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REFLECTION OF ACOUSTIC WAVES FROM A LAYER OF VARYING DENSITY IN RUBBERLIKE MATERIAL

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ABSTRACT

The paper gives a focus on the problem of acoustic waves reflection from a layer of varying density situated between two homogeneous materials. Presented results illustrate the dependence of reflection and transition coefficients on frequency and on the structure of the transition layer.

Key words: rubberlike materials, layered composites, acoustic waves reflection and transmission

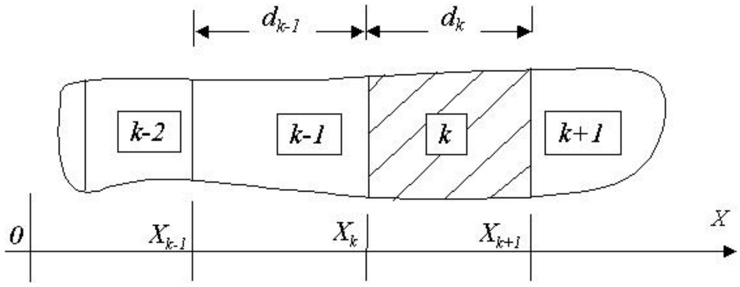
INTRODUCTION

It may be considered that the harmonic wave, propagating in layered media perpendicularly to the layers and the small motion u_i (i=1,3) in the k-th material, consists of two sinusoidal waves (Fig.1) the first running to the right and second one running to the left

$$\begin{aligned} u_1^{(k)}(X,t) &= A_k \exp i\omega\left(t - \frac{X - X_k}{c_k}\right) + B_K \exp i\omega\left(t + \frac{X - X_k}{c_k}\right) \\ u_3^{(k)}(X,t) &= \breve{A}_k \exp i\omega\left(t - \frac{X - X_k}{\breve{c}_k}\right) + \breve{B}_K \exp i\omega\left(t + \frac{X - X_k}{\breve{c}_k}\right) \end{aligned} \tag{1}$$

where c_k, $\breve{c}_k$ are the wave speeds for the longitudinal and transverse wave respectively. Here and bellow all quantities and functions connected with the propagation of the transverse wave have got the " "mark.

Fig. 1. Layered composite under consideration



CONSTUTIVE RELATIONS

The consideration the reflection transmission problem of acoustic waves from a layer of varying density in a compressible isotropic rubberlike solids is the aim of this paper. A purely mechanical theory is considered. The results are obtained for the special form of Blatz and Ko strain energy function [2], which is representative for foamed polyurethane rubber:

$$W(I_2, I_3) = \frac{\mu}{2}\left\{\frac{I_2}{I_3} - 3 + \frac{1-2\nu}{\nu}\left[I_3^{\frac{\nu}{1-2\nu}} - 1\right]\right\}, \tag{2}$$

where $I_1$and I_2 are the principal invariants of the deformation tensor, μ and ν are the shear modulus and Poisson's ratio for infinitesimal deformation from the natural reference state. It is easy to find ([2], [6]) since the value of the Poisson's ratio in this case is $\nu = 0.25$.

LINEARIZED EQUATIONS OF MOTION IN THE K-TH LAYER

For the motion defined by (1) the linearised system of equations of motion is reduced to two independent uncoupled wave equations with constant wave speeds c_k, $\breve{c}_k$:

$$u_{1,11}^{(k)} = \frac{1}{c_k^2}\ddot{u}_1^{(k)}, \quad u_{3,11}^{(k)} = \frac{1}{\breve{c}_k^2}\ddot{u}_3^{(k)} \tag{3}$$

where

$$c_k^2 = \frac{\lambda_k + 2\mu_k}{\rho_R^{(k)}}, \quad \breve{c}_k^2 = \frac{\mu_k}{\rho_R^{(k)}}$$

Suppose that two layers made of two different elastic materials are rigidly coupled at $X=X_k$ (Fig.1). The linearized boundary value problem is to consider. The equations of motion must be complemented with the continuity conditions at the interface. When two solids are in rigid contact, the displacement vector and stress one must be continuous from one medium to the other. The following conditions for the longitudinal wave are being obtained.

$$\begin{gathered} u_1^{(k-1)}(X_k, t) = u_1^{(k)}(X_k, t), \\ \rho_R^{(k-1)} c_{k-1}^2 u_{1,1}^{(k-1)}(X_k, t) = \rho_R^{(k)} c_k^2 u_{1,1}^{(k)}(X_k, t) \end{gathered}, \tag{5}$$

and for the transverse wave

$$\begin{gathered} u_3^{(k-1)}(X_k, t) = u_3^{(k)}(X_k, t), \\ \rho_R^{(k-1)} \breve{c}_{k-1}^2 u_{3,1}^{(k-1)}(X_k, t) = \rho_R^{(k)} \breve{c}_k^2 u_{3,1}^{(k)}(X_k, t) \end{gathered}. \tag{6}$$

