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VIBRATIONS OF STRINGS WITH ARBITRARY LARGE DISPLACEMENTS

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ABSTRACT

The paper considers arbitrary large deflections of strings and connected with it coupling of vertical and horizontal displacements. Strings are made of a linearly elastic material. The physical model of a tie rod, equations and algorithm of their solutions as well as examples of string analysis are presented. Obtained results are compared to analytical solutions already known but for small displacements. Presented examples are calculated using an Author's program.

Key words: structural mechanics, tie rod (tension member), string, cable - suspended structures, dynamics.

1. INTRODUCTION

Tie rod structures are widespread in different domains of technique. Increasing requirements given to contemporary engineering structures concerning their spans as well as economical considerations favour that cable-suspended structures which makes them more and more widespread. One of the reasons limiting their practical applications are some difficulties connected with calculation of such structures, which results from great non-linearity of equations which describe the tie rod systems. In this context, one of the main problems is to search for proper methods for solution of equations describing statics or dynamics of these systems.

Designing of tie rod structures is a subject of many scientific papers [1÷6]. The problem was repeatedly contemplated and continue to be the object of interesting works, which frequently link theoretical and experimental researches [7÷10]. The complexity of this problem is so high that univocal presentation in form of universal models, formulas and equations may not be possible. Hence the wealth of literature related to this problem.

The geometrical non-linearity causes that the analytical solution can be obtained only for elementary cases. The effective solutions are searched for by application of numerical methods using computers, e.g. [11÷15]. One of elementary examples of tie rod structures are strings. String is a single tie rod, preliminarily tightened, of a slight dead weight. Negligible dead load and preliminary tension cause that unloaded string lays along a straight line. The classical solution of the string is found by assumption of small deflections charged e.g. [13].

The subject of the paper is dynamics of a string made of an elastic material. The aim of the investigation is simulation of arbitrary large vibrations of the string.

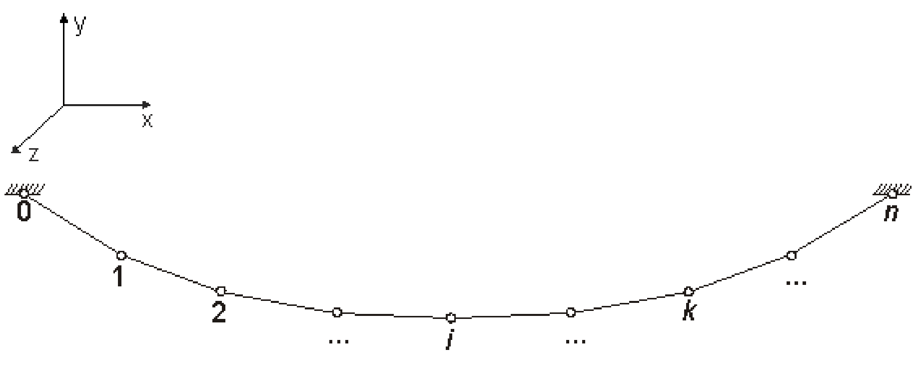
EQUATIONS OF MOTION

Differential equations of motion of tie rod systems were accurately discussed by the Author in [14]. The limp string made of an elastic material is considered. Each tension member of tie rod systems is fashioned in a shape of a chain composed of a finite number of straight bars connected in nodes by means of ideal pivot bearings (Fig. 1). This process may be called discretization of the tie rod, which leads up to division of the tie rod into finite elements FE (see [15]). The string is able to endure any possible displacement, however it is assumed, that its strains are infinitesimal. The loads acting upon a string can vary in space and time. The change of temperature may also cause a load of the string. The string may be preliminarily tensioned. We are in search for functions which describe mainly the axial forces $\underline{N}(x, y, z, t)$ and displacements $\underline{u}(x, y, z, t)$.

The following assumptions are made:

1) the material of the string is linearly elastic,
2) the string is able to transfer tensile forces only,
3) the string can assume arbitrary large displacements,
4) loads of the system form concentrated forces $P(x, y, z, t)$ acting in an arbitrary direction, temperature $T(x, y, z, t)$ and preliminary tension of the tie rod arrangement,
5) variability of boundary conditions in time and local changes connected e.g. with a change of the string length are admissible,
6) the preliminary configuration is known.

Fig. 1 Fashioned tie rod in a form of n straight bars



Equations of motion for an arbitrary node "i " of a single tie rod (string here) have the following form [14]:

$$
\begin{gathered}
\sum_{k=1}^{m} E_{ik} A_{ik} \left(\frac{l_{ik}^{t} - l_{ik}^{o}}{l_{ik}^{o}} - \alpha_{ik} T_{ik} - \varepsilon_{ik}^{\Delta} \right) \frac{x_k^t - x_i^t}{l_{ik}^t} + P_{x_i}^t = m_i \ddot{x}_i^t , \\
\sum_{k=1}^{m} E_{ik} A_{ik} \left(\frac{l_{ik}^{t} - l_{ik}^{o}}{l_{ik}^{o}} - \alpha_{ik} T_{ik} - \varepsilon_{ik}^{\Delta} \right) \frac{y_k^t - y_i^t}{l_{ik}^t} + P_{y_i}^t = m_i \ddot{y}_i^t , \\
\sum_{k=1}^{m} E_{ik} A_{ik} \left(\frac{l_{ik}^{t} - l_{ik}^{o}}{l_{ik}^{o}} - \alpha_{ik} T_{ik} - \varepsilon_{ik}^{\Delta} \right) \frac{z_k^t - z_i^t}{l_{ik}^t} + P_{z_i}^t = m_i \ddot{z}_i^t , \\
t = 1, 2, \ldots, T \ .
\end{gathered}
\tag{1}
$$

where: E_{ik} is the coefficient of elasticity (Young's modulus), A_{ik} - cross section area, l_{ik}^{0} - length of FE in the initial configuration, l_{ik}^{t} - length at a moment *t*, α_{ik} -coefficient of linear thermal expansion, T_{ik} - increase of temperature, $\varepsilon_{ik}^{\Delta}$ - preliminary deformation (initial), x_i^t, y_i^t, z_i^t - co-ordinates of a node, $P_{x\,i}^t, P_{y\,i}^t, P_{z\,i}^t$ - components of nodal forces.

Equations (1), which describe motion of the tie rod system nodes for all joints, ought to be composed, except from the supporting nodes. The boundary and initial conditions are the completion of this set of equations.

SOLUTION TO EQUATIONS OF MOTION FOR TIE ROD SYSTEMS – – NUMERICAL (COMPUTER) METHOD

Equations (1) are characterised by strong geometrical non-linearity. An iteration method for solving such systems has been elaborated.

The first stage of calculations is the determination of the initial configuration of the tie rod system. One assumed that it is a state of displacements of the system from the initial load acting statically (e.g. dead load, temperature of assembly, preliminary tension of the structure and others). In such a case, equations of motion (1) are changing into static equilibrium conditions of forces.

$$
\begin{gathered}
\sum_{k=1}^{m} E_{ik} A_{ik} \left(\frac{l_{ik} - l_{ik}^{o}}{l_{ik}^{o}} - \alpha_{ik} T_{ik} - \varepsilon_{ik}^{\Delta} \right) \frac{x_k - x_i}{l_{ik}} + P_{x_i} = 0, \\
\sum_{k=1}^{m} E_{ik} A_{ik} \left(\frac{l_{ik} - l_{ik}^{o}}{l_{ik}^{o}} - \alpha_{ik} T_{ik} - \varepsilon_{ik}^{\Delta} \right) \frac{y_k - y_i}{l_{ik}} + P_{y_i} = 0, \\
\sum_{k=1}^{m} E_{ik} A_{ik} \left(\frac{l_{ik} - l_{ik}^{o}}{l_{ik}^{o}} - \alpha_{ik} T_{ik} - \varepsilon_{ik}^{\Delta} \right) \frac{z_k - z_i}{l_{ik}} + P_{z_i} = 0.
\end{gathered}
\tag{2}
$$

