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VIBRATION OF TENSION MEMBERS

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ABSTRACT

The paper is concerned with the dynamics of tie rods. The ties are made of a linear elastic material. The model of a tie member in the shape of a chain composed of a finite number of straight bars is presented. The equations are derived and an example of analysis of a tie system is shown. The obtained equations are strongly nonlinear. For determination of the initial configuration, an iteration method for solution of static equilibrium conditions of forces has been elaborated. The solution of differential equations of motion has been found by the modified Newmark's method. The presented example is solved, using the author's program.

Key words: : Structural mechanics, tie rod (tension member), suspension structures, dynamics.

1. INTRODUCTION

Tie rod structures are widespread in different domains of techniques. Increasing requirements given to contemporary engineering structures favour their operational as well as economical features that are responsible for success of more and more sophisticated cable-suspended structures. One of the reasons, which limit their practical application is a difficulty connected with calculation of such structures, which results from strong non-linearity of equations that describe the tie rod systems. In this context, one of the main problems is the search for proper methods for solution of equations which describe statics and dynamics of such systems.

Designing of tie rod structures is a subject of many scientific papers [1-5]. The problem was repeatedly contemplated and continue to be the object of interesting works which frequently link theoretical and experimental studies together [6, 7]. The complexity of those problems is so high that univocal presentation in the shape of ready to use models, formulas and equations may not be possible. Hence there appears the wealth of literature relative to this problem. The problems touched in existing publications concern different types of constructions such as cable- suspended bridges, mast constructions, roofs and others [8-10], in which the tie rod system serve as load-carrying elements.

The geometrical non-linearity makes the analytical solution obtainable only for elementary cases. Effective solutions are searched for by application of numerical methods and using computers, e.g. [11-16]. Cable-suspended structures, which are subject to great displacements, have little mass and are very sensitive to variable, dynamical load. They are geometrically strongly non-linear systems, therefore, for instance, natural frequencies of those systems depend on the magnitude of displacements.

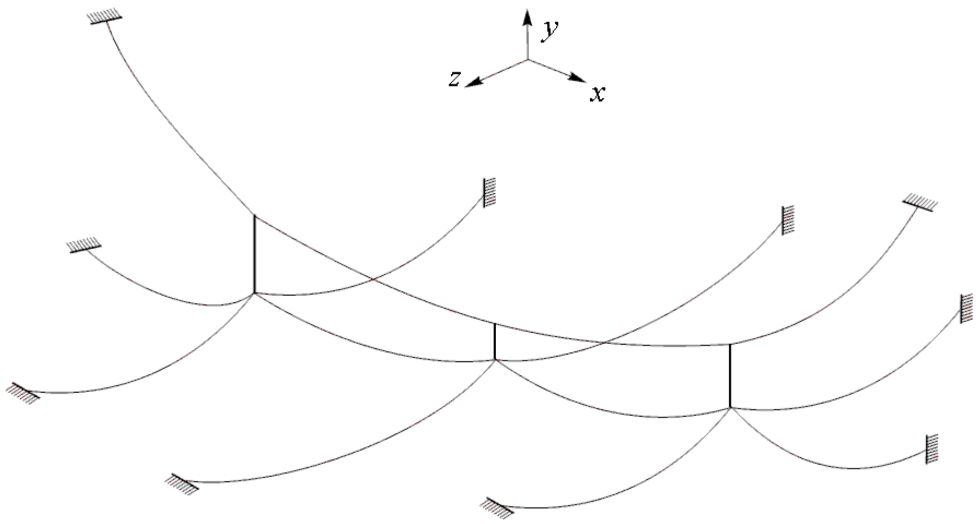
The subject of the paper is dynamics of tension members made of elastic materials. The aim of this publication is to develop an effective method of dynamical calculations of tension members and cable-suspended systems made of vicsous-elastic materials undergoing different variable loadings.

2. DIFFERENTIAL EQUATIONS OF MOTION OF TIE SYSTEMS

2.1. Formulation of the problem

A system of slim tie members (Fig. 2.1) is considered in this section. Component ties have an arbitrary curvature (sag) and are made of an elastic material.

Fig. 2.1. Considered tie system



The ties are able to endure arbitrary large displacements. It has been however assumed that the strains of these ties are infinitesimal. The loads acting on tension members can vary in space and time. The change of temperature may also cause a load of ties. The boundary conditions of tie systems are known. The flexibility of supports and its variability in time are also admissible. The tie system in its initial configuration can have a preliminary tension. We search for a function which describes the main axial forces $\mathrm{N}(x,y,z,t)$ and displacements $\mathrm{u}(x,y,z,t)$.

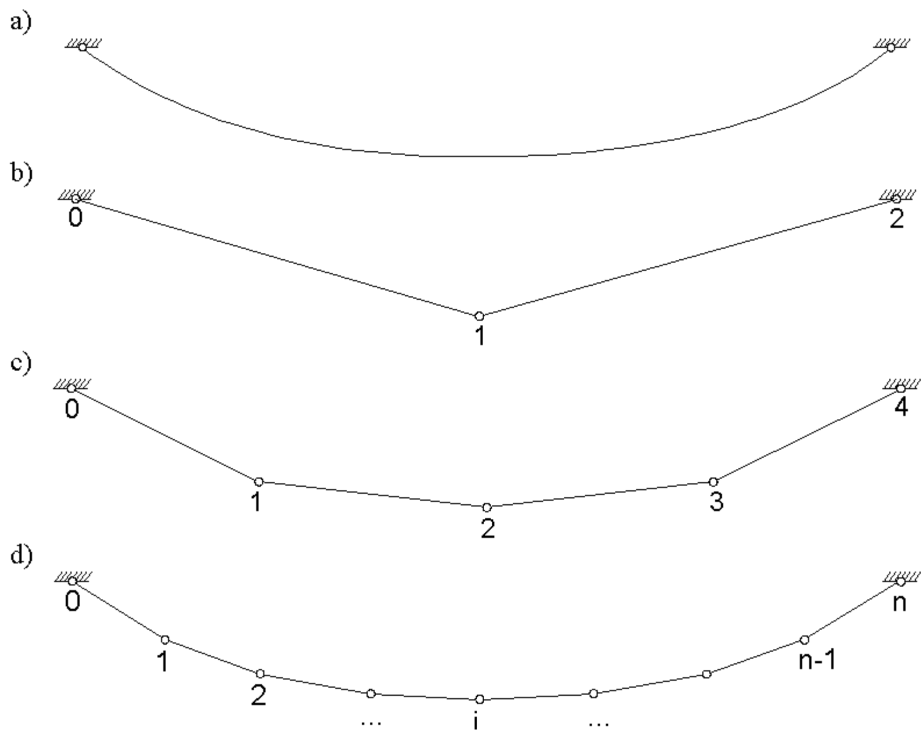
2.2. Assumptions

The following assumptions have been made:

1) the material of tie members is linearly elastic,
2) tie members are able to transfer tension forces only,
3) tie members can undergo arbitrary large displacements,
4) loads of the system constitute concentrated forces in any direction $P(x,y,z,t)$, temperature $T(x,y,z,t)$ and preliminary tension of the tie configuration,
5) time variability of boundary conditions and local changes connected with e.g. a change of length of individual ties are possible,
6) the initial tie configuration is given.

