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# STRESS DISTRIBUTION IN ANGLE JOINTS OF SKELETON FURNITURE

Jerzy Smardzewski, Tomasz Papuga
*Department of Furniture Design, The August Cieszkowski Agricultural University of Poznań, Poland*



## ABSTRACT

The presented study is of theoretical nature aiming to perform a numerical analysis of structural strength of skeleton furniture subjected to real useful loads. The objective of the performed investigations was to determine the state and distribution of stresses in the angle joint of a spatial structure of skeleton furniture. Experiments were carried out on dowel and tenon and mortise joints, which belong to the group of flat angle joints. The performed experiments allowed to develop a numerical model of a skeleton construction which was later used to perform calculations determining: reduced stresses according to the von Mises theory, tangential stresses in glue lines of dowel and tenon and mortise joints and tangential stresses in the wood of dowel joints. The results presented in this study are of cognitive nature. They determine values of stresses in joints of the loaded furniture piece and identify places with the greatest effort.

**Key words:** chair, joint, glue line, stress distribution.

## INTRODUCTION

Strength and stiffness of skeleton furniture depends, primarily, on constructional, material, technological and use factors. Each of them exerts some impact on durability and exploitation safety of a given piece of furniture, determines production costs and its market value. Therefore, the development of a suitable and correct design of furniture constructions requires carrying out appropriate strength analysis of the planned article. On the basis of such analysis, it is possible to design a piece of furniture which is characterised by appropriately strong elements and joints and, at the same time, meets all the aesthetic and useful requirements. First investigations concerning analytical methods of furniture design were undertaken by: Eckelman [4], Ganowicz and Nowak [5], Dzięgielewski and Smardzewski [3] who reduced the construction of skeleton furniture from multiple, statistically indeterminable, spatial structures to the form of flat frames putting forward, at the same time, procedures needed to solve these systems. However, the main problem was to determine a reliable strength of constructional intersections and the form of distribution of stresses in individual joints. The above problems were solved employing numerical methods for modelling 3D constructions of skeleton furniture.

However, the application of numerical methods for the strength analysis of joints of wooden constructions was confined to the determination of stresses in glue lines of corner joints subjected to shearing, torsion and bending: Wilczyński [14], Pellicane [7], Smardzewski and Dzięgielewski [11, 12], Smardzewski [9], Smardzewski and Dzięgielewski [13], Smardzewski [10], Nakai and Takemura [6] and Curtu et al. [1, 2].
It is evident from the above quoted literature that, so far, the character of work of corner joints in spatial constructions of skeleton furniture subjected to useful loads has not been determined. Therefore, it appears necessary to supplement results of experimental studies in this field by results of theoretical calculations, which would allow to correlate states of furniture loads with stress states in fasteners, couplings and joints using numerical calculations.