TRANSITION MATRICES

The conditions described above lead to the following relations between the amplitudes of the longitudinal and transverse waves in the regions $k-1$ and k respectively

$$\begin{bmatrix} A_k \\ B_k \end{bmatrix} = [M_k] \begin{bmatrix} A_{k-1} \\ B_{k-1} \end{bmatrix} \qquad \begin{bmatrix} \breve{A}_k \\ \breve{B}_k \end{bmatrix} = [\breve{M}_k] \begin{bmatrix} \breve{A}_{k-1} \\ \breve{B}_{k-1} \end{bmatrix} \tag{7}$$

where

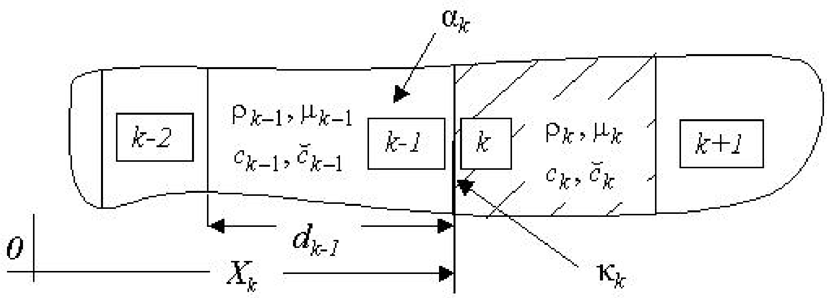
$$[\breve{M}_k] = \frac{1}{2} \begin{bmatrix} (1+\breve{\kappa}_k)\exp(-i\breve{\alpha}_k), & (1-\breve{\kappa}_k)\exp(i\breve{\alpha}_k) \\ (1-\breve{\kappa}_k)\exp(-i\breve{\alpha}_k), & (1+\breve{\kappa}_k)\exp(i\breve{\alpha}_k) \end{bmatrix},$$

$$[M_k] = \frac{1}{2} \begin{bmatrix} (1+\kappa_k)\exp(-i\alpha_k), & (1-\kappa_k)\exp(i\alpha_k) \\ (1-\kappa_k)\exp(-i\alpha_k), & (1+\kappa_k)\exp(i\alpha_k) \end{bmatrix},$$

$$\kappa_k = \frac{c_{k-1}\rho_R^{(k-1)}}{c_k \rho_R^{(k)}}, \quad \alpha_k = \omega \frac{X_k - X_{k-1}}{c_{k-1}} = \frac{\omega d_{k-1}}{c_{k-1}},$$

$$\breve{\kappa}_k = \frac{\breve{c}_{k-1}\rho_R^{(k-1)}}{\breve{c}_k \rho_R^{(k)}}, \quad \breve{\alpha}_k = \omega \frac{X_k - X_{k-1}}{\breve{c}_{k-1}} = \frac{\omega d_{k-1}}{\breve{c}_{k-1}}.$$

Fig. 2. Regions described by the parameters κ_k and α_k of the transitions matrix at the interface $X=X_k$



The complex valued matrices are the transition ones for the $X=X_k$ interface. They possess the following symmetries

$$(M_k)_{12} = (\overline{M}_k)_{21}, \quad (M_k)_{22} = (\overline{M}_k)_{11} \tag{9}$$

where the bar means the complex conjugate value. When only the density changes in every layer and the shear modulus and Poisson's ratio are constant, the impedance ratio κ is the same for the longitudinal and transverse wave and equals the density ratio

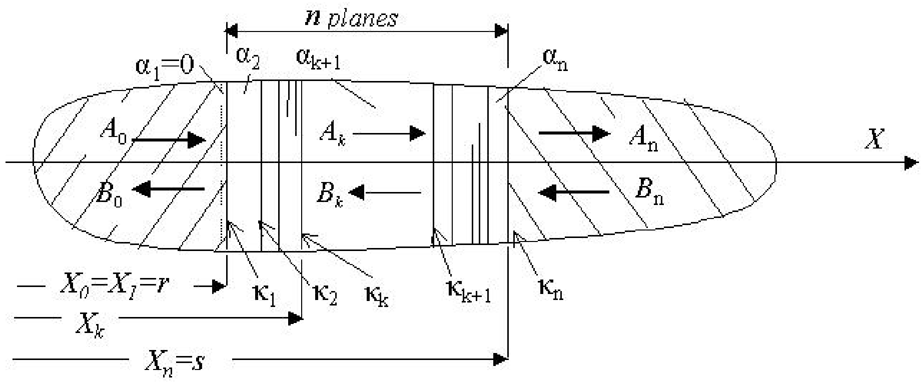
$$\mu_k = \mu_{k-1}, \ \nu_k = \nu_{k-1} \quad \Rightarrow \quad \kappa_k = \breve{\kappa}_k = \sqrt{\frac{\rho_R^{(k-1)}}{\rho_R^{(k)}}} \tag{10}$$