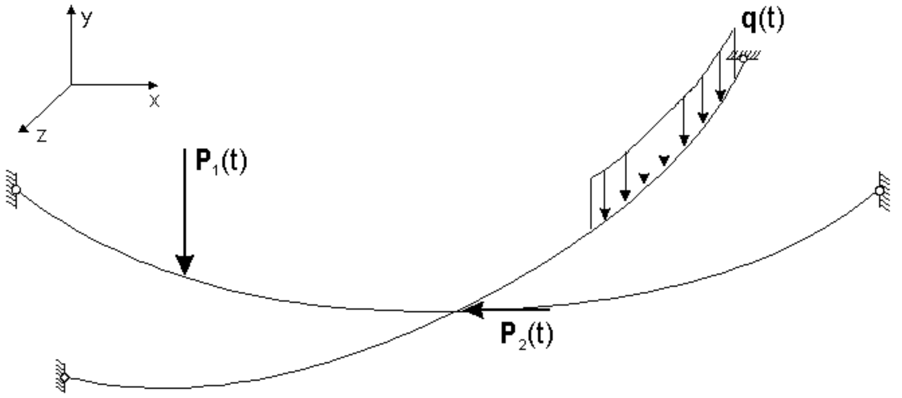
2.3. Physical model of a tie rod

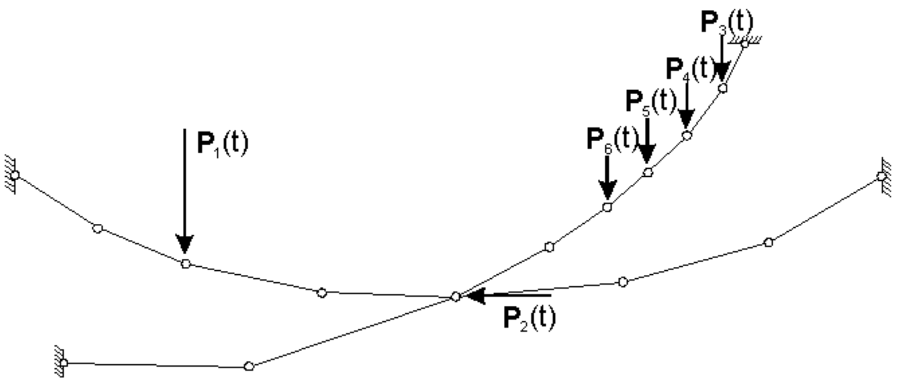
Fig. 2.2. Assumed model of a flexible tie:
a) actual tie rod, b) tie rod in the shape of two straight bars fashioned, c) tie rod in the shape of four straight bars fashioned, d) tie rod in the shape of *n* straight bars fashioned



Each tension member of the tie rod system in the shape of a chain composed of a finite number of straight bars is fashioned (Fig 2.2). This process, called discretezation of tie rod systems, leads up to division of the tie rod into finite elements ES (compare [16]). External loads in the form of concentrated forces are applied to joints of the discrete structure (Fig 2.3).

Fig. 2.3. Assumed model of the tie rod system: a) actual load, b) idealized loads

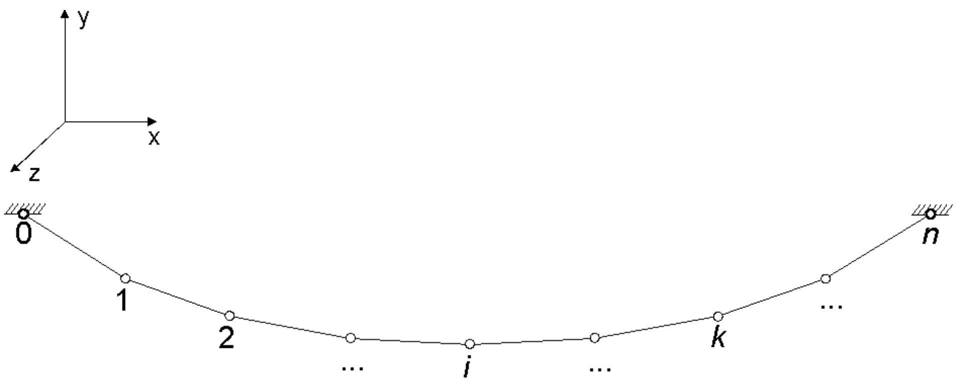




2.4. Geometrical equations

The configuration of a tie rod system in a unloaded (initial) condition is known. After discretezation of this system, according to principles given in 2.3, the co-ordinates of individual joints for this configuration are also known (Fig. 2.4).

Fig. 2.4. The tie rod in the initial configuration after discretezation



The lengths of individual bars l_{ik}^0 in the initial configuration are:

$$l_{ik}^0 = \sqrt{(x_k^0 - x_i^0)^2 + (y_k^0 - y_i^0)^2 + (z_k^0 - z_i^0)^2}, \quad i,k = 1,2,\ldots,n, \tag{2.1}$$

where: x_i^0, y_i^0, z_i^0 are known co-ordinates of the discretezated tie system. After application of loading, the length of the bars will change, (Fig. 2.5)

$$l_{ik}(t)=\sqrt{(x_k-x_i)^2+(y_k-y_i)^2+(z_k-z_i)^2}\,,$$
$$i,k=1,2,\ldots,n,\quad t\in\langle 0,\infty),\tag{2.2}$$

where x_i, y_i, z_i indicate unknown co-ordinates of joints of the discretezated tie system.

Geometrical equations for ES ending with „*ik*" are defined in the following form:

$$\varepsilon_{ik}(t)=\frac{l_{ik}-l_{ik}^0}{l_{ik}^0}+\alpha_{ik}T_{ik}+\varepsilon_{ik}^{\Delta},$$
$$i,k=1,2,\ldots,n,\quad t\in\langle 0,\infty),\tag{2.3}$$

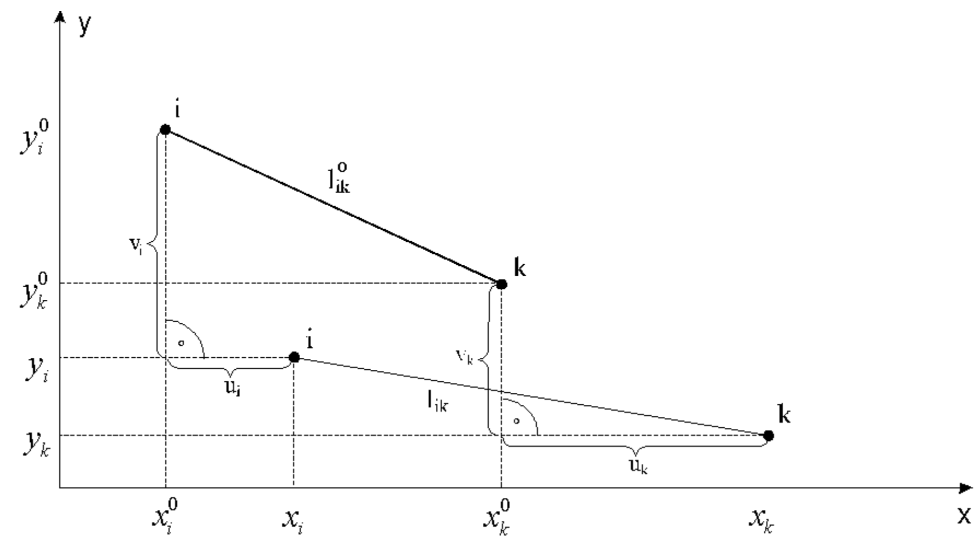
where:

α_{ik} - coefficient of linear thermal expansion,
T_{ik} - increase of temperature,
$\varepsilon_{ik}^{\Delta}$ - preliminary (initial) strains.

