Electronic Journal of Polish Agricultural Universities is the very first Polish scientific journal published exclusively on the Internet, founded on January 1, 1998 by the following agricultural universities and higher schools of agriculture: University of Technology and Agriculture of Bydgoszcz, Agricultural University of Cracow, Agricultural University of Lublin, Agricultural University of Poznan, University of Podlasie in Siedlce, Agricultural University of Szczecin and Agricultural University of Wroclaw.

ELECTRONIC JOURNAL OF POLISH AGRICULTURAL UNIVERSITIES

2005
Volume 8
Issue 4
Topic
CIVIL ENGINEERING

MONASTYRSKYY B., KACZYŃSKI A.. 2005. CONTACT BETWEEN AN ELASTIC LAYER AND AN ELASTIC SEMI-SPACE BOUNDED BY A PLANE WITH AXIALLY SYMMETRIC SMOOTH DIP
Electronic Journal of Polish Agricultural Universities, Civil Engineering, Volume 8, Issue 4.
Available Online http://www.ejpau.media.pl

CONTACT BETWEEN AN ELASTIC LAYER AND AN ELASTIC SEMI-SPACE BOUNDED BY A PLANE WITH AXIALLY SYMMETRIC SMOOTH DIP

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ABSTRACT

A problem of the theory of elasticity is considered for an elastic layer in frictionless contact with an elastic half-space bounded by a plane with an axially symmetric smooth dip. The method of the Hankel integral transforms is used to solve the problem. In this way we obtain a system of dual integral equations such that it can be reduced to a single Fredholm integral equation of the second kind. The numerical results are presented in the form of diagrams for particular cases of the dip. Besides we discuss the variation of the contact parameters depending on the layer thickness.

Keywords: discontinuous frictionless contact, an axially-symmetric problem, dual integral equations.

INTRODUCTION

In the classical contact problems of the theory of elasticity (see e.g. [1]) the contacting regions of solids are small as compared with the sizes of the bodies in contact. We call such cases "the contact of solids with unconformable boundaries". Then, as a rule it is assumed that there are no contact tractions before the application of loadings and that solids are in contact at a single point (in the case of the axial symmetry), or along a straight line (in two-dimensional cases). Another situation arises when one deals with contact of bodies which we call with "conformable" surfaces when the contact between the solids takes place along the whole region of contact except the areas of a local loss of contact. The absence of the contact can be generated by two factors either the two bounding surfaces do not coincide or there exists a local separation induced by external forces. Such types of interactions are less investigated in contrast to the classical contact of bodies, although contact of solids with conformed surfaces is typical for many cases known from practice.

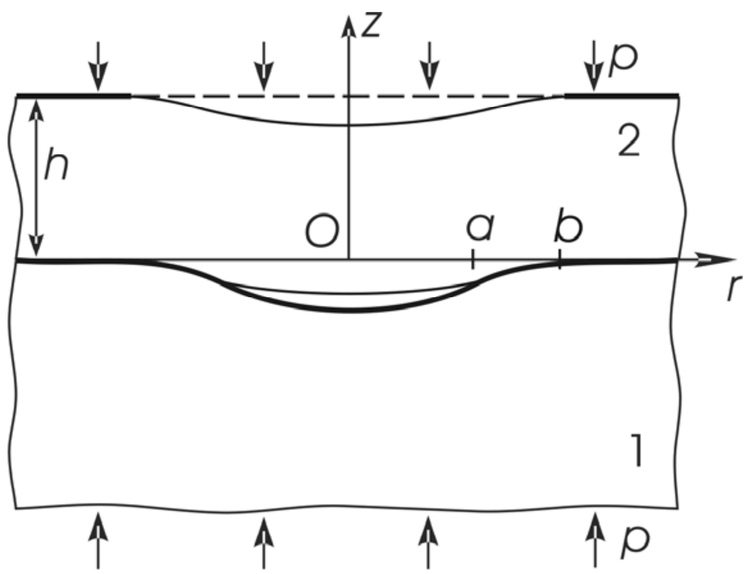
In this paper we study the elastic contact of two bodies with conformable boundaries, namely between a layer and a half-space, with allowance for the local absence of contact caused by an initial surface defect. The case of some specific shape of the dip with the second kind discontinuity of the surface curvature is analyzed in detail. A similar problem with the surface having the continuous curvature was solved in [2]. Some problems for two half-spaces or a half-space and a rigid foundation were considered in [3–7]. The plane contact problem for laminated half-spaces with interface flaws taken into account and the influence of sliding friction was treated in [8,9]. The literature on contact problems for elastic layers is very extensive. We mention here only the works, which are pertinent to the present study. Local separation between an elastic layer and a half-space was examined in [10–14]. Approaches employing contact of an elastic layer and a rigid substrate having cylindrical protrusion or pit can be found in [15, 16]. Some new results on the loss of contact between a layer and a foundation due to longitudinal compression are given in [17, 18].