REFLECTION AND TRANSMISSION COEFFICIENTS

Two different homogeneous materials marked with lines (Fig.3) are to be considered now. The complex amplitudes in these regions are respectively A_0, B_0, A_n and B_n. There are *n-1* different layers between the homogeneous regions and *n* interfaces. The reflection and transmission coefficients for composite layered body are defined as

$$\begin{aligned} B_n = 0 \Rightarrow \quad & r^{(0)} = \sqrt{B_0 \overline{B}_0} \,/\, \sqrt{A_0 \overline{A}_0}, \quad t^{(0)} = \sqrt{A_n \overline{A}_n} \,/\, \sqrt{A_0 \overline{A}_0} \\ A_0 = 0 \Rightarrow \quad & r^{(n)} = \sqrt{A_n \overline{A}_n} \,/\, \sqrt{B_n \overline{B}_n}, \quad t^{(n)} = \sqrt{B_0 \overline{B}_0} \,/\, \sqrt{B_n \overline{B}_n} \end{aligned} \tag{11}$$

Fig. 3. Geometry of the layered transition zone between two homogeneous regions

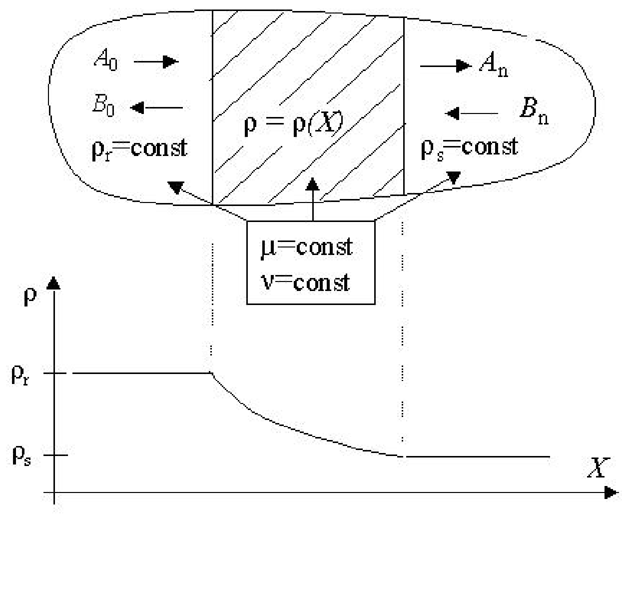


Taking now $X_0 = X_1 = r$ we obtain $\alpha_1 = 0$ and the other α_k ($k = 2,3...n$) coefficients take the different nonzero values. The density of the foam rubber is smaller then the density of solid rubber, but the elastic constants μ and ν can remain unchanged for modified foam rubber mixtures when the density $\rho = \rho(X)$ varies.

MODIFIED TRANSITION MATRICES

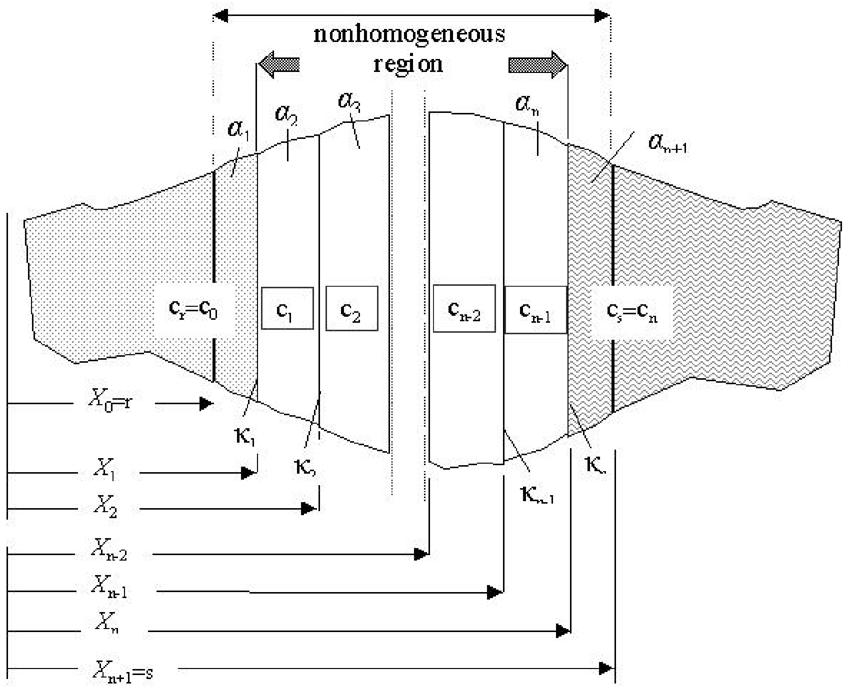
It may be assumed (Fig.4) that the composite body consists of two homogeneous regions with densities ρ_r and ρ_s and the inhomogeneous transition zone between, in which the density changes continuously from ρ_r to ρ_s. The propagation speed in the transition region also changes continuously with the variation of the density from c_r to c_s.