After taking advantage of (2.2), the geometrical equations will be described by the co-ordinates of tie system joints in current configuration.

$$\varepsilon_{ik}(t)=\frac{1}{l_{ik}^0}\sqrt{(x_k-x_i)^2+(y_k-y_i)^2+(z_k-z_i)^2}-1+\alpha_{ik}T_{ik}+\varepsilon_{ik}^{\Delta}\,.\tag{2.4}$$

Fig. 2.5. The „ *ik* " bar in the initial (non-loaded) configuration and in current (loaded) configuration



2.5. Physical equations

The tie rod is made of a linearly elastic material, hence physical equations for the bar „ *ik* " take the form:

$$N_{ik}=E_{ik}A_{ik}\varepsilon_{ik}\,,$$
$$i,k=1,2,\ldots,n,\quad t\in\langle 0,\infty),\tag{2.5}$$

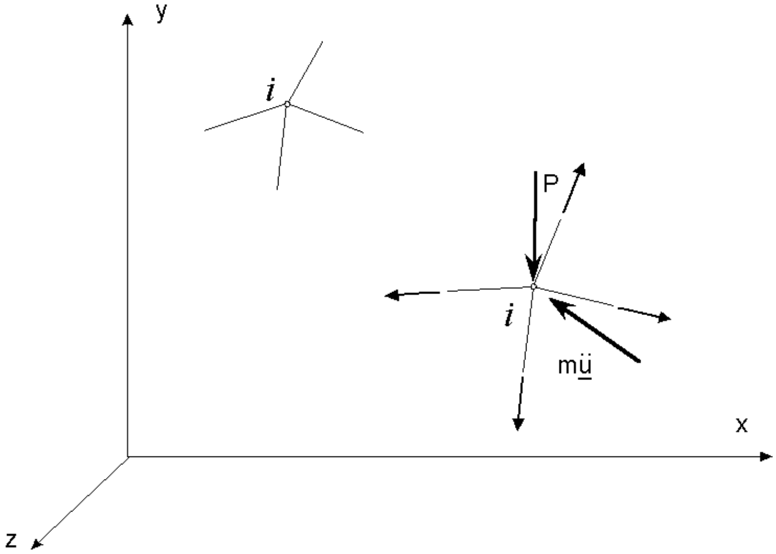
where:
N_{ik} - axial force,
A_{ik} - area of cross section,
E_{ik} - coefficient of elasticity (Young's modulus).

2.6. Differential equations of motion

Each joint „ i ”, where „ m ” bars meet, is described by three equilibrium equations (Fig. 2.6):

$$\begin{aligned} &\sum_{k=1}^{m} N_{ik} \cos\beta_{ik} + P_{x_i} = m_i \ddot{u}_i , \\ &\sum_{k=1}^{m} N_{ik} \cos\gamma_{ik} + P_{y_i} = m_i \ddot{v}_i , \\ &\sum_{k=1}^{m} N_{ik} \cos\eta_{ik} + P_{z_i} = m_i \ddot{w}_i , \end{aligned} \tag{2.6}$$

Fig.2.6. A joint of the tie rod system



where:

$$\cos\beta_{ik} = \frac{x_k - x_i}{l_{ik}}, \quad \cos\gamma_{ik} = \frac{y_k - y_i}{l_{ik}}, \quad \cos\eta_{ik} = \frac{z_k - z_i}{l_{ik}} \tag{2.7}$$

describe directions of individual bars. The values of u_i, v_i, w_i, which are components of the displacement vector of joint „ i ” are described by the following formulas (compare Fig. 2.5):

$$u_i(t) = \frac{x_i - x_i^0}{x_i^0}, \quad v_i(t) = \frac{y_i - y_i^0}{y_i^0}, \quad w_i(t) = \frac{z_i - z_i^0}{z_i^0}, \qquad t \in \langle 0, \infty) \quad , \tag{2.8}$$

where m_i represents mass concentrated in joint „ i ” and $\ddot{u}_i \equiv \frac{\partial^2 u_i}{\partial t^2}$.

Equations (2.6) which describe motion of tie systems of joints, so called equations of motion, ought to be set forth for all joints of the tie rod system, excluding the bearing joints. To complete this arrangement one makes use of equations (2.4) ÷ (2.5), and the boundary and initial conditions.

2.7. Boundary conditions

The character of bearing joint constraints „*i*" one described by the following equations:

$$\begin{aligned}
&\hat{x}_i(t) = \hat{x}_i^0 + \hat{u}_i^0(t) + R_{xi}(t) \cdot K_{xi}(t),\\
&\hat{y}_i(t) = \hat{y}_i^0 + \hat{v}_i^0(t) + R_{yi}(t) \cdot K_{yi}(t),\\
&\hat{z}_i(t) = \hat{z}_i^0 + \hat{w}_i^0(t) + R_{zi}(t) \cdot K_{zi}(t),\\
&t \in \langle 0, \infty), \quad i = 1,2,\ldots,n,
\end{aligned} \tag{2.9}$$

where:

$\hat{x}_i^0, \hat{y}_i^0, \hat{z}_i^0$ - known co-ordinates of bearing joints in the initial configuration,

$\hat{u}_i^0, \hat{v}_i^0, \hat{w}_i^0$ - known displacements of bearing joints, varying in time,

R_{xi}, R_{yi}, R_{zi} - components of bearing reactions,

K_{xi}, K_{yi}, K_{zi} - flexibility of bearing joints.

2.8. Initial conditions

Equations (2.8) are ordinary quadratic differential equations, hence their univocal solution requires double boundary conditions, e.g.:

$$u_i(0) = u_i^0, \quad \dot{u}_i(0) = \dot{u}_i^0, \quad v_i(0) = v_i^0, \quad \dot{v}_i(0) = \dot{v}_i^0, \quad w_i(0) = w_i^0, \quad \dot{w}_i(0) = \dot{w}_i^0. \tag{2.10}$$

3. SOLUTION OF EQUATIONS OF MOTION FOR TIE ROD SYSTEMS – COMPUTER (NUMERICAL) METHOD

On the ground of derived formulas, an algorithm of numerical calculations was elaborated, which was applied in a computer program afterwards. The program works in Win32 OS. It was prepared in C^{++} language and compiled and debugged by Microsoft Visual Studio v. 6.0 system.

The first stage of calculations is to assign the preliminary configuration of the tie rod system. It is assumed that it is a state of displacements of the system from the initial static loading (e.g. dead weight, temperature of assembling, preliminary tension of the structure or others). In such a case, equations of motion (2.8) change into static equations of force equilibrium:

$$\begin{aligned}
&\sum_{k=1}^{m} N_{ik} \cos\beta_{ik} + P_{x_i} = 0,\\
&\sum_{k=1}^{m} N_{ik} \cos\gamma_{ik} + P_{y_i} = 0,\\
&\sum_{k=1}^{m} N_{ik} \cos\eta_{ik} + P_{z_i} = 0.
\end{aligned} \tag{3.1}$$

After substitution of (2.5) and considering (2.3) to (3.1), one obtains:

$$\begin{aligned}
&\sum_{k=1}^{m} E_{ik} A_{ik} \left(\frac{l_{ik} - l_{ik}^o}{l_{ik}^o} - \alpha_{ik} T_{ik} - \varepsilon_{ik}^{\Delta} \right) \frac{x_k - x_i}{l_{ik}} + P_{x_i} = 0,\\
&\sum_{k=1}^{m} E_{ik} A_{ik} \left(\frac{l_{ik} - l_{ik}^o}{l_{ik}^o} - \alpha_{ik} T_{ik} - \varepsilon_{ik}^{\Delta} \right) \frac{y_k - y_i}{l_{ik}} + P_{y_i} = 0,\\
&\sum_{k=1}^{m} E_{ik} A_{ik} \left(\frac{l_{ik} - l_{ik}^o}{l_{ik}^o} - \alpha_{ik} T_{ik} - \varepsilon_{ik}^{\Delta} \right) \frac{z_k - z_i}{l_{ik}} + P_{z_i} = 0.
\end{aligned} \tag{3.2}$$