The paper is organized as follows. In section 2 we give the description and the mathematical formulation of the problem under study. Section 3 deals with a method of solving the resulting boundary value problem. With the help of Hankel integral representations of displacements and stresses for both the half-space and the layer the problem is reduced to a system of dual integral equations and next, to a Fredholm integral equation of the second kind. Numerical procedure of solution to this equation is described in Section 4. Moreover, the analysis of results obtained is carried out. The focus is made on the influence of the outer boundary of the layer on the contact parameters. Finally, some conclusions are made in Section 5.

THE STATEMENT OF THE CONTACT PROBLEM

Let us consider a frictionless contact problem for an elastic isotropic layer of thickness h and an elastic semi-infinitive region bounded by a plane (for $r > b$) and a smooth axially symmetric surface (for $r < b$), as shown schematically in Fig. 1. The axially symmetric surface is smooth, without any corner points. The solids are pressed together by pressure p applied at the upper boundary of the layer and at the infinity for the semi-infinite region. Since the bounding surfaces do not coincide over the whole boundary, a small gap of the radius a can appear. The problem lies in the determination of the fields of stresses and displacements within the bodies, the contact traction, the size of the interface gap and the threshold value of external load at which the gap disappears.

Fig. 1. Geometry of the problem



Let us use the cylindrical coordinate system (r, θ, z) with the center of the circular dip of radius a on the interface plane of contact $z = 0$. The layer (denoted by 2) occupies the region $\{(r, \theta, z):\ 0 \le r < \infty\ \wedge\ 0 \le \theta < 2\pi\ \wedge\ 0 \le z \le h\}$ and the half-space (denoted by 1) is defined by

$\{(r,\theta,z):\ 0\le r<\infty\ \wedge\ 0\le\theta<2\pi\ \wedge\ z\le 0\}$(see Fig. 1). These contacting bodies are isotropic, elastic, and homogeneous, of different shear moduli μ_i and Poisson's ratios $\nu_i, i=1,2$. Here, and in the sequel, the indices 1, 2 denote all the quantities (material constants, stresses, etc.) corresponding to the half-space and the layer, respectively.

The problem is an axially-symmetric one (independent of angle θ) with the only nonvanishing displacements u_r, u_z and components of the stress tensor $\sigma_{zz}, \sigma_{rz}, \sigma_{rr}, \sigma_{\theta\theta}$. To solve it, we apply the superposition principle and reduce to the two parts – for the layer and the half-space with bounding plain (without any dip) and a disturbance due to the existence of the surface dip. Since the first part is trivial, our attention is given now to the second problem. Bearing in mind all aforementioned assumptions we arrive at the following boundary conditions:

$$\sigma_{zz}^{(2)}(r,h)=0,\ \ \sigma_{zr}^{(2)}(r,h)=0,\qquad 0\le r<\infty, \tag{1}$$

$$\sigma_{zr}^{(1)}(r,0)=0,\ \ \sigma_{zr}^{(2)}(r,0)=0,\qquad 0\le r<\infty, \tag{2}$$

$$\sigma_{zz}^{(1)}(r,0)=p,\ \ \sigma_{zz}^{(2)}(r,0)=p,\qquad 0\le r<a, \tag{3}$$

$$\sigma_{zz}^{(1)}(r,0)=\sigma_{zz}^{(2)}(r,0),\ \ u_z^{(1)}(r,0)+f(r)=u_z^{(2)}(r,0),\qquad a<r<\infty, \tag{4}$$

$$\sigma_{zz}^{(1)}(r,-\infty)=0,\ \ \sigma_{zr}^{(1)}(r,-\infty)=0,\qquad 0\le r<\infty, \tag{5}$$

where $f(r)$ describes the initial shape of the dip and a is an unknown radius of the intercontact gap.