Fig. 4. Density profile in the inhomogeneous transition region



The calculations of the reflection and transmission coefficients simplify considerably when the transition zone [r,s] and speed interval [c_r, c_s] are divided into n layers of different thickness in such a way that the parameters α_k and κ_k ($k = 1,2,3...n$) have got the same values in every layer and all transition matrix M_k are equal $M_k = M = const$. Transition from one homogeneous material to another demands multiplication of n complex valued matrices M_k. It is possible to give simple analytic approach [1] for calculation of the product $(M)^n$ in this case. In this treatment it is necessary to include additionally two layers of homogeneous material on both sides of the transition zone, which thickness may be easy calculated (Fig.5).

Fig. 5. Geometry of the transition zone with constant parameters κ_k and α_k of the transition matrices at all interfaces



It is obtained respectively for the equal impedences ratio

$$\begin{gathered}\kappa_1 = \kappa_2 = ... \; .... = \kappa_n = \kappa \\ \frac{\rho_r c_r}{\rho_1 c_1} = \frac{\rho_1 c_1}{\rho_2 c_2} = ... \; .... = \frac{\rho_{n-1} c_{n-1}}{\rho_n c_n} = \kappa\end{gathered} \tag{12}$$

$$\kappa_T = \frac{c_s}{c_r} \qquad \kappa = (\kappa_T)^{1/n} \qquad c_k = \kappa^k c_r \tag{13}$$

and for the space dividing

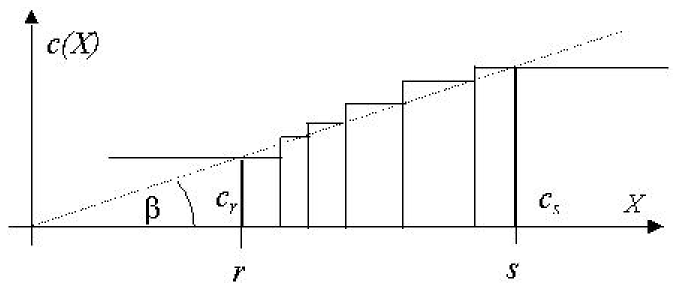
$$\begin{gathered}\alpha_1 = \alpha_2 = ... \; .... = \alpha_{n+1} = \alpha \\ \omega \frac{X_1 - r}{c_r} = \omega \frac{X_2 - X_1}{c_1} = ... \; .... = \omega \frac{s - X_n}{c_s} = \alpha\end{gathered} \tag{14}$$

$$\underbrace{c_r + c_1 + ... \; ...... + c_s}_{n+1} = \frac{\omega}{\alpha}(s - r) \tag{15}$$

$$\alpha = \frac{\omega}{p} \frac{(\kappa_T - 1)(\kappa - 1)}{\kappa_T \kappa - 1} \tag{16}$$

It may be taken into account in $(12)_2$ that $c_k^2 \rho_k = const$ and that the expression on the left hand side of (15) is the sum of the geometric progression with common ratio κ. The values of α and κ coefficients depend on the speed ratio $(13)_1$ and the number of layers *n*, the first one depends also additionally on the frequency of the incident wave and the parameter p, which describes the rate of speed changes in the transition zone and is defined below.

Fig. 6. Approximation of the speed c(X) in the transition zone by step function



$$p = \mathrm{tg}\beta = \frac{c_s - c_r}{s - r}, \quad c_r = pr, \quad c_s = ps \tag{17}$$

$$s \to r \;\Rightarrow\; tg\beta \to \infty, \;\; \alpha \to 0 \text{ jump discontinuity}$$