Equations (3.2) are valid for all the joints, except for the bearing joints, where $l_{ik}^0 = l_{ik}^0\left(x_i^0, y_i^0, z_i^0\right)$ presents the known length of a finite element in its initial configuration described by formula (2.1), and $l_{ik} = l_{ik}\left(x_i, y_i, z_i, t\right)$ is unknown length of the finite element of a tie member in its current configuration, as described by (2.2). In this system, the co-ordinates x_i, y_i, z_i are unknown. The thus formulated set of equations is characterised by strong non-linearity, hence determination of the solution to it is considerably difficult. For needs of the computer program, a certain method of solution procedure has been elaborated. Equations (3.2) have been rearranged to take the following form:

$$x_i = \frac{\sum_{k=1}^{m} x_k E_{ik} A_{ik}\left[\frac{1}{l_{ik}^0} - \frac{1}{l_{ik}}\left(1 + \alpha_{ik} T_{ik} + \varepsilon_{ik}^{\Delta}\right)\right] + P_{x_i}}{\sum_{k=1}^{m} E_{ik} A_{ik}\left[\frac{1}{l_{ik}^0} - \frac{1}{l_{ik}}\left(1 + \alpha_{ik} T_{ik} + \varepsilon_{ik}^{\Delta}\right)\right]},$$

$$y_i = \frac{\sum_{k=1}^{m} y_k E_{ik} A_{ik}\left[\frac{1}{l_{ik}^0} - \frac{1}{l_{ik}}\left(1 + \alpha_{ik} T_{ik} + \varepsilon_{ik}^{\Delta}\right)\right] + P_{y_i}}{\sum_{k=1}^{m} E_{ik} A_{ik}\left[\frac{1}{l_{ik}^0} - \frac{1}{l_{ik}}\left(1 + \alpha_{ik} T_{ik} + \varepsilon_{ik}^{\Delta}\right)\right]}, \tag{3.3}$$

$$z_i = \frac{\sum_{k=1}^{m} z_k E_{ik} A_{ik}\left[\frac{1}{l_{ik}^0} - \frac{1}{l_{ik}}\left(1 + \alpha_{ik} T_{ik} + \varepsilon_{ik}^{\Delta}\right)\right] + P_{z_i}}{\sum_{k=1}^{m} E_{ik} A_{ik}\left[\frac{1}{l_{ik}^0} - \frac{1}{l_{ik}}\left(1 + \alpha_{ik} T_{ik} + \varepsilon_{ik}^{\Delta}\right)\right]}.$$

Conjugation and non-linearity of relations (3.3) lies in the expression for $l_{ik} = l_{ik}\left(x_i, y_i, z_i\right)$. Equations (3.3) are then transformed into linear form. The form of (3.2) has evidently many others linear transformations. The choice of specific linearity has large influence on the effectiveness of iteration processes, i.e. upon convergence and accuracy of the solution to equations. The next step, i.e. solving the system of equations (3.3), is similar to Gauss-Seidle's iteration processes for linear equations. To accelerate the calculations, a kind of super-relaxation method used for linear equations can be introduced. For that purpose, from equations (3.4) one enumerates a series from which the unknowns can be found in consecutive iterations.

$$\Delta x_i = \frac{\sum_{k=1}^{m} x_k E_{ik} A_{ik}\left[\frac{1}{l_{ik}^0} - \frac{1}{l_{ik}}\left(1 + \alpha_{ik} T_{ik} + \varepsilon_{ik}^{\Delta}\right)\right] + P_{x_i}}{\sum_{k=1}^{m} E_{ik} A_{ik}\left[\frac{1}{l_{ik}^0} - \frac{1}{l_{ik}}\left(1 + \alpha_{ik} T_{ik} + \varepsilon_{ik}^{\Delta}\right)\right]} - x_i,$$

$$\Delta y_i = \frac{\sum_{k=1}^{m} y_k E_{ik} A_{ik}\left[\frac{1}{l_{ik}^0} - \frac{1}{l_{ik}}\left(1 + \alpha_{ik} T_{ik} + \varepsilon_{ik}^{\Delta}\right)\right] + P_{y_i}}{\sum_{k=1}^{m} E_{ik} A_{ik}\left[\frac{1}{l_{ik}^0} - \frac{1}{l_{ik}}\left(1 + \alpha_{ik} T_{ik} + \varepsilon_{ik}^{\Delta}\right)\right]} - y_i, \tag{3.4}$$

$$\Delta z_i = \frac{\sum_{k=1}^{m} z_k E_{ik} A_{ik}\left[\frac{1}{l_{ik}^0} - \frac{1}{l_{ik}}\left(1 + \alpha_{ik} T_{ik} + \varepsilon_{ik}^{\Delta}\right)\right] + P_{z_i}}{\sum_{k=1}^{m} E_{ik} A_{ik}\left[\frac{1}{l_{ik}^0} - \frac{1}{l_{ik}}\left(1 + \alpha_{ik} T_{ik} + \varepsilon_{ik}^{\Delta}\right)\right]} - z_i.$$

The new co-ordinates x_i^k, y_i^k, z_i^k in an iteration process, from formulas $x_i^k = x_i^{k-1} + \Delta x_i \cdot c$, $x_i^k = x_i^{k-1} + \Delta x_i \cdot c$, $x_i^k = x_i^{k-1} + \Delta x_i \cdot c$, are enumerated, where *c* is the coefficient of super-relaxation, *k*=1,2, ... denotes the number of consecutive iterations. On the basis of experience gained during calculations, depending on the considered problem, the coefficient *c* assumes a value from the interval <1, 2>. It has been observed that the calculation rate is approximately proportional to the assumed coefficient. This coefficient, for each separate problem, ought to be determined individually. A great influence upon the optimal value of it has preliminary tension of a tie member (the higher the tension is, the closer to 1 is the optimal coefficient). In the case of an average engineering tie rod system, there is no point of finding the exact value of that coefficient in calculations (it is enough to assume 1), as the time needed for its exact determination will not counterbalance the time of calculation. The assumed algorithm of calculations requires - apart from typical data such as length and stiffness of all members of a system, loads, boundary conditions - also the initial configuration, i.e. preliminary co-ordinates of all mobile joints of the system. The difference between this initial configuration and the determined one, has influence on the number of iterations and time of calculations. When even started from very "peculiar" initial configurations, the iteration process appears convergent and allows one to find a high accuracy solution quite quickly.

Calculations of the dynamical load are the next step. For this purpose we have a set of differential non-linear equations (2.6). These are ordinary coupled equations. To solve them, the numerical method of direct integration of a mobile equation, i.e. Newmark's method, has been applied. This method is very well elaborated for linear, mobile equations. In the case of non-linear equations, an adaptation of this method is necessary. The means using the so-called Newmark's incremental method for differential non-linear equations, to be find in literature, see e.g. [15]. In the case of equations (2.5) and (2.6), a special adaptation, considering their specifics, was indispensable. After substitution of (2.5), and taking into account (2.3) - (2.6), the final form of differential equations of motion for joint "i ", which links together „m " bars, is obtained:

$$
\begin{aligned}
&\sum_{k=1}^{m} E_{ik} A_{ik} \left(\frac{l_{ik}^t - l_{ik}^o}{l_{ik}^o} - \alpha_{ik} T_{ik} - \varepsilon_{ik}^{\Delta} \right) \frac{x_k^t - x_i^t}{l_{ik}^t} + P_{x_i}^t = m_i \ddot{x}_i^t , \\
&\sum_{k=1}^{m} E_{ik} A_{ik} \left(\frac{l_{ik}^t - l_{ik}^o}{l_{ik}^o} - \alpha_{ik} T_{ik} - \varepsilon_{ik}^{\Delta} \right) \frac{y_k^t - y_i^t}{l_{ik}^t} + P_{y_i}^t = m_i \ddot{y}_i^t , \\
&\sum_{k=1}^{m} E_{ik} A_{ik} \left(\frac{l_{ik}^t - l_{ik}^o}{l_{ik}^o} - \alpha_{ik} T_{ik} - \varepsilon_{ik}^{\Delta} \right) \frac{z_k^t - z_i^t}{l_{ik}^t} + P_{z_i}^t = m_i \ddot{z}_i^t , \\
&\qquad t = 1,2,\ldots,T \ .
\end{aligned}
\qquad (3.5)
$$