METHOD OF SOLUTION

Integral representations of the displacements and stresses

For a half-space with the boundary free of tangential tractions the displacements and stresses can be represented by a single harmonic function $\psi(r,z)$ [19]:

$$\begin{aligned}
u_r^{(1)} &= \frac{1}{2\mu_1} z\frac{\partial\psi}{\partial r}+\frac{1-2\nu_1}{2\mu_1}\int_z^\infty \frac{\partial\psi}{\partial r}dz, \qquad u_z^{(1)}=\frac{1}{2\mu_1} z\frac{\partial\psi}{\partial z}-\frac{1-\nu_1}{\mu_1}\psi,\\
\sigma_{rr}^{(1)} &= z\frac{\partial^2\psi}{\partial r^2}-2\nu_1\frac{\partial\psi}{\partial z}+(1-2\nu_1)\int_z^\infty \frac{\partial^2\psi}{\partial r^2}dz, \qquad \sigma_{zz}^{(1)}=z\frac{\partial^2\psi}{\partial z^2}-\frac{\partial\psi}{\partial z},\\
\sigma_{\theta\theta}^{(1)} &= \frac{z}{r}\frac{\partial\psi}{\partial r}-2\nu_1\frac{\partial\psi}{\partial z}+(1-2\nu_1)\frac{1}{r}\int_z^\infty \frac{\partial\psi}{\partial r}dz, \qquad \sigma_{rz}^{(1)}=z\frac{\partial^2\psi}{\partial r\partial z}.
\end{aligned} \tag{6}$$

Now we use the Hankel integral representation for the harmonic function ψ vanishing at $z=-\infty$ in the form

$$\psi(r,z)=\int_0^\infty A^{(1)}(\lambda)e^{\lambda z}J_0(\lambda r)\,d\lambda, \tag{7}$$

where $A^{(1)}$ is an unknown function to be determined, and $J_n(\lambda r)$ stands for the Bessel functions of the first kind, and order $n\ge 0$. It results from (6) that the normal displacements and normal stresses on the plane of contact $z=0$ can be expressed by a single function $A^{(1)}$ as

$$u_z^{(1)}(r,0) = -\frac{1-\nu_1}{\mu_1}\int_0^\infty A^{(1)}(\lambda) J_0(\lambda r)\,d\lambda, \quad \sigma_{zz}^{(1)}(r,0) = -\int_0^\infty \lambda A^{(1)}(\lambda) J_0(\lambda r)\,d\lambda. \tag{8}$$

Let construct similar relations for the layer. We start from the representation of solution to linear elasticity equations in axially symmetric case given by Weber in the form [20]

$$\begin{aligned}
&u_r^{(2)} = \frac{1}{2\mu_2}\frac{\partial}{\partial r}\left[\Phi + 2(1-\nu_2)\varphi\right], \qquad u_z^{(2)} = \frac{1}{2\mu_2}\frac{\partial}{\partial z}\left[\Phi - 2(1-\nu_2)\varphi\right],\\
&\sigma_{rr}^{(2)} = \frac{\partial^2}{\partial r^2}\left[\Phi + 2\varphi\right] + \frac{2\nu_2}{r}\frac{\partial \varphi}{\partial r}, \qquad \sigma_{zz}^{(2)} = -\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial \Phi}{\partial r}\right],\\
&\sigma_{\theta\theta}^{(2)} = \frac{1}{r}\frac{\partial}{\partial r}\left[\Phi + 2\varphi\right] + 2\nu_2\frac{\partial^2 \varphi}{\partial r^2}, \qquad \sigma_{rz}^{(2)} = \frac{\partial^2 \Phi}{\partial r \partial z}.
\end{aligned} \tag{9}$$