At the *n-th* iteration for the consecutive node „*i*", equations (2) reduce to the linear form through admission parts of co-ordinates of this node from the iteration *n*-1 (in the case of the first iteration, one accepts co-ordinates of the initial configuration):

$$x_i = \frac{\sum_{k=1}^{m} x_k E_{ik} A_{ik} \left[\frac{1}{l_{ik}^0} - \frac{1}{l_{ik}} \left(1 + \alpha_{ik} T_{ik} + \varepsilon_{ik}^{\Delta} \right) \right] + P_{x_i}}{\sum_{k=1}^{m} E_{ik} A_{ik} \left[\frac{1}{l_{ik}^0} - \frac{1}{l_{ik}} \left(1 + \alpha_{ik} T_{ik} + \varepsilon_{ik}^{\Delta} \right) \right]} \;,$$

$$y_i = \frac{\sum_{k=1}^{m} y_k E_{ik} A_{ik} \left[\frac{1}{l_{ik}^0} - \frac{1}{l_{ik}} \left(1 + \alpha_{ik} T_{ik} + \varepsilon_{ik}^{\Delta} \right) \right] + P_{y_i}}{\sum_{k=1}^{m} E_{ik} A_{ik} \left[\frac{1}{l_{ik}^0} - \frac{1}{l_{ik}} \left(1 + \alpha_{ik} T_{ik} + \varepsilon_{ik}^{\Delta} \right) \right]} \;, \qquad (3)$$

$$z_i = \frac{\sum_{k=1}^{m} z_k E_{ik} A_{ik} \left[\frac{1}{l_{ik}^0} - \frac{1}{l_{ik}} \left(1 + \alpha_{ik} T_{ik} + \varepsilon_{ik}^{\Delta} \right) \right] + P_{z_i}}{\sum_{k=1}^{m} E_{ik} A_{ik} \left[\frac{1}{l_{ik}^0} - \frac{1}{l_{ik}} \left(1 + \alpha_{ik} T_{ik} + \varepsilon_{ik}^{\Delta} \right) \right]} \;.$$

Based on the thus linearized equations, amendments of co-ordinates of the *i-th* node are designated. Changed co-ordinates in calculations of corrections of consecutive nodes are used. This process is repeated until equilibrium in all nodes of the system with the assumed accuracy would be secured. The thus accepted algorithm of calculations requires, apart from typical data such as length and rigidity of all bars of the system, loads and boundary conditions, also that the starting configuration, i.e. preliminary co-ordinates of all variable nodes are known. How this configuration differs from the determined one, the so called initial configuration, has influence upon the number of iterations and time of calculations. Starting even from very "curious" initial configuration, the iteration process occurs convergent and allows one to find quick and highly accurate solution.

The next stage are calculations caused by dynamical loads. A system of non-linear differential equations has to be solved (1). These are coupled ordinary differential equations. For solution of sets of equations, a numerical method of direct integration equations of motion, i.e Newmark's method was applied. This is a well developed method for solving linear systems of equations. In the case of non-linear equations, an adaptation of this method is indispensable. The methods using the so called Newmark's incremental method for solving differential non-linear sets of equations one can find in literature, e.g. [13]. In the case of equations (1), a special adaptation considering specificity of this arrangement is indispensable. Expressions $m_i \ddot{u}_i, m_i \ddot{v}_i, m_i \ddot{w}_i$ are substituted by equivalent expressions $m_i \ddot{x}_i, m_i \ddot{y}_i, m_i \ddot{z}_i$. Equations (1) are set for all nodes, excluding bearing joints. These are non-linear ordinary differential equations of the second order. The unknown in these equations are values of co-ordinates x_i^t, y_i^t, z_i^t, $i = 1,2,\ldots,n$ – nodes of tie rod systems, $t = 1,2,\ldots,T$ - nodes at the discretized time axis. At the moment $t = 0$, the initial boundary conditions referred to strains and forces are known. In equations (1), accelerations of displacements appear. These quantities are described by approximation used in Newmark's method.

$$\dot{f}_j^{\,t} = \frac{\delta}{ah}\left(f_j^{\,t} - f_j^{\,t-1}\right) + \left(1 - \frac{\delta}{\alpha}\right)\dot{f}_j^{\,t-1} + h\left(1 - \frac{\delta}{2\alpha}\right)\ddot{f}_j^{\,t-1},$$
$$\ddot{f}_j^{\,t} = \frac{1}{\alpha h^2}\left[f_j^{\,t} - f_j^{\,t-1} - \dot{f}_j^{\,t-1} h + h^2\left(\frac{1}{2} - \alpha\right)\ddot{f}_j^{\,t-1} \right], \qquad (4)$$
$$j = i,k \,, \qquad f = x,y,z \,,$$

where *h* is the step of integration, α, δ are parameters of the method.

Next formulae (4) one substitutes to (1) which yields the following recurrent formulae:

$$f_i^{\,t} = F_i^{\,t}\left(f_i^{\,t}\right) + G_i^{\,t-1}\left(f_i^{\,t-1}, \dot{f}_i^{\,t-1}, \ddot{f}_i^{\,t-1}, N_{ik}^{t-1}, \dot{N}_{ik}^{t-1}\right) + H_i^{\,t}\left(P^t\right),$$
$$t = 1,2,\ldots,T, \quad f = x,y,z. \qquad (5)$$

For properly selected parameters α, δ, the recurrent process is unconditionally stable. Complexity of these formulae results from non-linearity which arises from the presence of unknown co-ordinates f_i^t from the left and right sides of the formulae. The function G_i^{t-1} is a known function, determined at the moment $t-1$, but the function H_i^t depends upon known forces which cause vibration of tie rod systems. In the case of a geometrically linear problem, the function F_i^t equals to zero. A somewhat more accurate algorithm was presented in [14]. The method presented above has been used in Author's computer program. This program allows one to intercept strains of nodes and forces in bars of tie rod systems as a function of time *t*.

EXAMPLES OF CALCULATIONS

Example 1.

A preliminarily tensioned elastic string (Fig. 2) loaded in the mid-span by time variable force $P(t) = P_0\left[1 - H(t)\right]$ is considered, where $P_0 = 20$ N, $H(t)$ is Heaviside's function:

$$H(t) \overset{def}{=} \begin{cases} 0 & dla \quad t < 0 \\ 1 & dla \quad t > 0 \end{cases}.$$

String's data:

a) span (length) – l = 60 cm,
b) cross-section area (ϕ = 0.1 mm) – $A = 3.1416 \cdot 10^{-8}$m^2,
c) Young's modulus E = 205 GPa,
d) string mass $\mu = 2.46615 \cdot 10^{-4}$ kg/m (total mass $1.47969 \cdot 10^{-4}$ kg),
e) preliminary string tension - 1.13652 kN.

Fig. 2. Scheme of the string

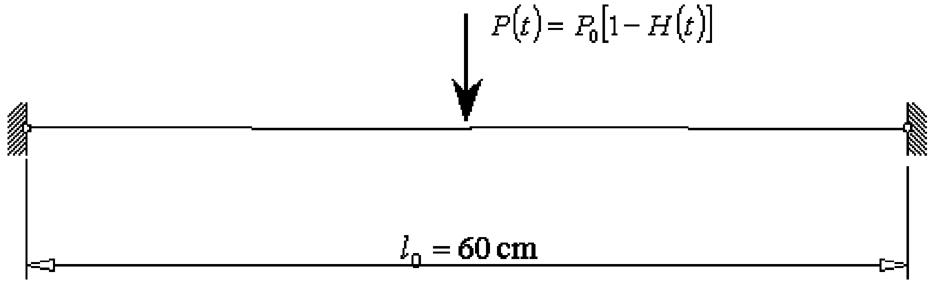
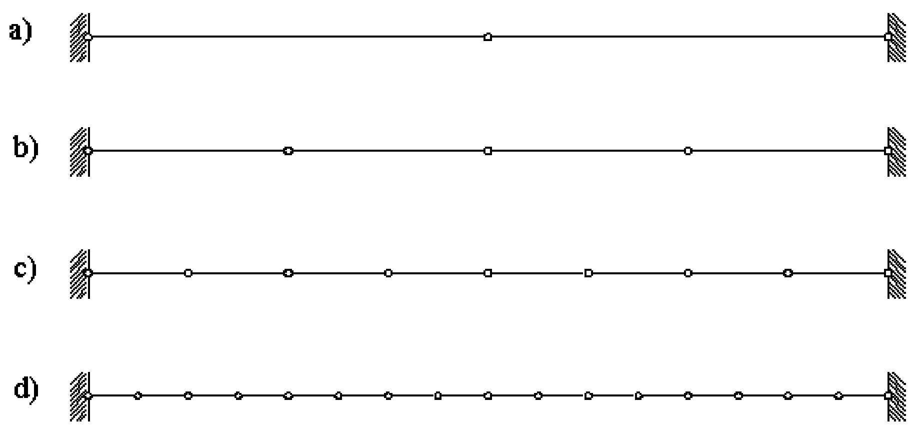