The expressions $m_i \ddot{u}_i, m_i \ddot{v}_i, m_i \ddot{w}_i$ with equivalent expressions $m_i \ddot{x}_i, m_i \ddot{y}_i, m_i \ddot{z}_i$ have been replaced. Equations (3.5) for all joints, excluding the bearing joints, are settled down. These are non-linear, ordinary, heterogeneous differential equations of the second order. The unknown quantities in these equations are the co-ordinates x_i^t, y_i^t, z_i^t, of $i = 1,2,\ldots,n$ – joints of the tie rod system and $t = 1,2,\ldots,T$ - joints along the discretized axis of time. In the moment $t = 0$, the initial conditions concerning displacements (2.12) and forces (2.14) are known. In equations (3.10), velocities and accelerations of displacements appear. These magnitudes, by the approximation used in Newmark's method, are described

$$
\begin{aligned}
&\dot{f}_j^{\,t} = \frac{\delta}{\alpha h}\left(f_j^{\,t} - f_j^{\,t-1}\right) + \left(1 - \frac{\delta}{\alpha}\right)\dot{f}_j^{\,t-1} + h\left(1 - \frac{\delta}{2\alpha}\right)\ddot{f}_j^{\,t-1} \\
&\ddot{f}_j^{\,t} = \frac{1}{\alpha h^2}\left[f_j^{\,t} - f_j^{\,t-1} - \dot{f}_j^{\,t-1} h + h^2 \left(\frac{1}{2} - \alpha\right)\ddot{f}_j^{\,t-1} \right], \\
&j = i,k \ , \quad f = x,y,z \ ,
\end{aligned}
\qquad (3.6)
$$

where: h is the integration step, α, δ - parameters of the method.

Next, formulas (3.6) are introduced to (3.5). For known initial conditions (2.12) and (2.15), the recursion formulas are obtained as follows:

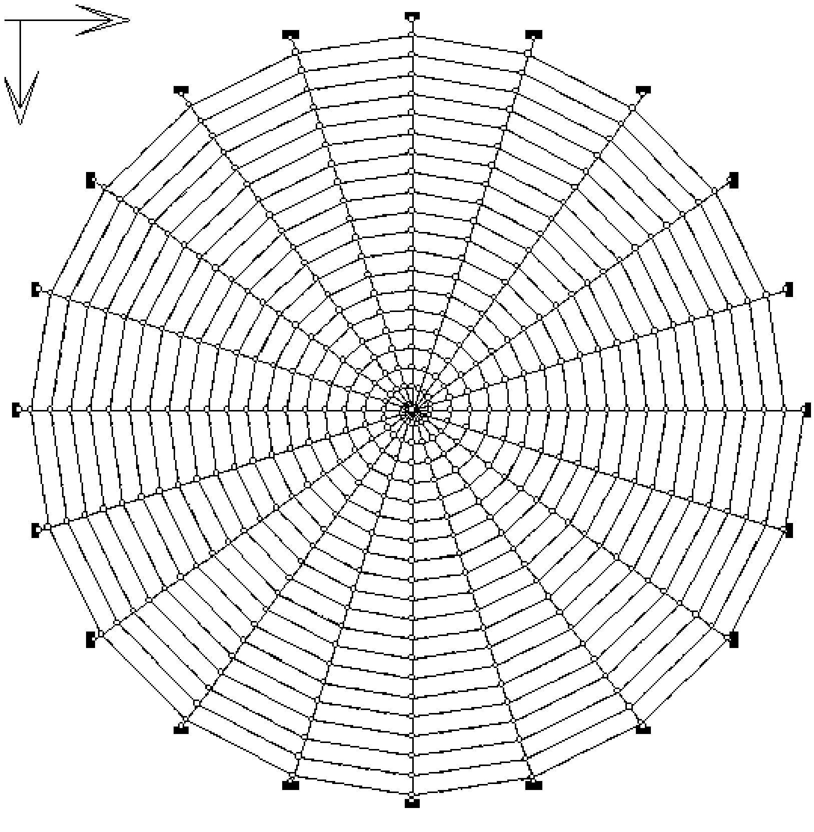
$$f_i^t = F_i^t\left(f_i^t\right) + G_i^{t-1}\left(f_i^{t-1}, \dot{f}_i^{t-1}, \ddot{f}_i^{t-1}, N_{ik}^{t-1}, \dot{N}_{ik}^{t-1}\right) + H_i^t\left(P^t\right),$$
$$t = 1,2,\ldots,T \ , \quad f = x, y, z \ . \tag{3.7}$$

By proper selection of parameters α, δ, the process of recursion (3.7) is unconditionally stable. The complexity of formula (3.7) results from the non-linearity which rests upon the existence of unknown co-ordinates f_i^t from the left and right hand sides of the formula. The function G_i^{t-1} is known, determined at the moment $t-1$, and the function H_i^t depends upon a known forcing which causes vibration of the tie rod system. In the case of a geometrically linear problem, the function F_i^t is zero.

4. EXAMPLES OF CALCULATION

The considered system consists of 817 elastic tie bars stretched between 420 joints, distributed on one plane whose shape is similar to a spider-web (Fig.4.1). The spider-web is composed of 20 radial tie rods, each 5m in length and fixed at the ends. Between the radial ties, paralleled ties are inserted, which forms a spiral.

Fig. 4.1. Spider-web



Data of the system:

a) span of the system - 10 m,
b) total mass of the system 3559.08 kg,
c) longitudinal rigidity of radial members – *EA* = 224.4 MN,
d) longitudinal rigidity of parallel members – *EA* = 139.055 MN.

Preliminary tension by progressive shortening of all parallel tie rods is realised in steps as follows: by 0,1% of their length on the first external layer (20 tie rods), 0,2% on the second layer (next 20 tie rods), and so on, up to 2% for the middle tie rods. The load represents a force acting in the central point of the spider-web, which is perpendicular to the system plane. It is variable in time $P(t) = P_0\left[1 - H(t)\right]$, where P_0 = 2500 kN, and *H*(*t*) is a Heaviside's function.

A discrete model of the tie system presented in Fig. 4.1 was applied in calculations. Distributed mass of individual tie members was concentrated in points of joints. An integration step of the value $h = 1 \cdot 10^{-4}$ *s* was assumed. The time of observation was 100 seconds (1 000 000 instants). Consecutive figures present diagrams of vertical vibrations (Fig. 4.2) and horizontal ones (Fig. 4.3). In next figures (4.4 ÷ 4.8), successive forms of spider-web deformations are shown. These figures, by means of colours and different thickness of lines, illustrate the forces in tie rods – the red colour indicates the maximal value of force, the blue one -force near to 0 kN, and the violet - members excluded form co-operation as a result of compression.

On the basis of carried out calculations, the following conclusions are formulated:

1. The bars in the central part of the spider-web, as a result of compression are often out of co-operation. Because of that, integration with a very little time step ($\Delta t = h = 1 \cdot 10^{-4}$ *s*.) was indispensable. Otherwise, the results of calculations will considerably deviate from expected values.
2. The maximum tensile forces occur in radial bars close to supports. The forces in parallel tie rods are much smaller than the radial ones.
3. The maximum vertical displacements appear obviously in the middle of the spider-web. The drop of these displacements along the radial tie rods is non-linear. The reason of it are mostly the forces acting in the parallel tie members.