Now all the transition matrices M_k are equal and take the form

$$[M_k] = [M] = \frac{1}{2}\begin{bmatrix} (1+\kappa)\exp(-i\alpha), & (1-\kappa)\exp(i\alpha) \\ (1-\kappa)\exp(-i\alpha), & (1+\kappa)\exp(i\alpha) \end{bmatrix} \tag{18}$$

where κ and α are defined by $(13)_2$ and (16) . The matrix M with symmetries (9) may be transformed to the following form [1]:

$$[M] = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} = \sqrt{m}\begin{bmatrix} \dfrac{M_{11}}{\sqrt{m}} & \dfrac{M_{12}}{\sqrt{m}} \\ \dfrac{M_{21}}{\sqrt{m}} & \dfrac{M_{22}}{\sqrt{m}} \end{bmatrix} = \sqrt{m}\begin{bmatrix} \cos\varphi - iE\sin\varphi & (C+iD)\sin\varphi \\ (C-iD)\sin\varphi & \cos\varphi + iE\sin\varphi \end{bmatrix} \tag{19}$$

where $m = \det\lfloor M_{ij} \rfloor$ on condition that

$$-1 < \mathrm{Re}\left(\frac{M_{11}}{\sqrt{m}}\right) < 1, \quad \cos\varphi = \mathrm{Re}\left(\frac{M_{11}}{\sqrt{m}}\right) = \frac{1+\kappa}{2\sqrt{\kappa}}\cos\alpha \tag{20}$$

If the condition is satisfied, all constant φ, C, D, E in the representation $(19)_2$ are real and may be calculated if compared with $(19)_1$. If the condition $(20)_1$ is replaced by following one

$$\mathrm{Re}\left(\frac{M_{11}}{\sqrt{m}}\right) > 1, \quad \cosh\varphi = \mathrm{Re}\left(\frac{M_{11}}{\sqrt{m}}\right) = \frac{1+\kappa}{2\sqrt{\kappa}}\cos\alpha \tag{21}$$

the matrix (19) may be transformed to more convenient form

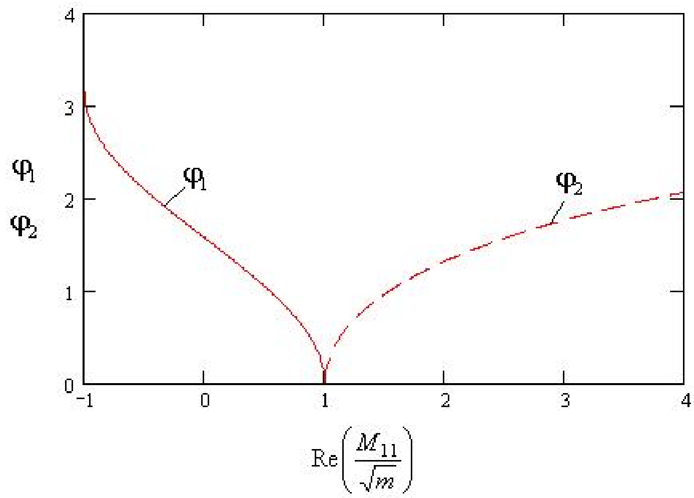
$$[M]=\sqrt{m}\begin{bmatrix}\cosh\varphi - iE\sinh\varphi & (C+iD)\sinh\varphi \\ (C-iD)\sinh\varphi & \cos\varphi + iE\sinh\varphi\end{bmatrix} \tag{22}$$

$$\mathrm{Re}\left(\frac{M_{11}}{\sqrt{m}}\right)=\frac{1+\kappa}{2\sqrt{\kappa}}\cos\alpha=\frac{1+\kappa}{2\sqrt{\kappa}}\cos\left[\frac{\omega}{p}\frac{(\kappa_T-1)(\kappa-1)}{\kappa_T\kappa-1}\right] \tag{23}$$

The real value of the expression $M_{11}/\sqrt{m}$ may be smaller or greater than 1. φ_1 or φ_2 respectively (Fig.7) may be calculated what leads to the adequate form of the transition matrix

$$\varphi_1=\arccos\left[\mathrm{Re}\left(\frac{M_{11}}{\sqrt{m}}\right)\right] \quad or \quad \varphi_2=\arccos\mathrm{h}\left[\mathrm{Re}\left(\frac{M_{11}}{\sqrt{m}}\right)\right] \tag{24}$$

Fig. 7. Functions φ_1 φ_2 determining the form of the transition matrix



To find the relationship between the amplitudes of the waves in both homogeneous regions connected by the zone of *n-1* virtual layers with n interfaces, the matrix M (in the form (19)$_2$ or (22)) should be first raised to the $n-th$ power.

$$([M])^n=\left[M_{ij}^{(n)}\right]=(m)^{n/2}\begin{bmatrix}\cos n\varphi - iE\sin n\varphi & (C+iD)\sin n\varphi \\ (C-iD)\sin n\varphi & \cos\varphi + iE\sin n\varphi\end{bmatrix} \tag{25}$$