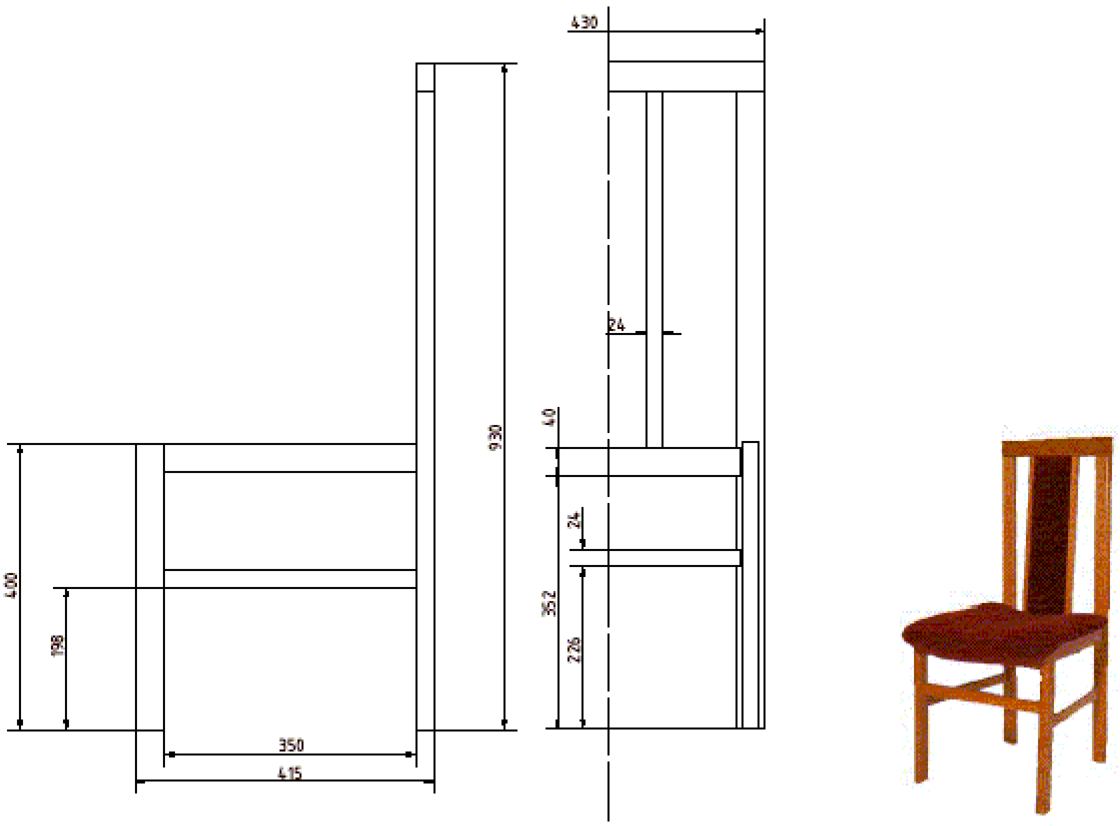
## OBJECTIVE OF INVESTIGATIONS

The objective of the undertaken investigations was to determine the distribution of stresses in 3D corner dowel and tenon and mortise joints of the spatial structure of skeleton furniture. The analysis of tangential stresses in glue lines of the joints was assumed as significant. The expected research results should expand and enhance our current knowledge associated with stresses in glue lines and 3D joints of corner connections allowing optimal design of spatial structures of skeleton furniture under use loads.

## METHODOLOGY

The analysis of stresses in angle 2D joints was carried out in the skeleton construction of a chair mass-produced in one of Polish chair factories (Fig. 1).

**Fig. 1. Geometry and model of the examined chair**



The basic constructional intersections of the examined piece of furniture were connections in a dowel joint (Fig. 2) used to connect legs and elements of the underframe and the backrest with the rear leg. Tenon and mortise joints (Fig. 3) were proposed to connect the bar and connectors with legs.