Fig. 3. Discretization of the string: a) in 2 finite elements, b) in 4 finite elements, c) in 8 finite elements, d) in 16 finite elements



For calculations, a discretizated model of the string is applied (Fig. 3). Division into 2, 4, 8, 16, 60 i 120 bar sections (finite elements) is considered. The distributed mass of the string is concentrated in nodes. The step of integration $h = 2 \cdot 10^{-5}$ *s* is assumed. Time of string observations amounted to 2,5 seconds (125 000 moments).

A case of string transverse vibrations with small deflections was examined in paper [13]. The exact analytical solution of such a case expresses a function of vertical displacements of the following form:

$$w(x,t)=\frac{2P}{Sl}\sum_{n=1}^{\infty}\frac{\sin\alpha_n x\sin\alpha_n\xi}{\alpha_n^2}\cos\omega_n t\,,$$

where:

$\omega_n = c\alpha_n$, $c=\sqrt{\frac{S}{\rho A}}$, $\alpha_n = \frac{n\pi}{l}$, $n = 1,2,\dots$,

S – preliminary string tension,
ρ – density of the string material,
A – area of the string cross-section,
l – span of the string.

To secure that Author’s calculations correspond to assumptions of analytical calculations, the value of the force P_0 is selected in such a way that the maximum deflection does not exceed $1/200$ of its length. On the basis of this premise a force of P_0 = 20 N has been assumed in calculations.

The comparison of concordance of vibration periods as a function of numbers of finite elements refered to analytical solutions is presented in Tab. 1

Table 1. Vibration periods of the string [s]

Number of finite elements	Period of vibrations		Coincidence of calculations [%]
	Analitical solutions [13]	Computer solutions	
2	0.017677721	0.019633	9.96
4		0.019051	7.21
8		0.017420	1.48
16		0.017855	0.99
60		0.017826	0.83
120		0.017671	0.04

Table 2 contains a comparison of amplitudes of the middle string point (amplitude diagrams of 1 and 5 peaks $w(t)$) that refers to the analytical solution as well.

Table. 2. Vertical displacements of the string (middle point) [m]

Number of finite elements	Initial configuration			Current configuration					
				Amplitude of peak 1			Amplitude of peak 5		
	Analytical solution [120]	Computer solution	Coincidence of calculations [%]	Analytical solution [120]	Computer solution	Coincidence [%]	Analytical solutions [120]	Computer solution	Coincidence [%]
2	0.00262	0.00264	0.79	-0.00255	-0.00264	3.37	-0.00253	-0.00264	4.21
4		0.00264	0.79		-0.00232	9.65		-0.00193	30.88
8		0.00264	0.79		-0.00244	4.35		-0.00212	19.31
16		0.00264	0.79		-0.00244	4.39		-0.00234	7.94
60		0.00264	0.79		-0.00256	0.58		-0.00250	1.00
120		0.00264	0.79		-0.00259	1.46		-0.00253	0.16

Vertical displacements of the string middle point are presented in Figs. 4 ÷ 6. Vertical displacements of the point situated in $1/4$ of the string span are presented in Figs. 7 ÷ 9.

Based on the calculations made the following conclusions may be formulated:

1. The number of elements of the discretized system affects accuracy of calculations. For 16 finite elements, an error of the period is about 1 %, and the amplitude error is 4 ÷ 8 %. For 60 elements, the amplitude error falls down below 1.5 %. In numerical calculations the amplitudes error increases and the period error stabilises.
2. In the compatibility analysis, the numerical results are referred to analytical solutions, however one has to remark that the analytical solution was obtained for two important simplifications assumed:
   - only transverse vibrations of the string appear,
   - longitudinal vibrations of the string are neglected.

   Such simplifications can only be assumed for small deflections of the string. In Author's numerical solution, a preliminary coupling of transverse and longitudinal vibrations appears. It means that, in this scope, the mathematical model applied by the Author is more accurate than the model applied in the analytical solution [13].
3. As a result of observations of vibration diagrams of the point situated in $\frac{1}{4}$ of the string span, characteristic flattening of the amplitude is noticed. Simultaneously, in more accurate numerical solutions (with discretization into 60 and 120 elements) gradual decay of the "flattening" is observed. The question is if the decay of "flattening" in the obtained solution may be a consequence of discretizations, accretion of errors in numerical recurential Newmark's processes, or it is a natural phenomenon whose deficiency in the analytical solution results from some idealisation of the mathematical model (see conclusions p. 2). In two consecutive examples, a trial of further testing of this phenomenon is to be presented.

Fig. 4. Comparison of middle point vibrations of the string divided into 60, 120 finite elements and the accurate solution [13] at time 0÷0.25 s

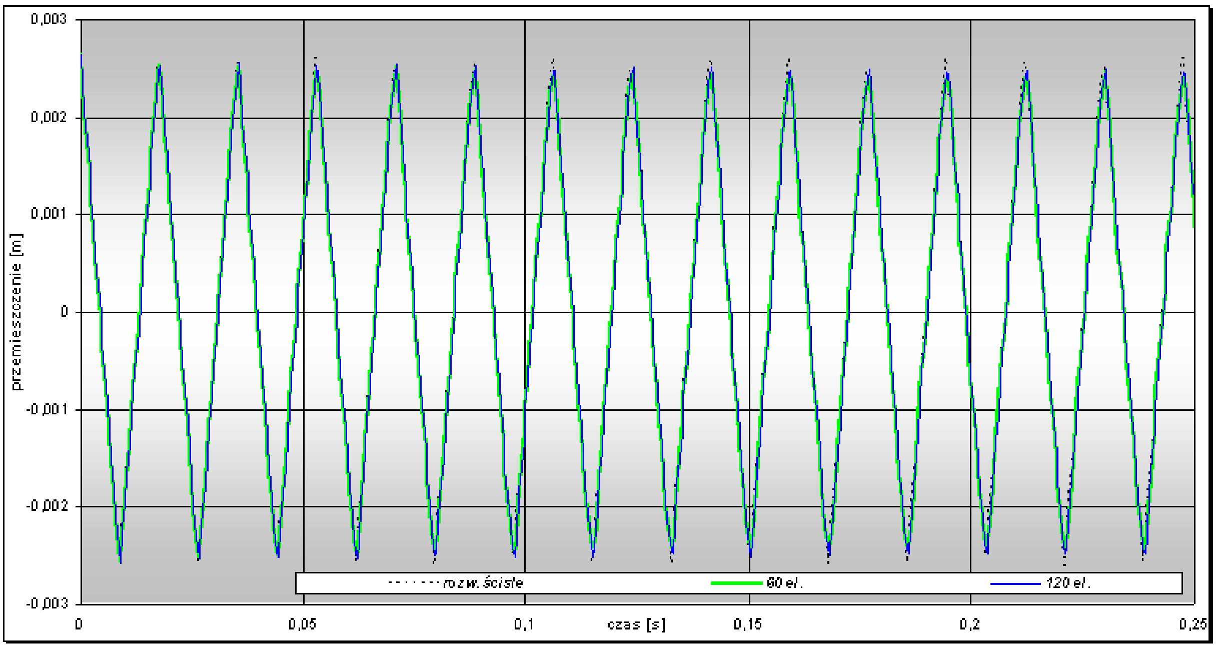


Fig. 5. Comparison of middle point vibrations of the string divided into 60, 120 finite elements and the accurate solution [13] at time 1÷1.25 s