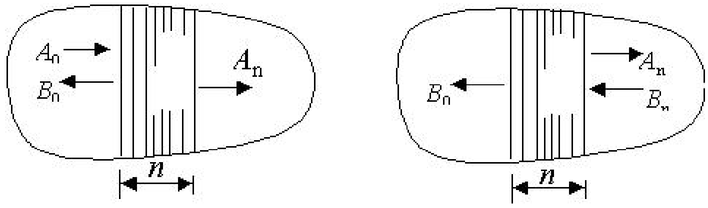
The result is very simple (comp.[1]), because constant multipler $\sqrt{m}$ and all arguments of the trygonometric functions become multiplied *n* times. Analogous result as above may be obtained for the representation (22). The amplitudes of the waves in both homogeneous regions satisfy also the following equation

$$\begin{bmatrix}A_n \\ B_n\end{bmatrix}=\left[M_{ij}^{(n)}\right]\begin{bmatrix}A_0 \\ B_0\end{bmatrix} \tag{26}$$

Now $B_n=0$ may be taken and A_0, B_0, A_n considered as the amplitudes of the incident reflected and transmitted waves. The reflection and transmission coefficients (comp.(11)) are equal respectively

$$r^{(0)}=\sqrt{\frac{B_0\overline{B}_0}{A_0\overline{A}_0}}=\sqrt{\frac{M_{21}^{(n)}\overline{M}_{21}^{(n)}}{M_{22}^{(n)}\overline{M}_{22}^{(n)}}} \qquad t^{(0)}=\sqrt{\frac{A_n\overline{A}_n}{A_0\overline{A}_0}}=\sqrt{\frac{(\kappa)^{2n}}{M_{22}^{(n)}\overline{M}_{22}^{(n)}}} \quad . \tag{27}$$

Fig. 8. Incident wave and assumed reflection – transmission patterns



If we take $A_0 = 0$ in analogous way the results are

$$r^{(n)} = \sqrt{\frac{A_n \overline{A}_n}{B_n \overline{B}_n}} = \sqrt{\frac{M_{12}^{(n)} \overline{M}_{12}^{(n)}}{M_{22}^{(n)} \overline{M}_{22}^{(n)}}} \qquad t^{(n)} = \sqrt{\frac{B_0 \overline{B}_0}{B_n \overline{B}_n}} = \sqrt{\frac{1}{M_{22}^{(n)} \overline{M}_{22}^{(n)}}} \tag{28}$$

After calculations of all terms of the matrix $\left[M_{ij}^{(n)}\right]$ the final results are

$$r^{(n)} = r^{(0)} = \frac{1}{\sqrt{1 + \left[\left(\frac{1+\kappa}{1-\kappa}\right)^2 - 1\right]\left(\frac{\sin\varphi}{\sin n\varphi}\right)^2}},$$

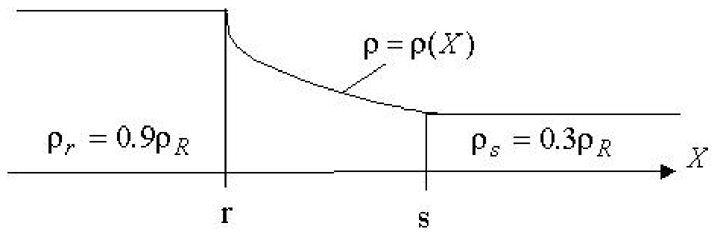
$$t^{(n)} = \frac{t^{(0)}}{\sqrt{(\kappa)^{2n}}} = \sqrt{\frac{\kappa_T \left[\left(\frac{1+\kappa}{1-\kappa}\right)^2 - 1\right]\left(\frac{\sin\varphi}{\sin n\varphi}\right)^2}{1 + \left[\left(\frac{1+\kappa}{1-\kappa}\right)^2 - 1\right]\left(\frac{\sin\varphi}{\sin n\varphi}\right)^2}} \tag{29}$$

It follows from (29) that both expressions depend only on the speeds ratio κ_T in both homogeneous regions, number n of the layers, and on the ratio ω / p (comp.(24)). For *n=1*, it follows from (29) the well known results for the jump discontinuity

$$r^{(n)} = r^{(0)} = \frac{1-\kappa}{1+\kappa}, \qquad t^{(n)} = \frac{t^{(0)}}{\kappa} = \frac{2}{1+\kappa} \tag{30}$$