**Fig. 2. Dimensions of the connection with a dowel joint**

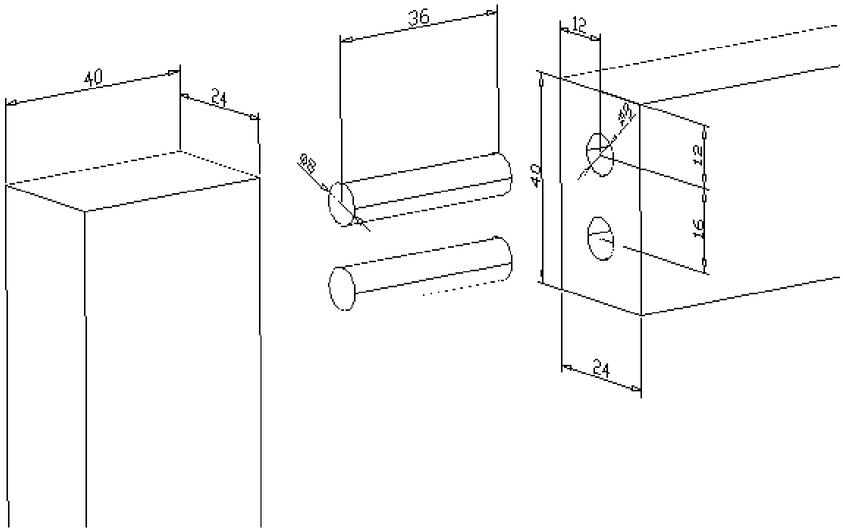



**Fig. 3. Dimensions of the connection with a tenon and mortise joint**

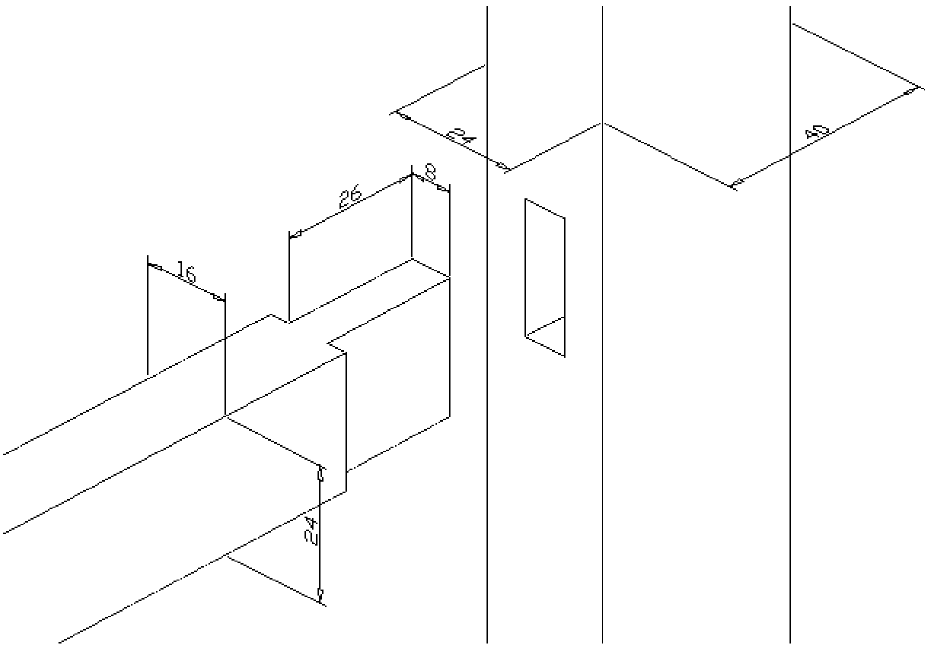



**Table 1. Dimensions of cross sections of elements**

| Elements | Dimension a [mm] | Dimension b [mm] |
|---|---|---|
| 1 | 24 | 40 |
| 2 | 16 | 24 |

Dowel joints were modelled by regular polygons representing the cross section of the dowel and two layers of the joint circumscribed on these polygons (Fig. 4). For tenon and mortise joints, rectangular prisms were built formed by cubical finite elements with a two-layered glue line glued on (Fig. 5).