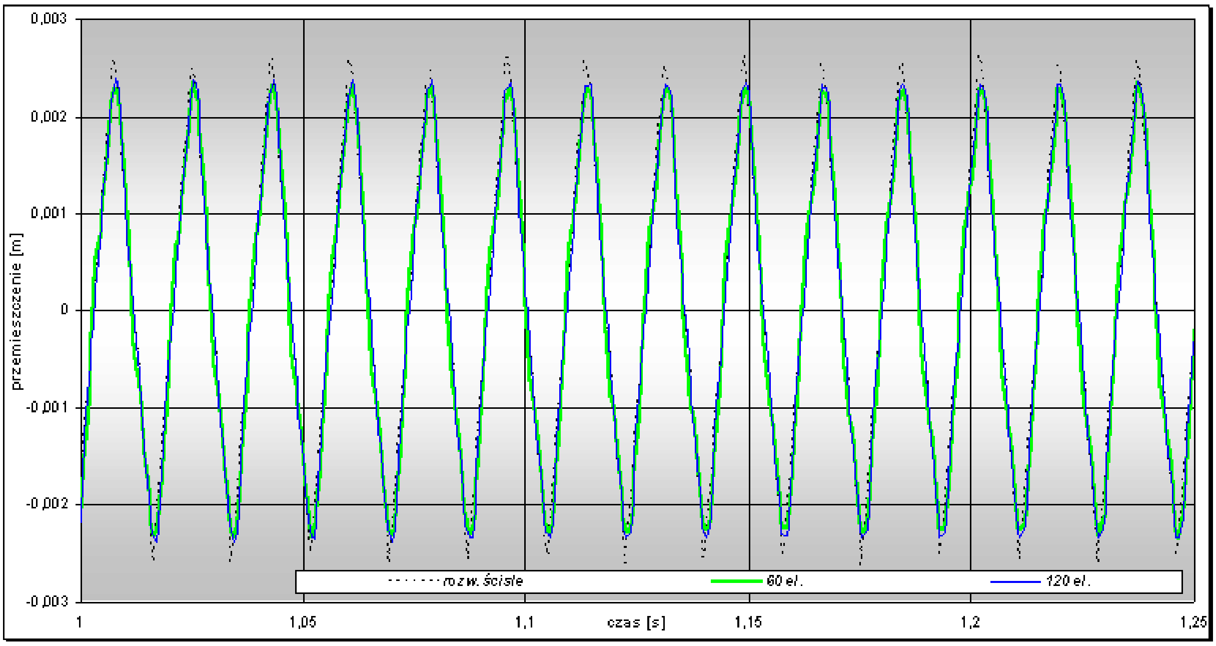


Fig. 6. Comparison of middle point vibrations of the string divided into 60, 120 finite elements and the accurate solution [13] at a time 2.25÷2.5 s

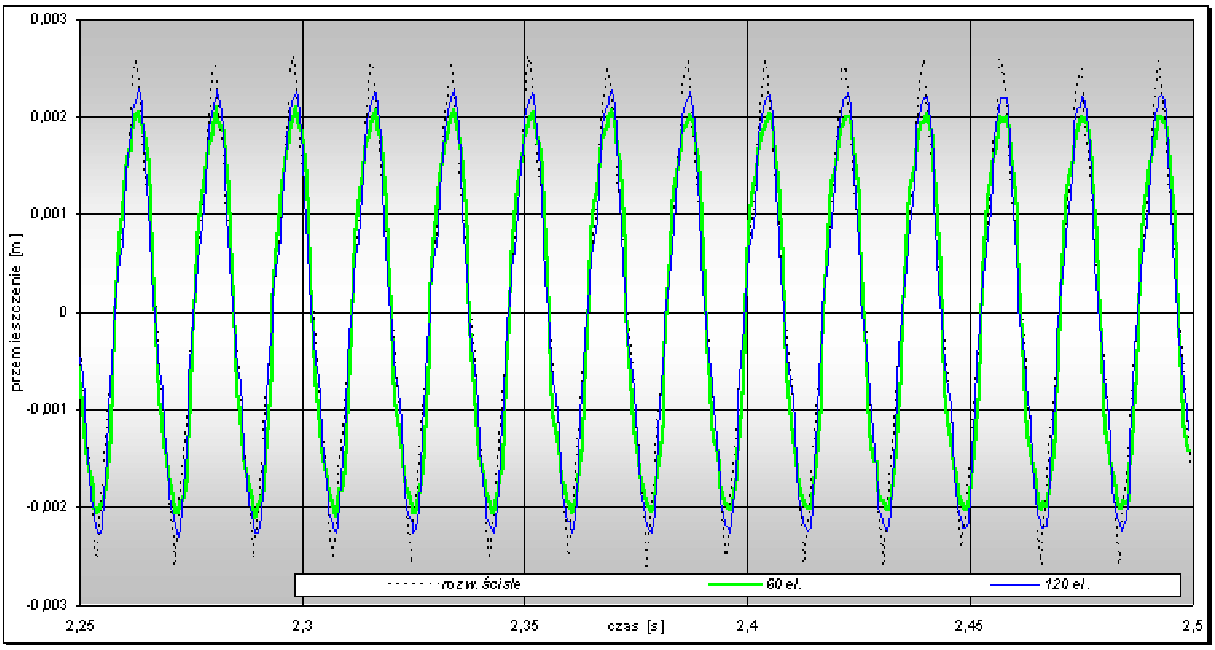


Fig. 7. Comparison of vibrations in the point situated in ¼ of the span of the string divided into 60, 120 finite elements and the accurate solution [13] at time 0÷0.25 s

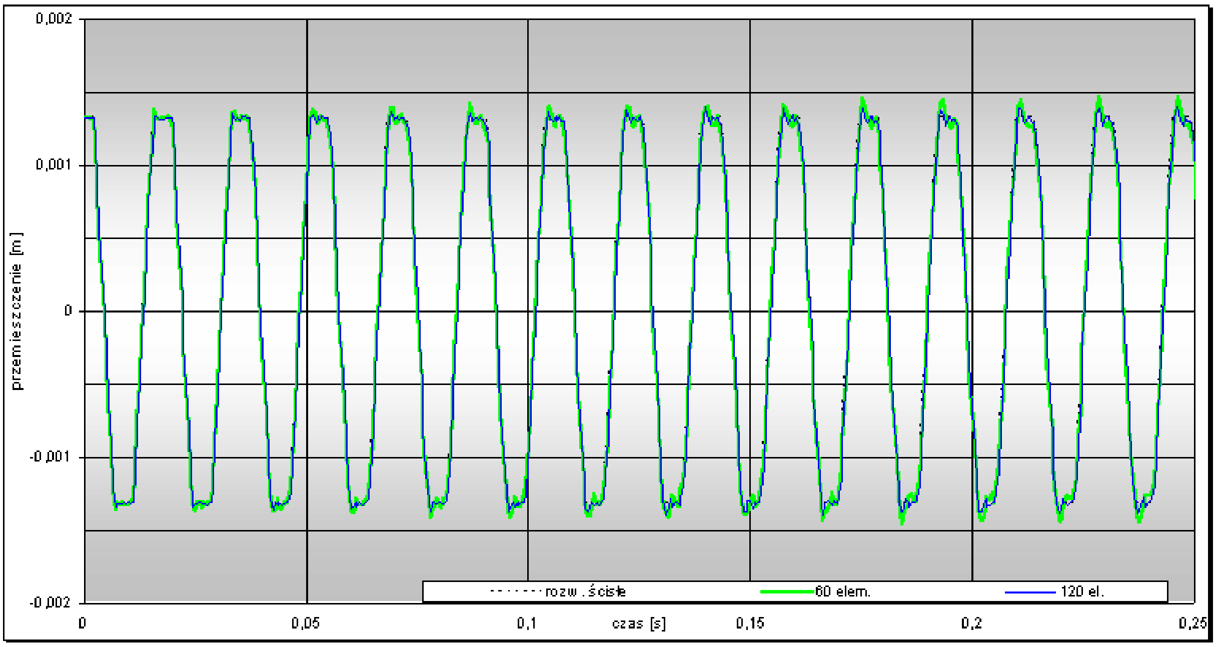


Fig. 8. Comparison of vibrations in the point situated in ¼ of the span of the string divided into 60, 120 finite elements and the accurate solution [13] at time 0÷1.25 s