The functions $\Phi \equiv \Phi(r,z)$ and $\varphi \equiv \varphi(r,z)$ satisfy the equations

$$\Delta\varphi = 0, \quad \Delta^2\Phi = 0, \quad \Delta\Phi = 2\frac{\partial^2\varphi}{\partial z^2}, \tag{10}$$

where $\Delta \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$ is the Laplacian operator. For these functions we assume the following integral representations:

$$\begin{aligned}
&\Phi = \int_0^\infty \left\{\lambda^{-1}\left[A^{(2)}(\lambda) + B^{(2)}(\lambda)\lambda z\right]\operatorname{ch}\lambda z + \lambda^{-1}\left[C^{(2)}(\lambda) + D^{(2)}(\lambda)\lambda z\right]\operatorname{sh}\lambda z\right\} J_0(\lambda r)\,d\lambda,\\
&\varphi = \int_0^\infty \lambda^{-1}\left[D^{(2)}(\lambda)\operatorname{ch}\lambda z + B^{(2)}(\lambda)\operatorname{sh}\lambda z\right] J_0(\lambda r)\,d\lambda.
\end{aligned} \tag{11}$$

Here $A^{(2)}$, $B^{(2)}$, $C^{(2)}$, $D^{(2)}$ stand for unknown functions of λ. Now, after substitution of (11) into (9), the components of stress and displacement can be expressed as follows

$$\begin{aligned}
u_r^{(2)} = &-\frac{1}{2\mu_2}\int_0^\infty \left\{\left[A^{(2)} + 2(1-\nu_2)D^{(2)} + B^{(2)}\lambda z\right]\operatorname{ch}\lambda z + \left[C^{(2)} + 2(1-\nu_2)B^{(2)} + D^{(2)}\lambda z\right]\right\} J_1(\lambda r)\,d\lambda,\\
u_z^{(2)} = &\frac{1}{2\mu_2}\int_0^\infty \left\{\left[A^{(2)} - (1-2\nu_2)D^{(2)} + B^{(2)}\lambda z\right]\operatorname{sh}\lambda z + \right.\\
&\left. + \left[A^{(2)} - (1-2\nu_2)D^{(2)} + B^{(2)}\lambda z\right]\operatorname{sh}\lambda z\right\} J_0(\lambda r)\,d\lambda,\\
\sigma_{rr}^{(2)} = &-\int_0^\infty \lambda\left\{\left[[A^{(2)} + 2D^{(2)} + B^{(2)}\lambda z]\operatorname{ch}\lambda z + [C^{(2)} + 2B^{(2)} + D^{(2)}\lambda z]\operatorname{sh}\lambda z\right] J_0(\lambda r)\,d\lambda + \right.\\
&\left. -\left[[A^{(2)} + 2(1-\nu_2)D^{(2)} + B^{(2)}\lambda z]\operatorname{ch}\lambda z + [C^{(2)} + 2(1-\nu_2)B^{(2)} + D^{(2)}\lambda z]\operatorname{sh}\lambda z\right]\frac{J_1(\lambda r)}{\lambda r}\right\} d\lambda,
\end{aligned}$$

$$\sigma_{\theta\theta}^{(2)} = -\int_0^\infty \lambda \Big\{ 2\nu_2 [D^{(2)} \operatorname{ch} \lambda z + B^{(2)} \operatorname{sh} \lambda z] J_0(\lambda r) +$$

$$+ \Big[[A^{(2)} + 2(1-\nu_2) D^{(2)} + B^{(2)} \lambda z] \operatorname{ch} \lambda z +$$

$$+ [C^{(2)} + 2(1-\nu_2) B^{(2)} + D^{(2)} \lambda z] \operatorname{sh} \lambda z \Big] \frac{J_1(\lambda r)}{\lambda r} \Big\} d\lambda,$$

$$\sigma_{zz}^{(2)} = \int_0^\infty \lambda \Big\{ \big[A^{(2)} + B^{(2)} \lambda z \big] \operatorname{ch} \lambda z + \big[C^{(2)} + D^{(2)} \lambda z \big] \operatorname{sh} \lambda z \Big\} J_0(\lambda r)\, d\lambda,$$

$$\sigma_{rz}^{(2)} = -\int_0^\infty \lambda \Big\{ \big[A^{(2)} + D^{(2)} + B^{(2)} \lambda z \big] \operatorname{sh} \lambda z + \big[C^{(2)} + B^{(2)} + D^{(2)} \lambda z \big] \operatorname{ch} \lambda z \Big\} J_1(\lambda r)\, d\lambda. \tag{12}$$

Using the boundary conditions (1) and the second one of (2) we obtain

$$B^{(2)} = -C^{(2)} = \frac{\operatorname{sh}(\lambda h) \operatorname{ch}(\lambda h) + \lambda h}{\operatorname{sh}^2(\lambda h) - (\lambda h)^2} A^{(2)}, \quad D^{(2)} = \big[1 - \operatorname{ch}^2(\lambda h) \big] A^{(2)}. \tag{13}$$