Fig. 4.2. Vertical vibrations of the central point of the spider-web

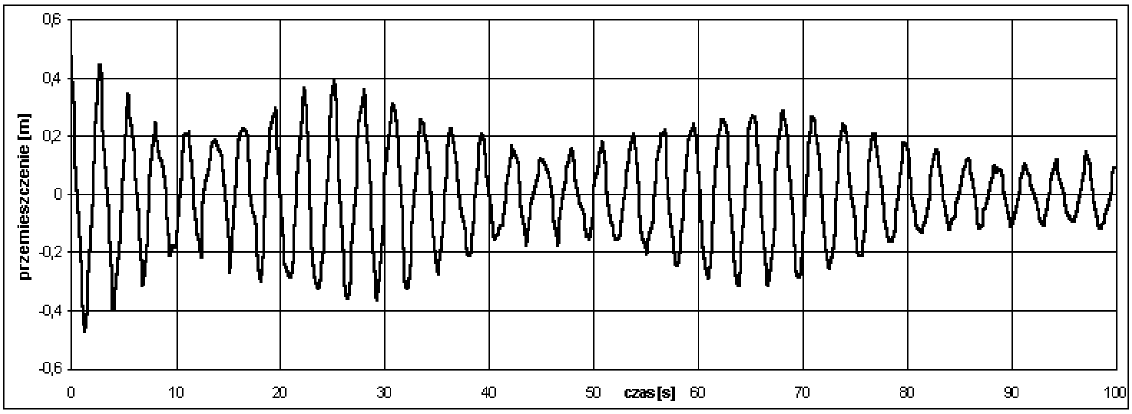


Fig. 4.3. Horizontal vibrations of the central point of the spider-web

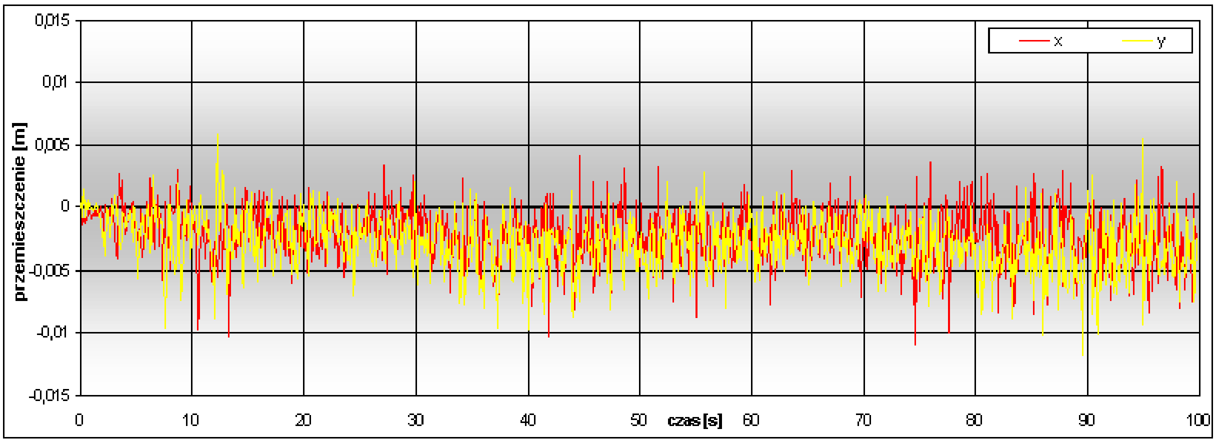
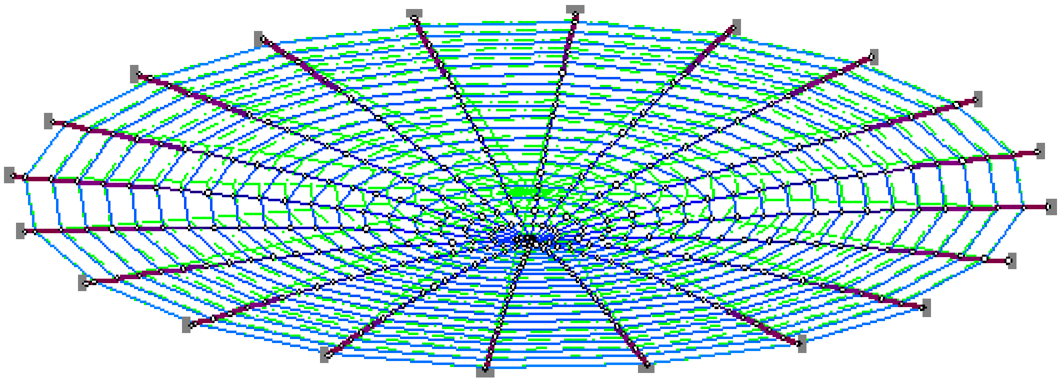
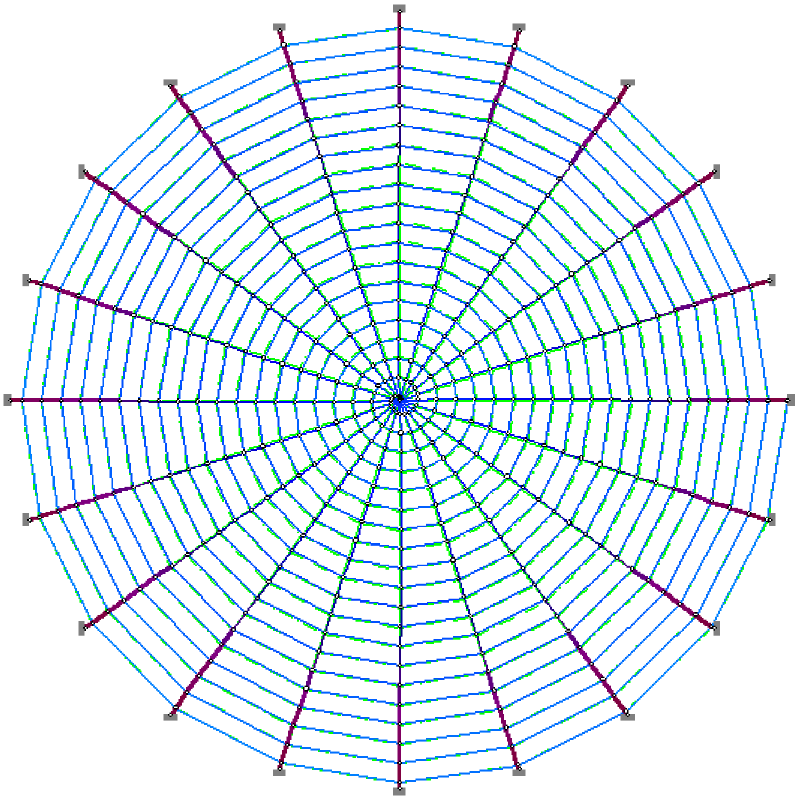


Fig. 4.4. Spider-web at t = 0+, just after application of the force *P*(*t*), a) axonometry, b) top view , c) side view

a)



b)



c)

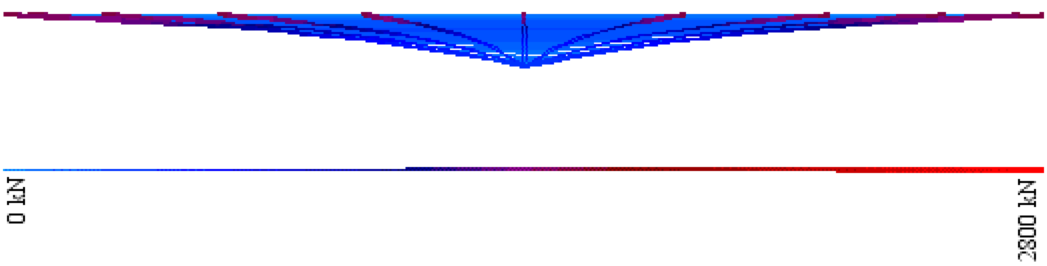


Fig. 4.5. Spider-web at $t = 6 \cdot 10^{-1}$ *s*, a) axonometry, b) top view, c) side view