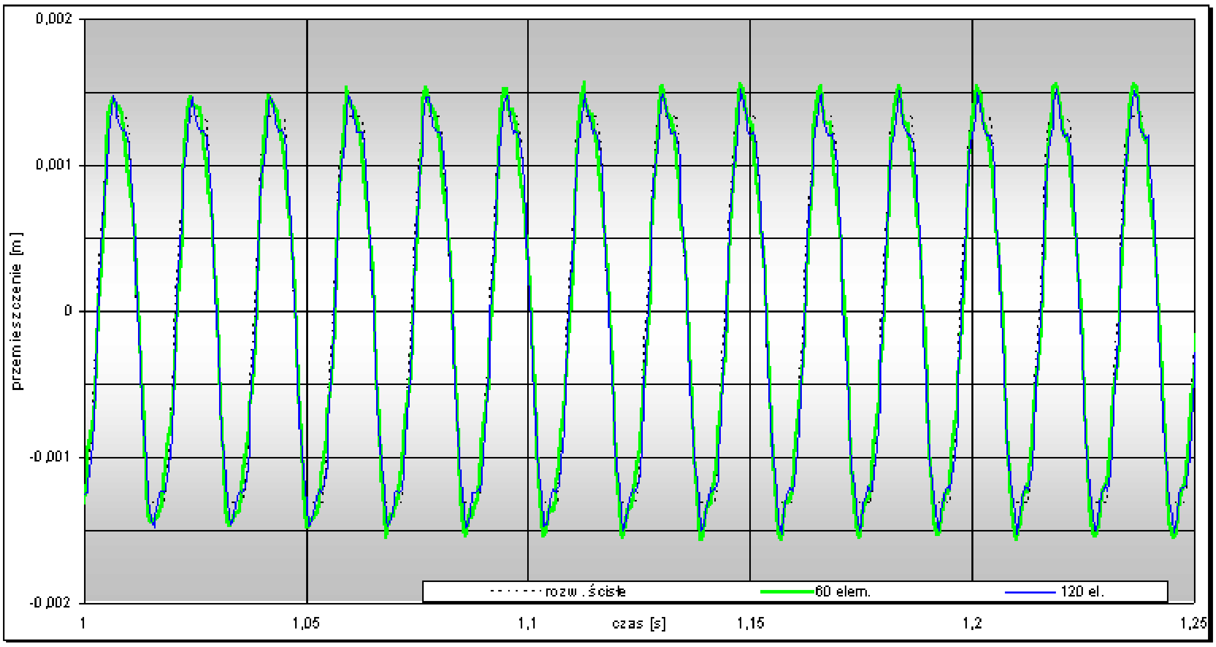
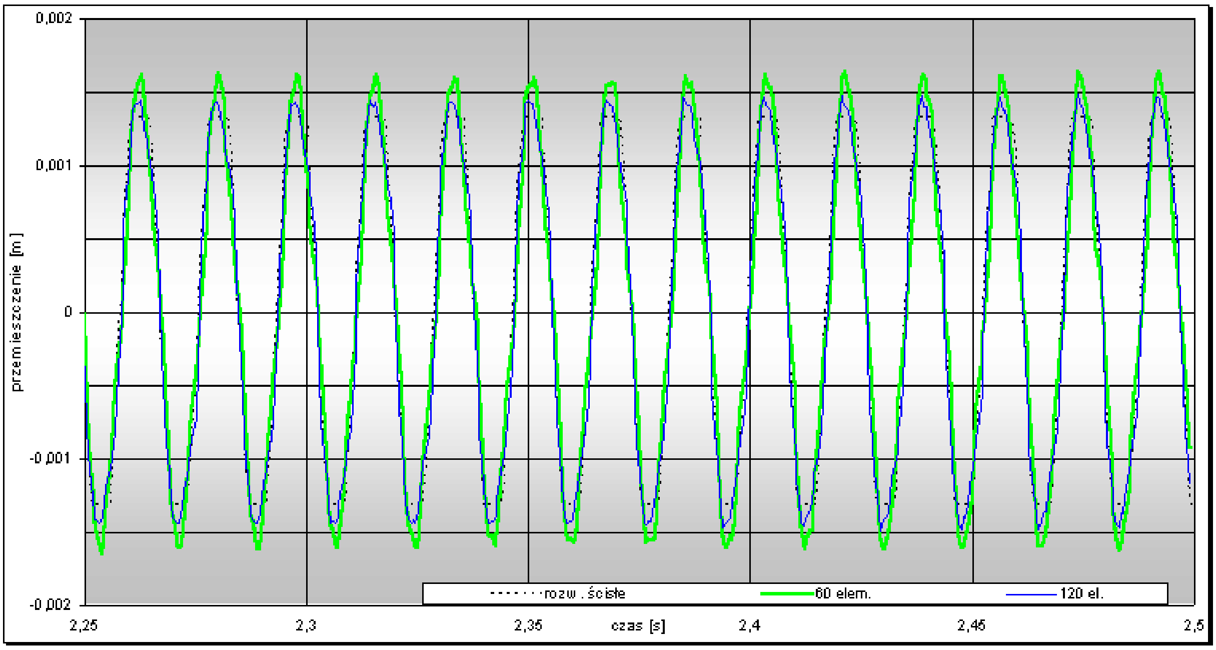


Fig. 9. Comparison of vibrations in the point situated in ¼ of the span of the string divided into 60, 120 finite elements and the accurate solution [13] at time 2.25÷2.5 s



Example 2

A preliminarily tensioned elastic string loaded at $\frac{1}{4}$ of the span by a time variable force $P(t) = P_0\left[1 - H(t)\right]$ is considered, where $P_0 = 5$ N.

The string data (identical with example 1):

a) span (length) – $l = 60$ cm,
b) cross-section area ($\phi = 0.1$ mm) – $A = 3.1416 \cdot 10^{-8}$ m^2,
c) Young's modulus $E = 205$ GPa,
d) String mass $\mu = 2.46615 \cdot 10^{-4}$ kg/m (total mass $1.47969 \cdot 10^{-4}$ kg),
e) Preliminary string tension - 1.13652 kN.

For calculations, a discretized model of the string is applied (Fig. 3). Division into 4, 8, 16, 60 and 120 rod elements (finite elements) is considered. The distributed mass of the string is concentrated in points of nodes. The step of integration $h = 2 \cdot 10^{-5}$ *s* is assumed. The time of strings observations is 2,5 seconds (125 000 moments).

The aim of calculations is the examination of "flattening effects" for very small amplitudes. The load P_0 is selected in such a way that the maximum string deflection does not exceed $\frac{l}{1000}$. On the basis of this premise, $P_0 = 5$ N is assumed in the calculations.

The comparison of accuracy of calculation of vibration periods with respect to assumed number of finite elements is presented in Tab. 3.

Table 3. Periods of string vibrations [s]

Number of finite elements	Period of vibrations		Coincidence [%]
	Analytical solution [13]	Computer solution	
4	0.01767587	0.01936727	8.73
8		0.01782161	0.82
16		0.01752763	0.85
60		0.01765412	0.12
120		0.01769333	0.10

A comparison of displacements at the middle point of the string is given in Tab. 4 (amplitudes of 1 and 5 peaks $w(t)$).

Vertical displacements of the string intermediate points are presented Figs. 10 ÷ 12. Vertical displacements of the point located in $\frac{1}{4}$ of the string span are shown in Figs. 13 ÷ 15.

