequency by the tolerance model, related to the fluctuation of the plate rotation, are bigger than values of the second lower frequency, but differences between them are small.

Certain other dynamic problems of medium thickness plates resting on a periodic Winkler's subsoil will be studied in the forthcoming papers.

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