NUMERICAL RESULTS

Fig. 9. Density values assumed in the homogeneous regions



Some experimental results for several elastomers were presented in [6]. The density of the solid rubber is $\rho_R = 911\ \mathrm{kg/m^3}$. In agreement with the main conclusions of this paper, the following elastic constants and densities for the foam rubber: μ=0.221 MPa, ν=0.25 were assumed. The density of the foam rubber is smaller then the density of solid rubber $\rho_r < \rho_R$, $\rho_s < \rho_R$ and the elastic constants μ and ν can remain unchanged for modified foam rubber mixtures when the density varies. The calculations were made for two values of the density ratios $\rho_r / \rho_R = 0.9$ and $\rho_s / \rho_R = 0.3$ in both homogeneous regions (Fig.9). In the transition zone the density $\rho = \rho(X)$ changes continuously from ρ_r to ρ_s. The assumption about density ratios reflects properly the true attributes of foam rubber produced in this day and age.The physical properties of four foam rubber composites were studied experimentally in [1]. The ratio of densities (shear moduli) of every pair of two different patterns chosen arbitrarily from the whole series, ranges from $\rho_r / \rho_s = 1.25$ ($\mu_r / \mu_s = 1$) to $\rho_r / \rho_s = 3.27$ ($\mu_r / \mu_s = 1.35$). The speeds of the transverse wave propagation in m/sec are equal $\breve{c}_r = 16.41$, $\breve{c}_s = 28.43$ respectively and $\breve{\kappa}_T = \breve{c}_s / \breve{c}_r = \sqrt{0.9/0.3} = \sqrt{3}$. The speeds of the longitudinal waves are diffrent, but because of the same elastic constant in both homogeneous regions and in the transition zone the ratio of $\kappa_T = c_s / c_r = \sqrt{0.9/0.3} = \sqrt{3}$ has the same value as previous. For the jump discontinuity

$$n = 1, \quad \kappa_T = \kappa = \sqrt{3}, \quad r^{(1)} = r^{(0)} = 0.268, \quad t^{(1)} = 0.732 \ . \tag{31}$$

was obtained.

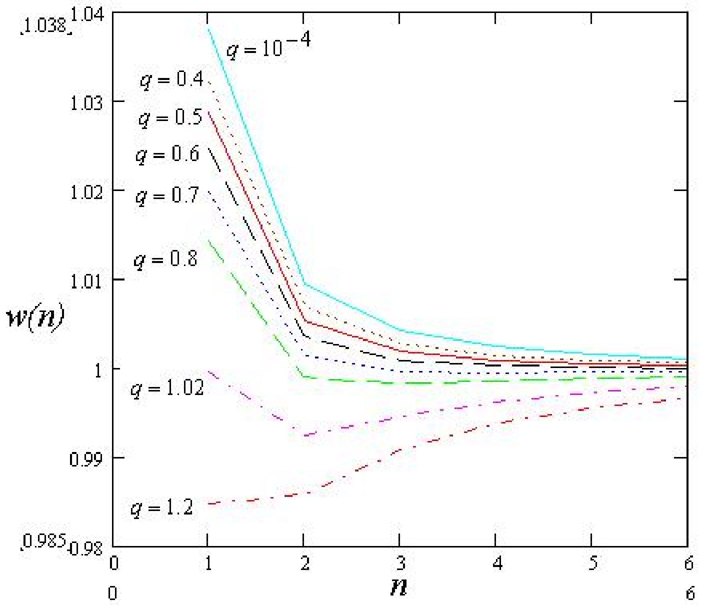
The case of nonhomogeneous transition zone, modelled as layered region of *n-1* virtual layers with *n* interfaces and equal transition matrices $M_k = M$, ($k = 1,2..n$) was considered. It was denoted bellow as *w(n),* depending on *n* expression on the right hand side of (23).

$$w(n) = \frac{1+\kappa}{2\sqrt{\kappa}} cos\left(q \frac{(\kappa_T - 1)(\kappa - 1)}{\kappa_T \kappa - 1} \right) \tag{32}$$

where (comp. $(13)_{1,2}$) $\kappa_T = c_s / c_r = const$, $\kappa = (\kappa_T)^{1/n}$ and $q = \omega / p$.

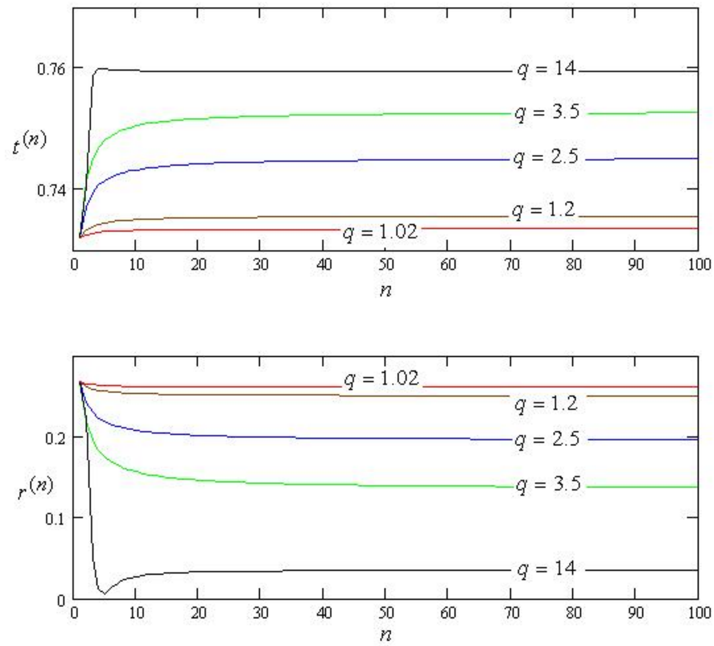
The function $w(n)$ with variable parameter *q* (comp. (23)) for all $n \to \infty$ takes the values greater or smaller than 1 depending on the value of *q* (Fig. 10). The form of the transition matrix $(19)_2$ or (22) respectively may be chosen.