Table 4. Amplitudes of verical displacements of the string (middle point) [m]

Number of finite elements	Initial configurations			Current configuration					
				Amplitude of peak 1			Amplitude of peak 5		
	Analytical splution [13]	Computer solution	Coincidence [%]	Analytical solution [13]	Computer solution	Coincidence [%]	Analytical solution [13]	Computer solution	Coincidence [%]
4		0.000495	1.07		-0.000251	33.95		0.000392	21.01
8		0.000495	1.07		-0.000223	25.65		0.000405	17.13
16	0.000490	0.000495	1.07	-0.000166	-0.000186	10.86	0.000474	0.000433	9.55
60		0.000495	1.07		-0.000169	1.90		0.000464	2.23
120		0.000495	1.07		-0.000166	0.12		0.000474	0.08

Based on the calculations made one formulates the following conclusions:

1. The number of elements of the discretized system effects the accuracy of calculations. Yet with 8 finite elements, the period error becomes less than 1 %. The amplitude error for 16 finite elements amounts to 10 ÷ 11 %, and for 60 elements about 2 %. In numerical calculations, the amplitude error increases and the period error stabilises.
2. In coincidence analyses, the numerical results are assigned to the analytical solution. It was obtained by some assumptions presented in example 1. One can state that in result of load reduction (P_0), which consequently resulted in reduction of amplitude, considerably smaller divergence between vibration periods calculated according to [13] and numerically was obtained. Slight worsening of the amplitude results can be explained by a load applied otherwise.
3. For a force acting in ¼ of the string span, characteristic flattening of amplitude peaks in vibration diagrams of individual points of the string are much more clearer (especially near the central point) than those observed in example 2. Simultaneously, a phenomenon of decay of the flattening observed in numerical solutions of example 2 is considerably slower. It is connected with better realisation of assumptions for analytical solutions (small amplitudes of vibrations). It is to be emphasised at the same time that the density of partition of the string into finite elements depends on degree of the flattening decay. The denser is the division of the structure, the larger are deflections beyond the flat segment of vibration diagram (compare diagrams of vibrations for division into 60 and 120-elements).

Fig. 10. Comparison of vibrations of the middle point of the string divided into 60, 120 finite elements and the accurate solution [13] at time 0÷0.25 s

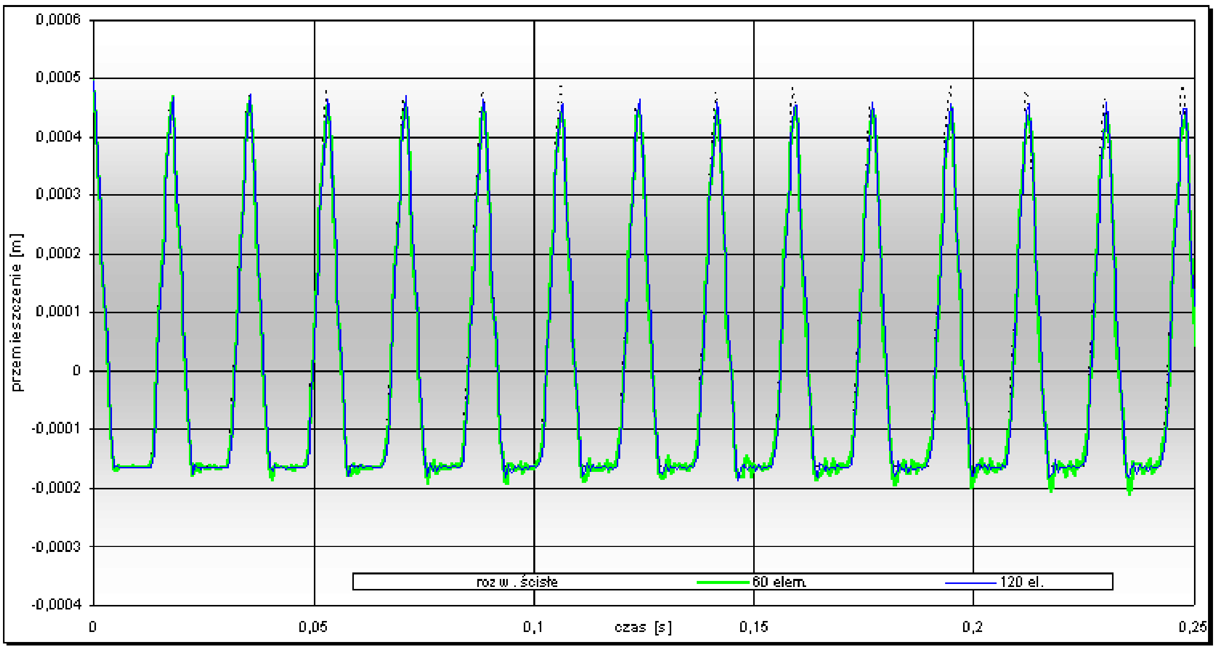


Fig. 11.Comparison of vibrations of the middle point of the string divided into 60, 120 finite elements and the accurate solution [13] at time 1÷1.25 s

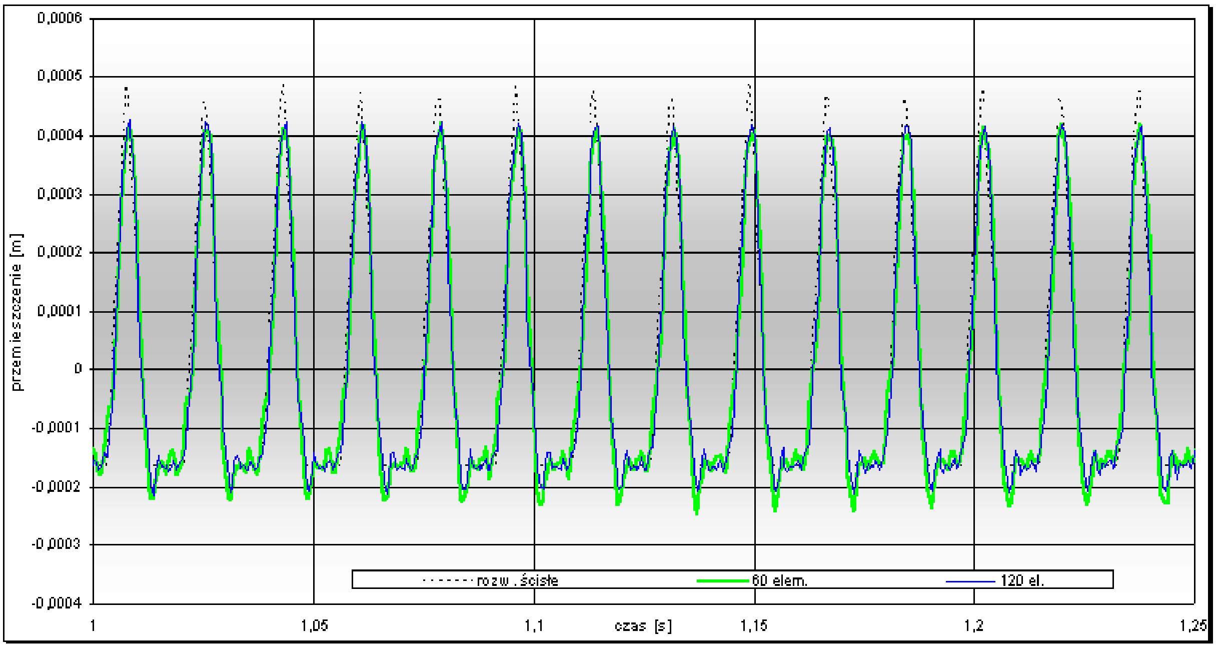


Fig. 12.Comparison of vibrations of the middle point of the string divided into 60, 120 [13] 120 finite elements and the accurate solution at time 2.25÷2.5 s