Thus, substituting relations (13) into (12) we arrive at the integral representation of the stresses and displacements within the layer through the single function $A^{(2)}$. In particular, the expressions for the quantities of interest on the plane $z = 0$ take the following form

$$u_z^{(2)}(r,0) = -\frac{1-\nu_2}{\mu_2} \int_0^\infty k(\lambda) A^{(2)}(\lambda) J_0(\lambda r)\, d\lambda, \quad \sigma_{zz}^{(2)}(r,0) = \int_0^\infty \lambda A^{(2)}(\lambda) J_0(\lambda r)\, d\lambda, \tag{14}$$

where

$$k(\lambda) = \frac{\operatorname{sh}(\lambda h) \operatorname{ch}(\lambda h) + \lambda h}{\operatorname{sh}^2(\lambda h) - (\lambda h)^2}. \tag{15}$$

Reduction of the problem to a system of dual integral equations

It is clear from expressions (6), (12), (13) that the boundary conditions (1), (2), (5) are identically satisfied. Now, using (8) and (14), the remaining boundary conditions (3) and (4) lead to the following system of dual integral equations:

$$\begin{aligned}
&-\int_0^\infty \lambda A^{(1)}(\lambda)\, J_0(\lambda r)\, d\lambda = p, \qquad 0 \le r < a, \\
&\int_0^\infty \lambda A^{(2)}(\lambda)\, J_0(\lambda r)\, d\lambda = p, \qquad 0 \le r < a, \\
&f(r) - \frac{1-\nu_1}{\mu_1} \int_0^\infty A^{(1)}(\lambda)\, J_0(\lambda r)\, d\lambda = -\frac{1-\nu_2}{\mu_2} \int_0^\infty k(\lambda) A^{(2)}(\lambda)\, J_0(\lambda r)\, d\lambda, \quad a < r < \infty, \\
&-\int_0^\infty \lambda A^{(1)}(\lambda)\, J_0(\lambda r)\, d\lambda = \int_0^\infty \lambda A^{(2)}(\lambda)\, J_0(\lambda r)\, d\lambda, \qquad a < r < \infty.
\end{aligned} \tag{16}$$

With the choice of $A^{(2)}(\lambda)=-A^{(1)}(\lambda)$ equations $(16)_{1\text{-}2}$ and $(16)_4$ are automatically satisfied, so the set (16) of the dual integral equations can be rewritten in the following form:

$$\begin{aligned}&\int_0^\infty \lambda A^{(1)}(\lambda)\,J_0(\lambda r)\,d\lambda=-p, \qquad 0\le r<a,\\&\int_0^\infty\left(\frac{1}{m_1}+\frac{k(\lambda)}{m_2}\right)A^{(1)}(\lambda)\,J_0(\lambda r)\,d\lambda=f(r), \qquad a<r<\infty\end{aligned} \tag{17}$$

where $m_i=\mu_i/1-\nu_i$, $i=1,2$. Introducing a new function

$$C(\lambda)=A^{(1)}(\lambda)-B(\lambda) \tag{18}$$

provided

$$B(\lambda)=\lambda\left(\frac{1}{m_1}+\frac{k(\lambda)}{m_2}\right)^{-1}F(\lambda), \quad F(\lambda)=\int_0^\infty r\,f(r)\,J_0(\lambda r)\,dr, \tag{19}$$

the dual equations (17) reduce to the following form:

$$\begin{aligned}&\int_0^\infty \lambda C(\lambda)\,J_0(\lambda r)\,d\lambda=P(r), \qquad 0\le r<a,\\&\int_0^\infty\frac{M}{1+l(\lambda)}C(\lambda)\,J_0(\lambda r)\,d\lambda=0, \qquad a<r<\infty,\end{aligned} \tag{20}$$

where

$$l(\lambda)=\frac{1-k(\lambda)}{\frac{m_2}{m_1}+k(\lambda)}, \quad \frac{1}{M}=\frac{1}{m_1}+\frac{1}{m_2}, \quad P(r)=-p-\frac{1}{M}\int_0^\infty\lambda^2\left[1+l(\lambda)\right]F(\lambda)\,J_0(\lambda r)\,d\lambda. \tag{21}$$