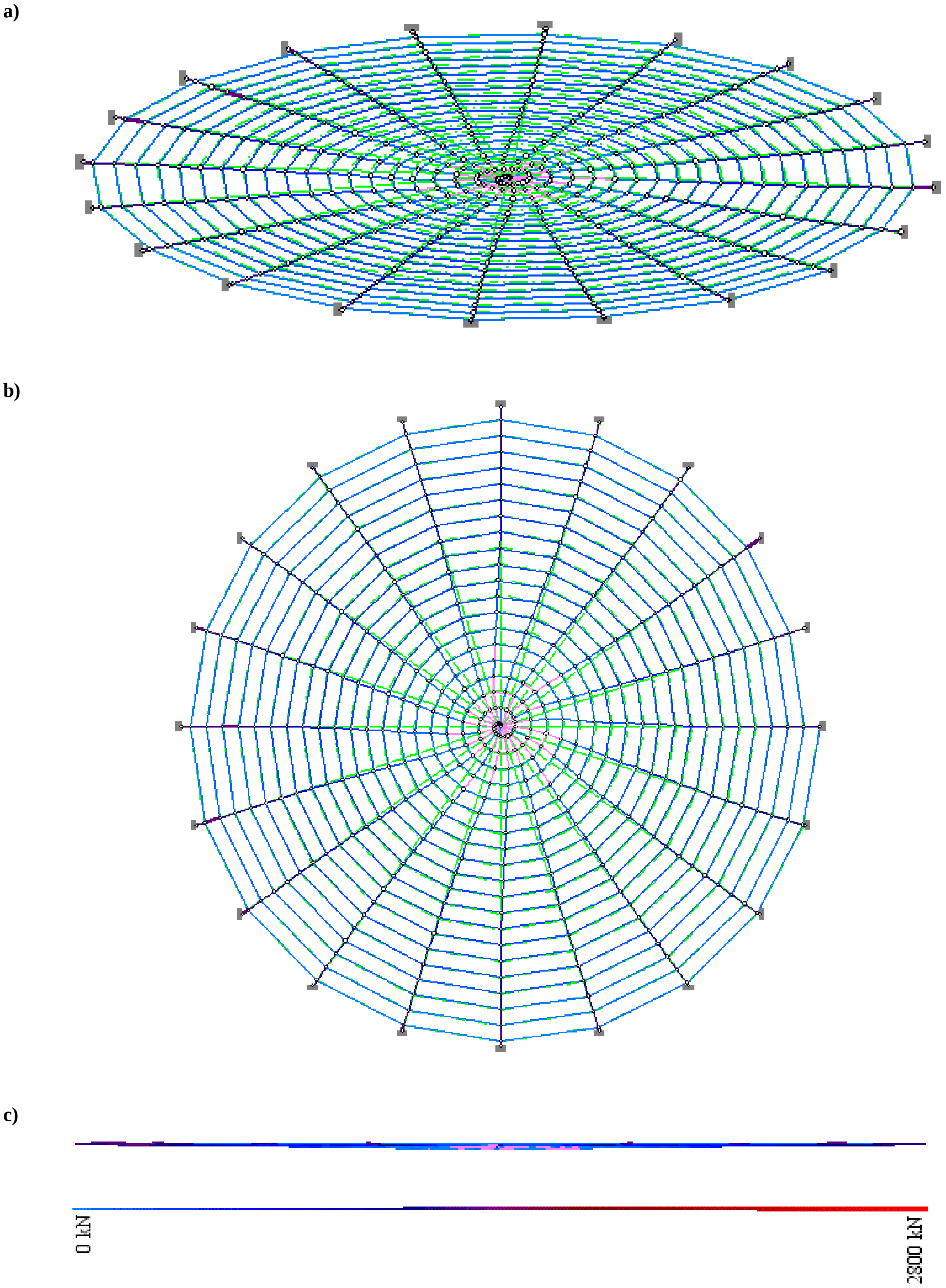


Fig. 4.6. Spider-web at t = 1.3 s, a) axonometry, b) top view, c) side view

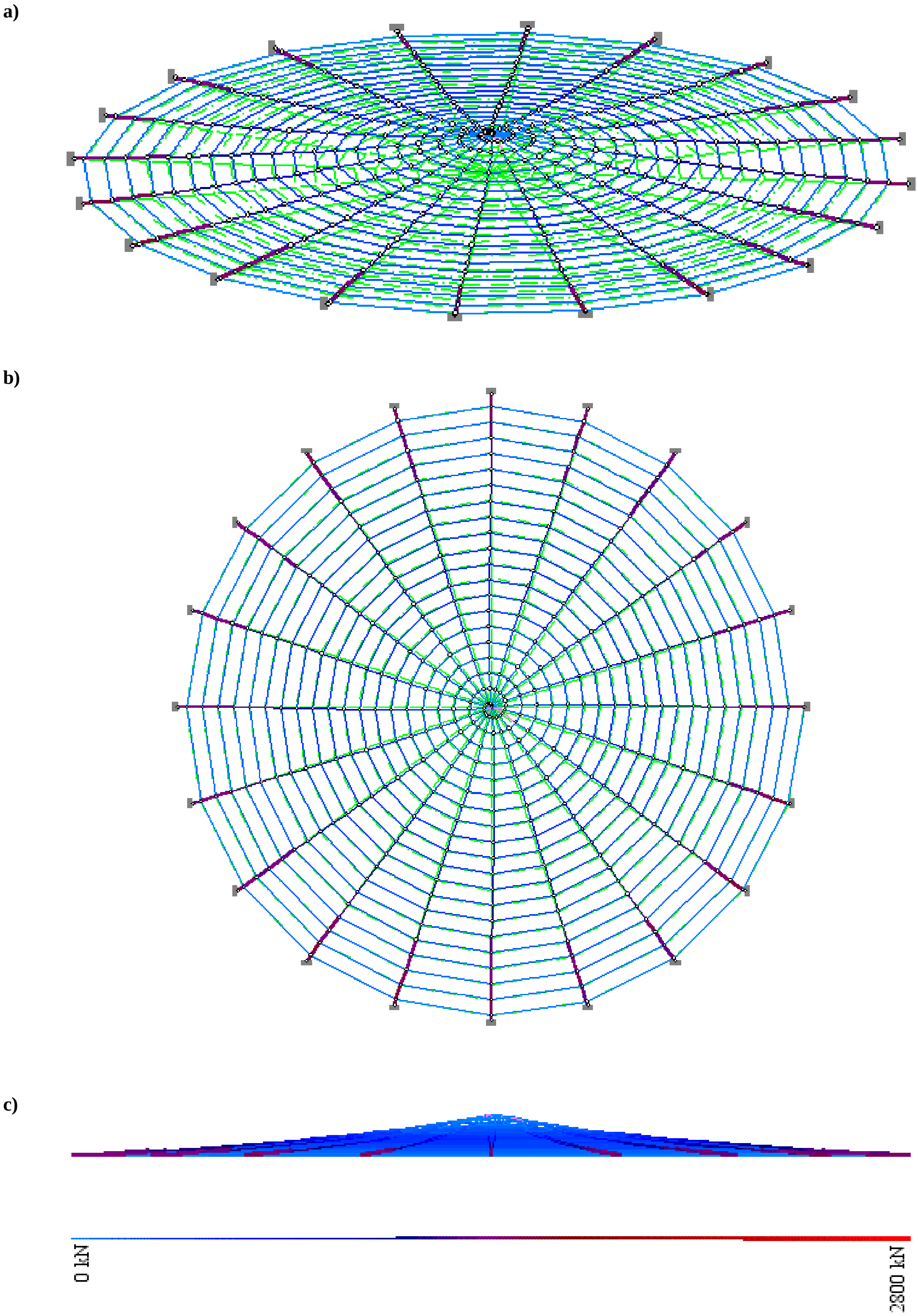
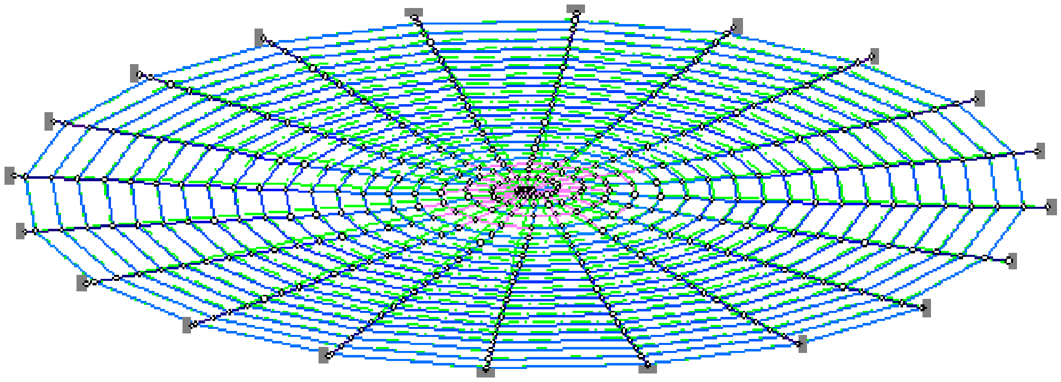
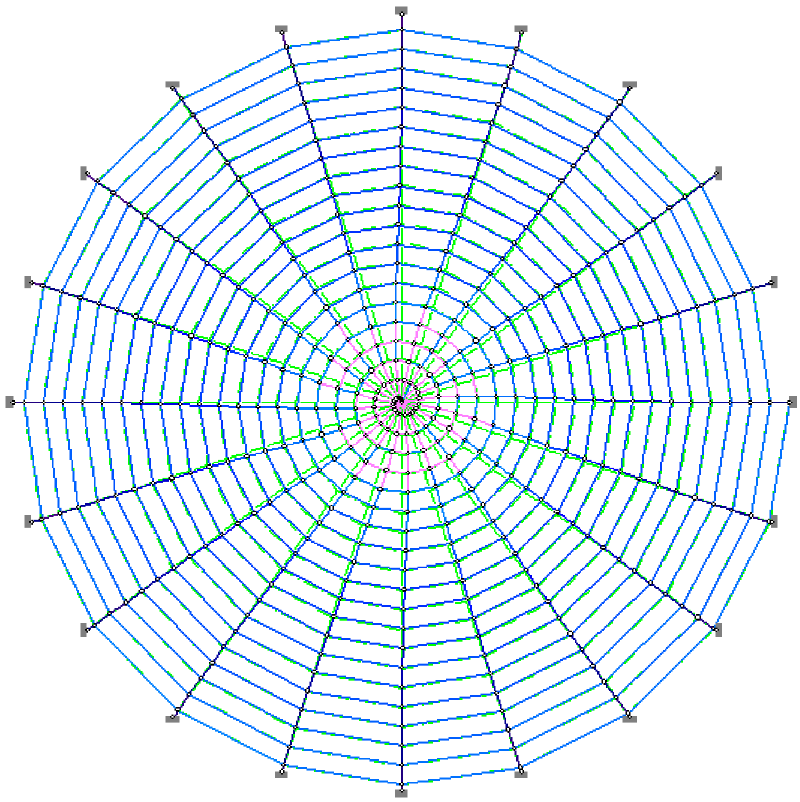