Fig. 10. Plot of the function $w(n) = Re\left(M_{11} / \sqrt{m}\right)$ for different values of the parameter *q*



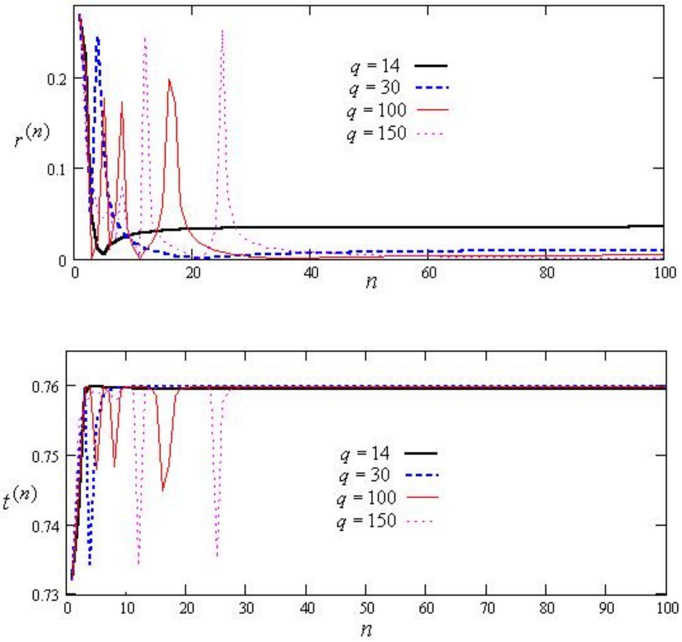
For $\kappa_T = \breve{\kappa}_T = const$ the form of the matrix depends only on the value of the parameter *q*. It is easy to see that for *q*>1.02 the matrix has the form $(19)_2$. If *q* changes in the limits $1.02 \le q \le 14$ (Fig.11) small fluctuations of the value of the transmission coefficient from the value 0.732 which is representative for the jump discontinuity may be observed.

Fig. 11. Variations of the tramsmission and reflection coefficients for $1.02 \le q \le 14$



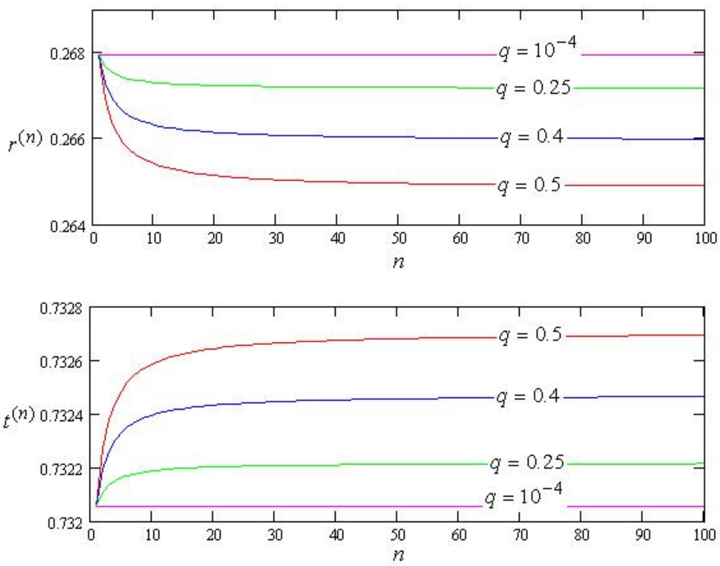
The reflection coefficient decreases with the growth of *q*. If q takes the values greather than *q*=30 (Fig.12) the reflection coefficient rapidly vanishes and the transmission coefficient has got stable value equal ≈ 0.76.

Fig. 12. Variations of the tramsmission and reflection coefficients for $14 \le q \le 150$



It may bee assumed now that q changes in the limits $10^{-4} \le q \le 0.5$. The fluctuations in these case are extremaly small (Fig.13). Both coefficient have got practically the values equivalent to the ones for jump discontinuity. It is easy to made the generalisation about the results described above. We take the constant finite value for p. For short waves, when ω increases, the inhomogeneous transition zone puts out the reflected wave and reduces the transmission wave.

Fig. 13. Variations of the tramsmission and reflection coefficients for $10^{-4} \le q \le 0.5$



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