To solve (20), we follow the Lebedev-Uflyand technique [21]. If we define $C(\lambda)$ in terms of continuous function $\gamma(t)$ by

$$C(\lambda)=\frac{1+l(\lambda)}{M}\int_0^a\gamma(t)\sin(\lambda t)\,dt \tag{22}$$

and then substitute the expression into equations (20), we find that equation $(20)_2$ is satisfied identically, and the remaining one, after some manipulations, results in the Fredholm integral equation of the second kind for the unknown function γ

$$\gamma(r)+\frac{2}{\pi}\int_0^a\gamma(t)\mathrm{K}\,(r,t)\,dt=-\frac{2}{\pi}Mpr-\frac{2}{\pi}\int_0^\infty\lambda\left[1+l(\lambda)\right]F(\lambda)\sin(\lambda r)\,d\lambda, \quad 0\le r<a \tag{23}$$

with the kernel given by

$$\mathrm{K}\;(r,t)=\int_0^\infty l(\lambda)\sin(\lambda t)\sin(\lambda r)\,d\lambda. \tag{24}$$

The derived equation (23) can be solved numerically for particular shapes of the dip.

Contact characteristics in terms of function γ

In context of contact mechanics, we are particularly interested in calculating the contact normal pressure distribution $\sigma_{zz}(r,0),\ r>a$, the height of the created gap $H(r)=u_z^{(2)}(r,0)-u_z^{(1)}(r,0)-f(r)$ and the radius of the gap a. It can be shown that these parameters are determined by the solution γ to the Fredholm integral equation (23). Thus, the contact traction and the gap's height are given by

$$\begin{aligned}
\sigma_{zz}&=-p-\frac{1}{M}\int_0^\infty \lambda^2[1+l(\lambda)]F(\lambda)J_0(\lambda r)\,d\lambda-\frac{1}{M}\int_0^a\frac{\gamma'(t)\,dt}{\sqrt{r^2-t^2}}+\\
&\quad+\frac{1}{M}\frac{\gamma(r)}{\sqrt{r^2-a^2}}-\frac{1}{M}\int_0^a\gamma(t)\int_0^\infty \lambda\, l(\lambda)\sin(\lambda t)J_0(\lambda r)\,d\lambda\,dt,\\
H(r)&=\int_r^a\frac{\gamma(t)}{\sqrt{t^2-r^2}}dt.
\end{aligned} \tag{25}$$

In order to evaluate a, we use the condition of smooth passage of gap's faces $H'(a)=0$ that yields

$$\gamma(a)=0. \tag{26}$$

It is relevant to remark here that this condition ensures boundedness of contact traction at $r=a$.

NUMERICAL RESULTS

Let us present a method for the numerical solution of governing integral equation (23) for the case of initial dip in the form

$$f(r)=\begin{cases}-H_0\left(1-\dfrac{r^2}{b^2}\right)^{3/2}, & 0\le r\le b\\ 0, & r\ge b\end{cases},\qquad H_0 \ll b. \tag{27}$$

It is worth to note here that for this shape the curvature $\kappa=f''(r)\left[1+\left(f'(r)\right)^2\right]^{-3/2}$ has a discontinuity of the second kind at $r=b$ since $\lim\limits_{r\to b-}\kappa=+\infty,\ \lim\limits_{r\to b+}\kappa=0$.

The unknown function $\gamma(r)$ is sought in the class of polynomials of odd degree, i.e.

$$\gamma(r)\approx\gamma_k(r)=c_1r+c_2r^3+\ldots+c_kr^{2k-1}. \tag{28}$$

The reason for the choice of such approximation functions is that for the limiting case when the layer degenerates into a half-space ($h=\infty$) the equation (23) has a known analytical solution (see [4]) expressed by the polynomial of 3rd degree

$$\gamma(r) = \frac{2}{\pi} Mpr + \frac{3H_0}{2b}\left(r - \frac{r^3}{b^2}\right). \tag{29}$$