**Fig. 4. Network of finite elements for the dowel and glue line**

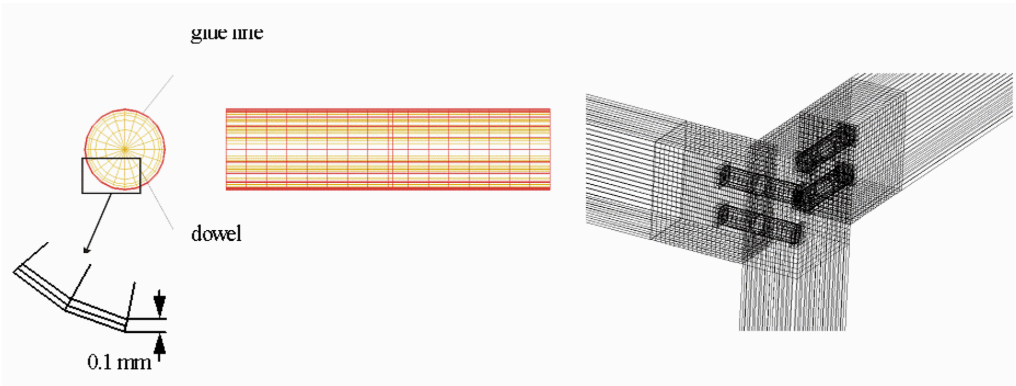
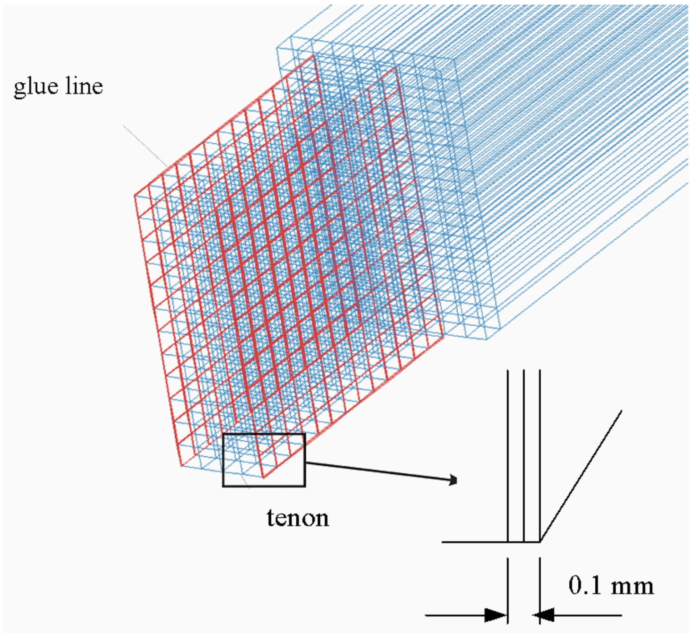
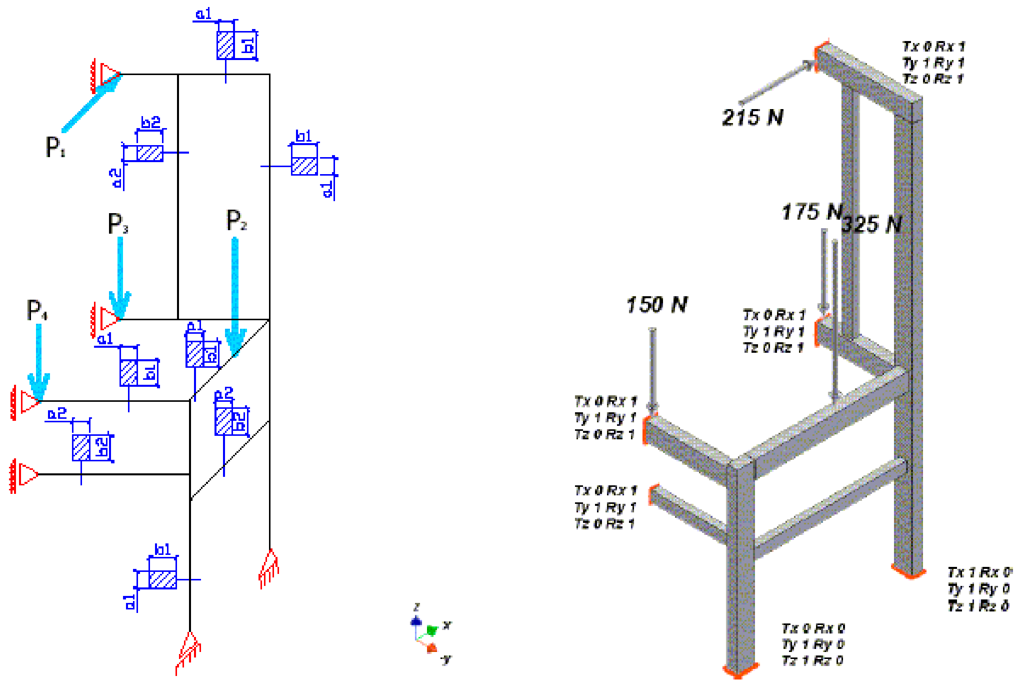



**Fig. 5. Network of finite elements for the tenon and glue line**




The strength analysis of the furniture construction was performed only for the symmetrical half of the spatial structure, loading it in accordance with the stipulations of the PN-ISO 7173:1994 standard (Fig. 6).

**Fig. 6. Static diagram of the furniture frame**




Spatial orthotropic elements from the 3D Continium Elements containing 21 nodes (Fig. 7) were employed as calculating elements. On the other hand, glue lines were treated as isotropic material having the same deformation values in different directions (Fig. 8).