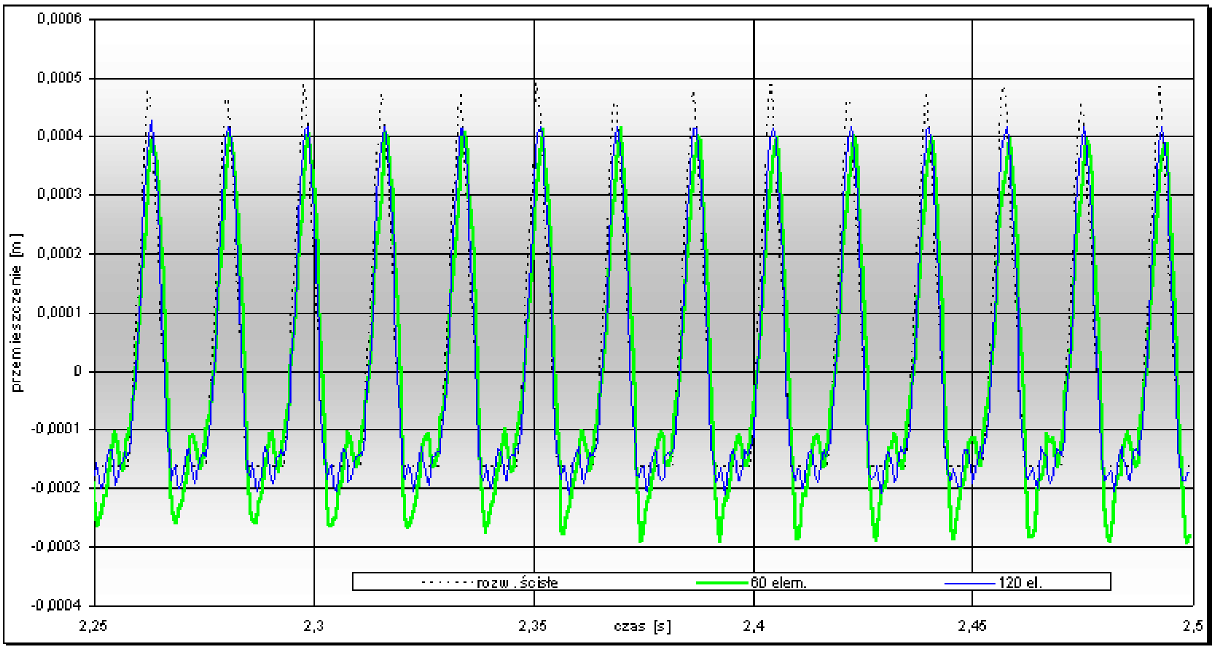


Fig. 13. Comparison of vibrations of the point situated in ¼ of the span of the string divided into 60, 120 finite elements and the accurate solution [13] at time 0÷0.25s

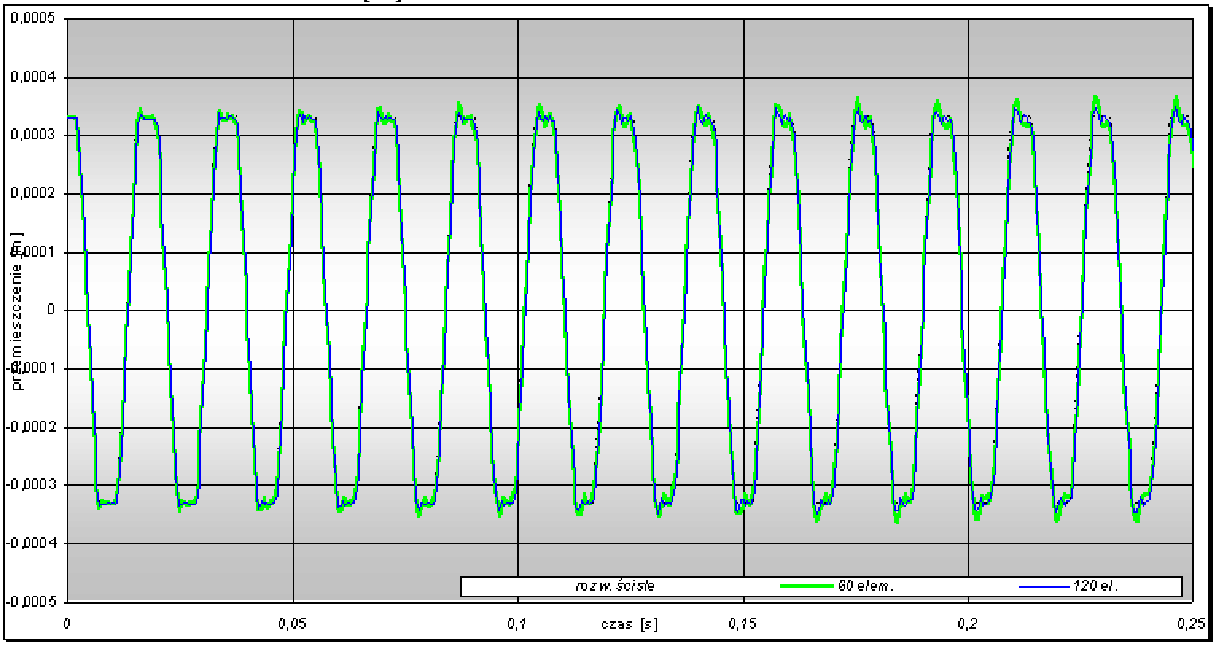


Fig. 14. Comparison of vibrations of the point situated in ¼ of the span of the string divided into 60, 120 finite elements and the accurate solution [13] at time 1.0÷1.25 s

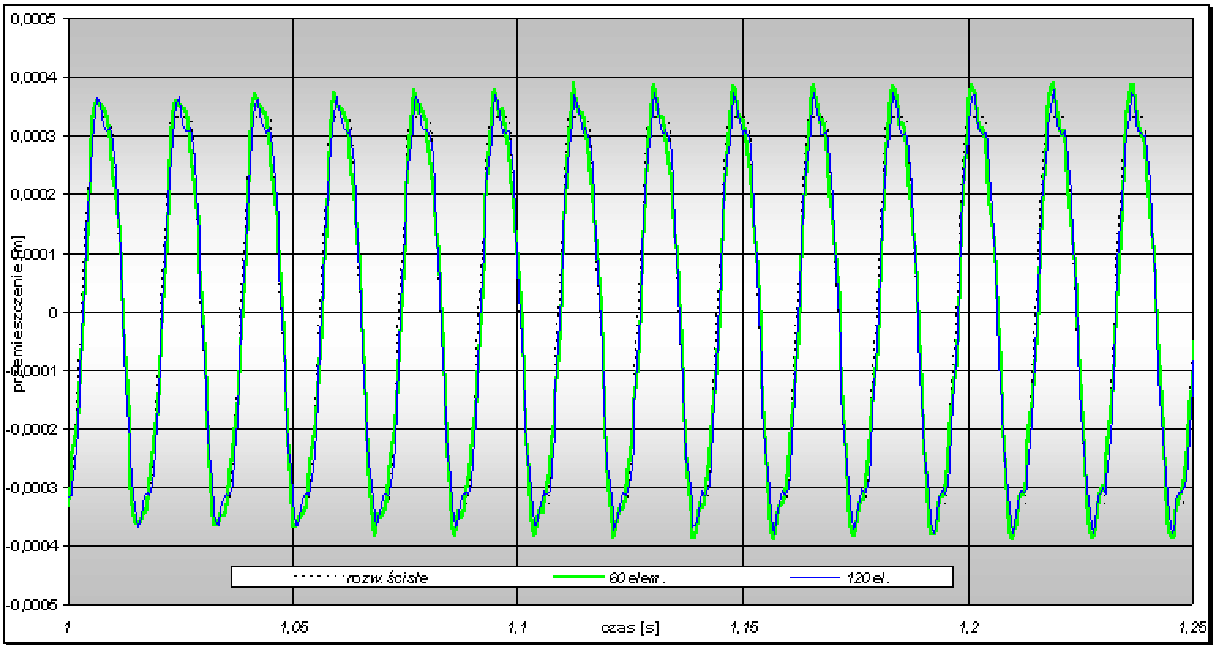
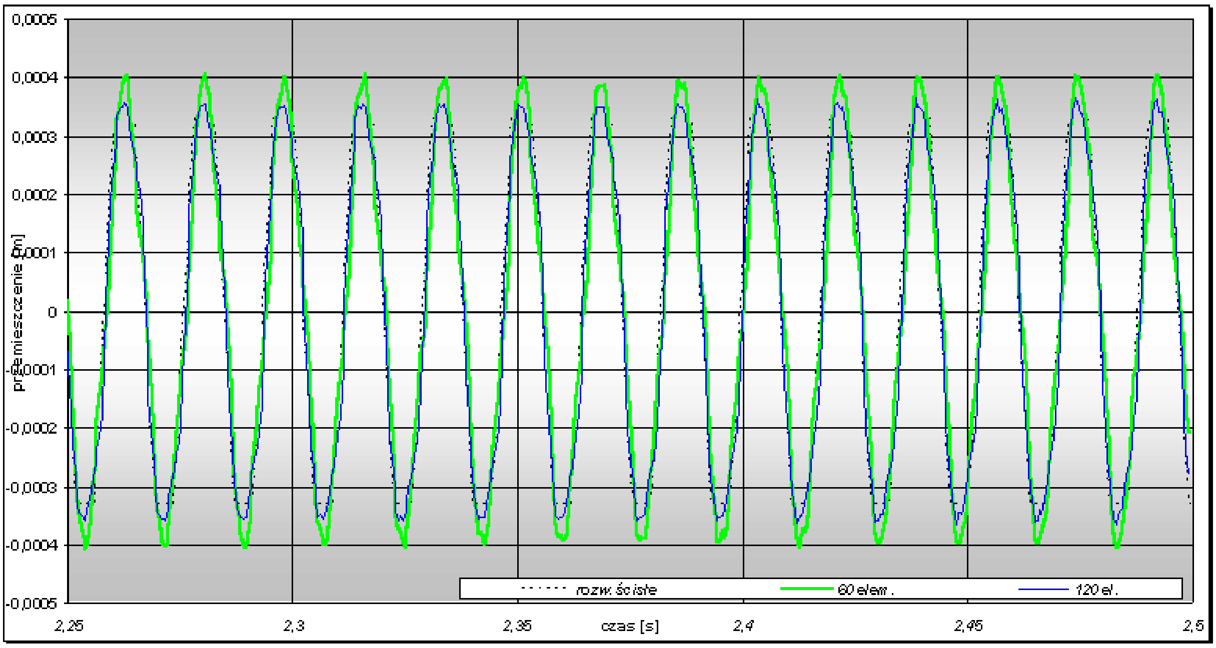