The unknown coefficients of the polynomial (28) can be determined from a system of k linear algebraic equations by employing the collocation method. These equations are obtained by satisfying the integral equation (23) in k collocation points chosen uniformly in the interval $(0, a)$. The accuracy of approximation is achieved by the iteration process. For every step of the iteration the approximation polynomial is found. Going to the next step, the degree of the polynomial increases by two. The iteration process is continued until the following condition is satisfied

$$\max_{r\in(0,\,a)} \left| \frac{\gamma_k(r) - \gamma_{k+1}(r)}{\gamma_k(r)} \right| < 0.005 \,. \tag{30}$$

In Table 1, for the given accuracy, the degrees of the approximation polynomial $n = 2k - 1$ have been arranged depending on the thicknesses of the layer h. It is seen that the thinner is the layer, the higher is the degree of the approximation polynomial. Conversely, with an increase in h the degree n can be smaller.

Table 1. The approximation polynomial degree n versus the thickness parameter h/b

h/b	0.4÷0.6	0.7÷0.9	1	2	∞
n	11	9	5	3	3

Some numerical results for contact parameters are illustrated in Figures 2-4. All calculations were carried out for geometric quantities related to the radius of the initial recess b and for the pressure and stresses related to m_2. Moreover, the simplifying assumption of identical materials $m_1 = m_2$ have been used.

Fig. 2 shows the relationship between the threshold load p_{cr} (i.e. the smallest value of the external pressure for which the gap disappears) and the thickness of the layer. It is seen that as the parameter h increases, p_{cr} monotonically increases and asymptotically approaches the value $1.1781 \cdot 10^{-3}\, m_2$ (the dotted line) that correspond to the case when the layer degenerates into a half-space. One may also note that for $h/b = 2$ the value of the threshold pressure differs from that of the half-space less than 1%.

Fig. 2. Dependence of the threshold pressure on the thickness of the layer

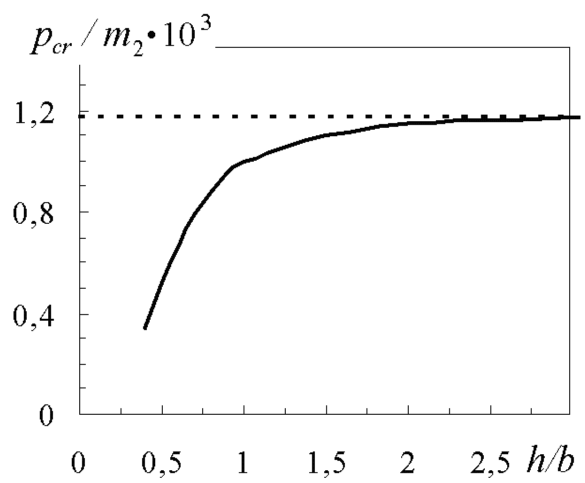


Fig. 3 demonstrates the change of gap radius a under the external load. It can be seen that for the thinner layer the gap becomes smaller for a fixed external load.

Fig. 3. Variations of gap radius under external load for different thicknesses of the layer

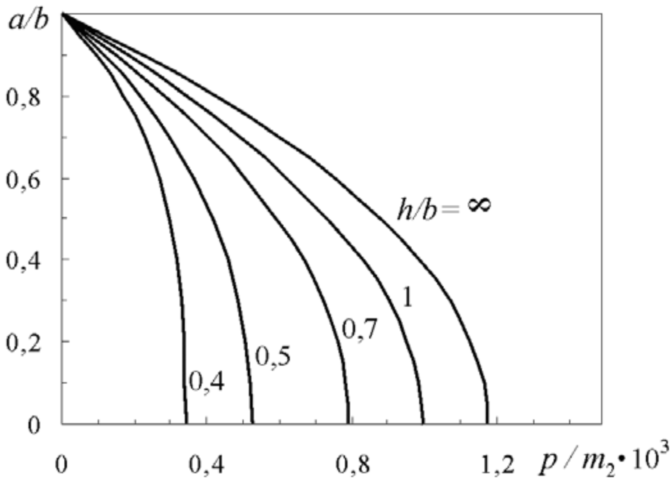
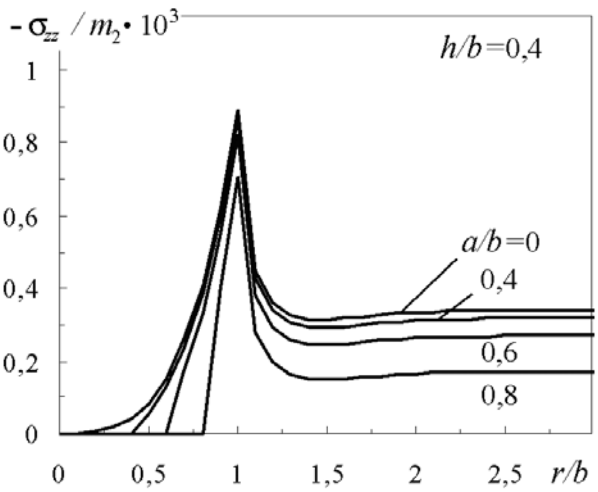