Fig. 4.7. Spider-web at *t* = 2,1 *s*, a) axonometry, b) top view, c) side view

a)



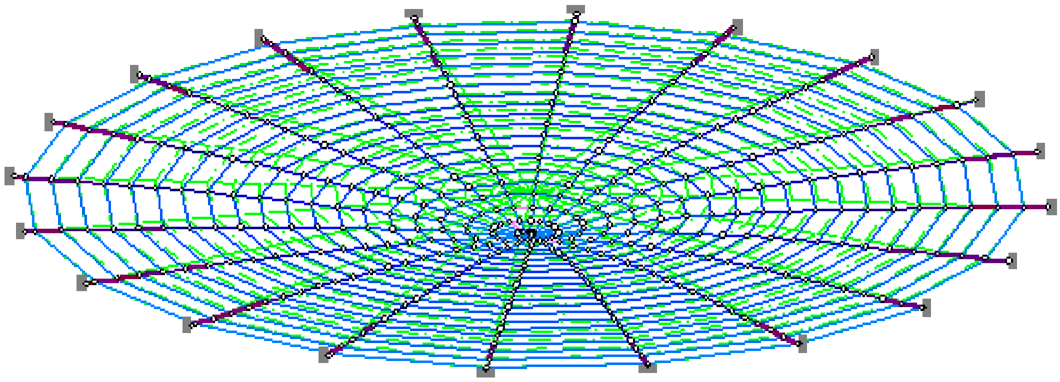
b)



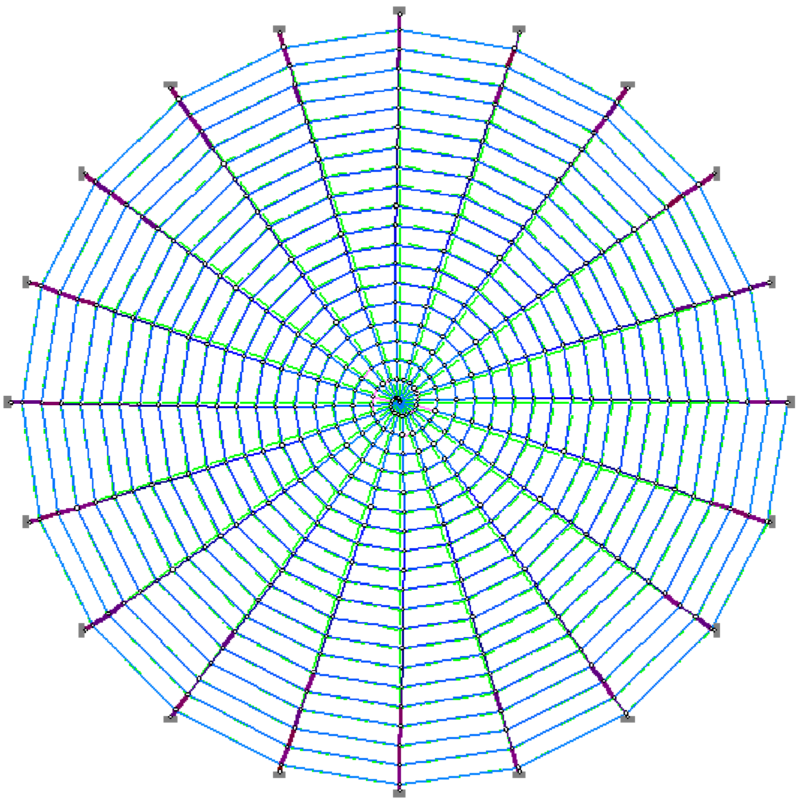
c)

Fig. 4.8. Spider-web at t = 2,7 s, a) axonometry, b) top view, c) side view

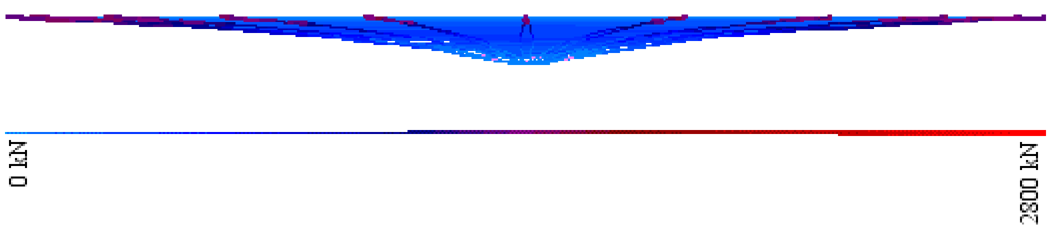
a)



b)



c)



5. SUMMARY

In this paper, the application of discretezation of a single flexible tie rod with arbitrary deflection (sag) with respect to straight segments for dynamical analysis of an arbitrary tie rod system is presented. For the modelled structure, a set of strongly non-linear algebraic equations, has been obtained. The original iteration method, which is based on linearization of equations in the iteration process, has bee elaborated. This method, on account of high accuracy of the solution to equations of motion as well as short time of calculations, appears very effective. This method can be successfully used for solving miscellaneous tie rod systems with arbitrarily large displacements of joints and applied loads of any variation in time.

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