Fig. 15.Comparison of vibrations of the point situated in ¼ of the span of the string divided into 60, 120 finite elements and the accurate solution [13] at time 2.25÷2.5 s



Example 3

A preliminarily tensioned elastic string loaded in the mid-span by a time variable force $P(t) = P_0[1 - H(t)]$ is considered, where $P_0 = 200$ N.

The string data (identical as in Example 1):

a) span (length) – l = 60 cm,

b) cross-section area (Φ = 0.1 mm) – $A = 3.1416 \cdot 10^{-8}$ m^2,

c) Young's modulus E = 205 GPa,

d) string mass $\mu = 2.46615 \cdot 10^{-4}$ kg/m (total mass $1.47969 \cdot 10^{-4}$ kg),

e) preliminary string tension - 1.13652 kN.

For calculations, a discretizated model of the string is applied (Fig. 3). Division into 2, 4, 8, 16, 60 and 120 bar sections (finite elements) is considered. The distributed mass of the string is concentrated in nodes. The step of integration $h = 2 \cdot 10^{-5}$ s is assumed. Time of spring observations amounted to 2.5 seconds (125 000 moments). The aim of calculations is observation of "flattening effects" for large amplitudes (when assumption of small displacements is not fulfilled) and comparison of vibration diagrams with analogous diagrams from example 2. On the basis of such a premise in a force of P_0 = 200 N has been assumed in calculations.

Figure 16 presents a comparison of analogous vibration diagrams of several parts of the string from example 2 (for the force P_0 = 20 N, displacements in example 2 are enlarged tenfold) with current diagrams and the analytical solution [13].

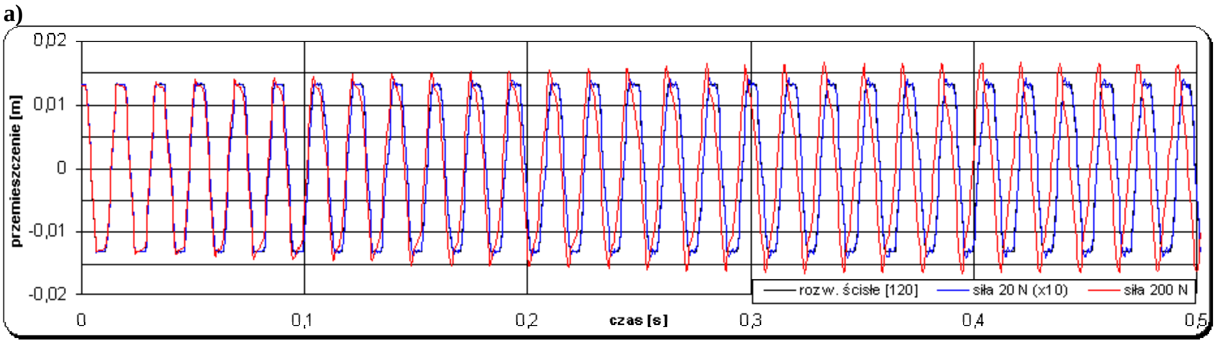
Based on the carried out calculations, one formulates the following conclusions:

1. Numerical solutions obtained for discretizations into 2, 4, 8 i 16-elements do not accurately reflect analysed vibrations of strings, hence it is not possible to formulate detailed conclusions.
2. Diagrams of vibrations of the point situated in ¼ of the string span, obtained on the basis of numerical calculations for discretization into 60 and 120 elements, allow one to observe a relationship between the magnitude of amplitudes and velocity of oblates. The greater the amplitude, the faster the discussed process.

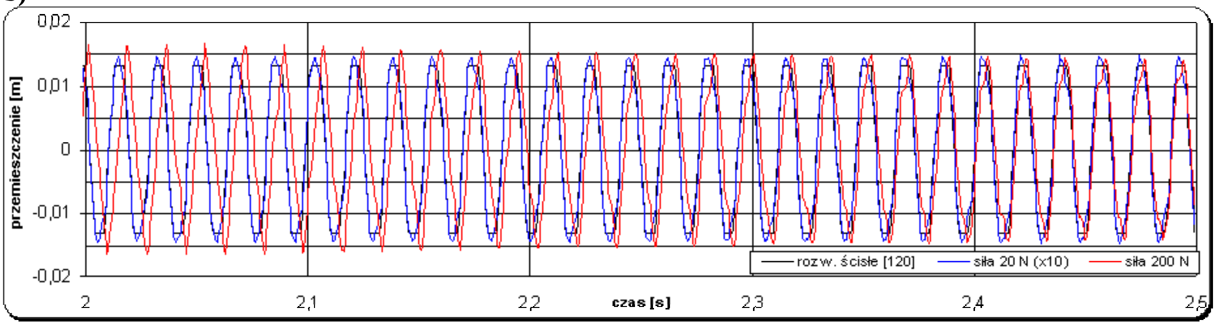
3. One can not univocally answer the question whether the expiring of oblates is a natural phenomenon, whose lock in the analytical solution ought to be explained by an over-idealised mathematical model of accepted assumptions, or it is the result of numerical mistakes. The results of numerical calculations in examples 1, 2 and 3, however, induce the Author to a statement that it is a natural phenomenon in reality.

Fig. 16. Diagrams of vibrations of the point situated in ¼ of the span of the string partitioned in 120 finite elements, a) at time 0÷0.5 s, b) at time 2÷2.5 s

a)



b)



SUMMARY

It is known that the so-far elaborated analytical solutions refer to strings undergoing small displacements only. In the present paper, numerical solutions pay respect to strings exhibiting arbitrary large displacements. For small displacements, a simplified physical model is assumed, hence also a mathematical model is reduced to the assumption that transverse vibrations of strings are independent of longitudinal vibrations. The paper - on the contrary- considers a more precise model of computations indicating the coupling between the transverse and longitudinal vibrations. In the case proposed in the paper, the numerical model is more precise than the model applied in analytical solutions. For small displacements of a string, the analytical and numerical solutions are approximate, especially with a dense discretization of the numerical model. For large displacements the numerical solutions are considerably different from the analytical ones.

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