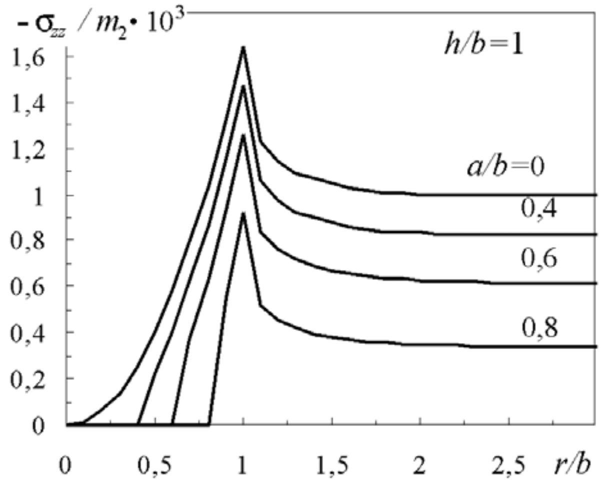
Fig. 4a-c show the contact pressure distributions for various values $h/b: 0,4;\ 1;\ \infty$, respectively, and for different values of gap's parameters $a/b: 0;\ 0,4;\ 0,6;\ 0,8$. It is worth noting that for the thin layers (Fig. 4a) the contact pressure has a local minimum at $r > b$. This can be explained by the bending of the layer in the vicinity of the initial dip tip and is not observed for the thicker layer (Fig. 4b). A qualitative similarity in graphs of the contact pressure between the thick layers (Fig. 4b) and the half-space (Fig. 4c) is noticed as well. Note that the contact stresses $\sigma_{zz}(r,0)$ (see Fig. 4 a-c) exhibit "sharp peaks" at $r = b$. Such unexpected behavior can be explained by the specific geometry of the considered dip surface. Probably the reason is that the point $r = b$ is a point of discontinuity of the second kind of the curvature of function (27). This statement has not been strictly proved yet. However, it is confirmed by the analytical solution to the similar contact problem of two half-spaces, obtained in [6]. Moreover, in the case of the dip surface with the continuous curvature at $r = b$ such "sharp peaks" are not observed (see [2]).

Fig. 4. Dependences of the contact stress for different values of gap sizes and thicknesses of the layer: a) $h/b = 0,4$, b) $h/b = 1$, c) $h/b = \infty$

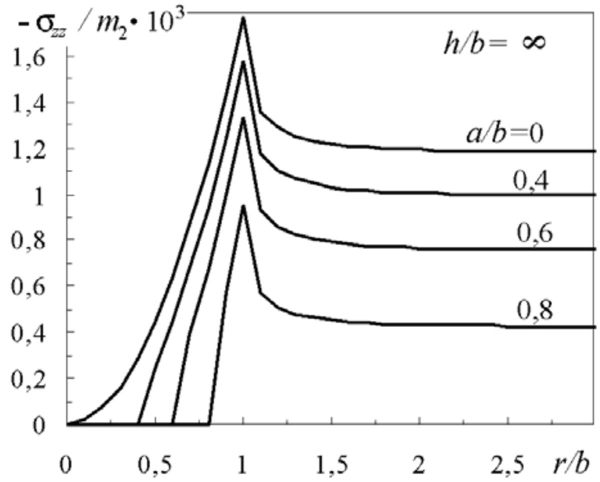
a)



b)



c)



CONCLUSIONS

This paper shows how the outer boundary of the layer can influence on the frictionless layer-half space contact behavior. The effect of the layer thickness on the size of the gap and the distribution of contact stresses has been analyzed. Numerical analysis of the solution to the problem revealed that the essential differences in contact characteristics were manifested for thinner layers.

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