**Fig. 7. Spatial orthotropic element for wood**

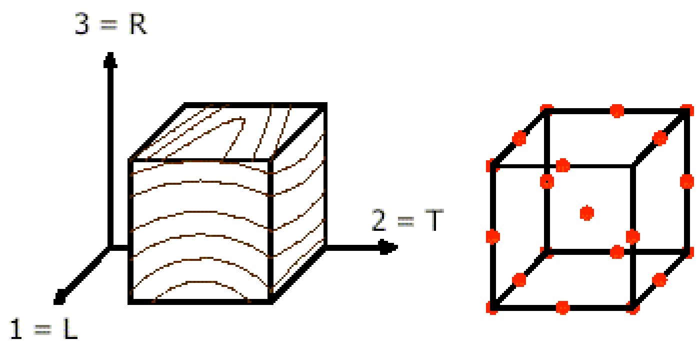
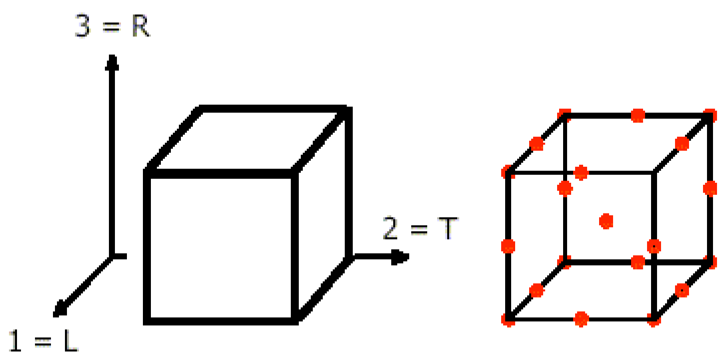



**Fig. 8. Spatial isotropic element for glue line**



In the case of wood, which is an orthotropic material characterised by appropriate interrelationships between $\varepsilon_L$, $\varepsilon_T$, $\varepsilon_R$ deformations and $\sigma_L$, $\sigma_T$, $\sigma_R$ stresses, the following correlations were used for stress tensor:

$$\sigma_i = E_i * \varepsilon_i$$
$$\tau_{ij} = G_{ij} * \gamma_{ij}$$

where for wood:

$$\varepsilon_L = \frac{1}{E_L}\sigma_L - \frac{v_{TL}}{E_T}\sigma_T - \frac{v_{RL}}{E_R}\sigma_R$$

$$\varepsilon_T = -\frac{v_{LT}}{E_L}\sigma_L + \frac{1}{E_T}\sigma_T - \frac{v_{RT}}{ER}\sigma_R$$

$$\varepsilon_R = -\frac{v_{LR}}{E_L}\sigma_L - \frac{v_{TR}}{E_T}\sigma_T + \frac{1}{E_R}\sigma_R$$

and

$$\gamma_{LT} = \frac{1}{G_{LT}}\tau_{LT}$$
$$\gamma_{TR} = \frac{1}{G_{TR}}\tau_{TR}$$
$$\gamma_{RL} = \frac{1}{G_{RL}}\tau_{RL}$$

whereas

$$\frac{v_{LT}}{E_L} = \frac{v_{TL}}{E_T};\ \frac{v_{LR}}{E_L} = \frac{v_{RL}}{E_R};\ \frac{v_{TR}}{E_T} = \frac{v_{RT}}{E_R}.$$

The following values of elasticity characteristics were adopted for beech wood [14]: Young's module: $E_L$ = 15400 MPa, $E_R$ = 2060 MPa, $E_T$ = 1120 MPa, Kirhchoff's module: $G_{LR}$ =1530 MPa, $G_{LT}$ = 1170 MPa, $G_{RT}$ = 450 MPa, as well as Poisson's coefficients: $v_{LR}$ = 0.41; $v_{LT}$ = 0.54; $v_{RL}$ =0.055; $v_{RT}$ = 0.66; $v_{TL}$ = 0.037; $v_{TR}$ = 0.35.

Glue lines were treated as isotropic material having identical elastic properties in different directions and the following values were adopted after Wilczyński [14]: $E$ = 400 MPa, $G$ = 148 MPa, $v$ = 0.3.

## DESCRIPTION OF RESEARCH

All numerical calculations were performed with the assistance of the Algor® package which employs the algorithm of finite elements. Results of stresses for individual joints were collected as follows:

- in the case of dowel joints, either the top or bottom dowels were analysed, depending on where higher stress values occurred. The author used the von Mises hypothesis to determine them. In the case of longitudinal cross sections of glue lines, consecutive places in the centre of two bond layers were indicated determining normal and tangential stresses (Fig. 9). On the other hand, stresses in the dowel were collected on the longitudinal cross section at the distance of 0.5 mm from the dowel centre (Fig. 10).
- tangential stresses in the glue line of the mortise joint were also determined in the centre of two layers on the entire surface of the glue line (Fig. 11).

**Fig. 9. The way stress values in the glue line of the dowel joint were collected**

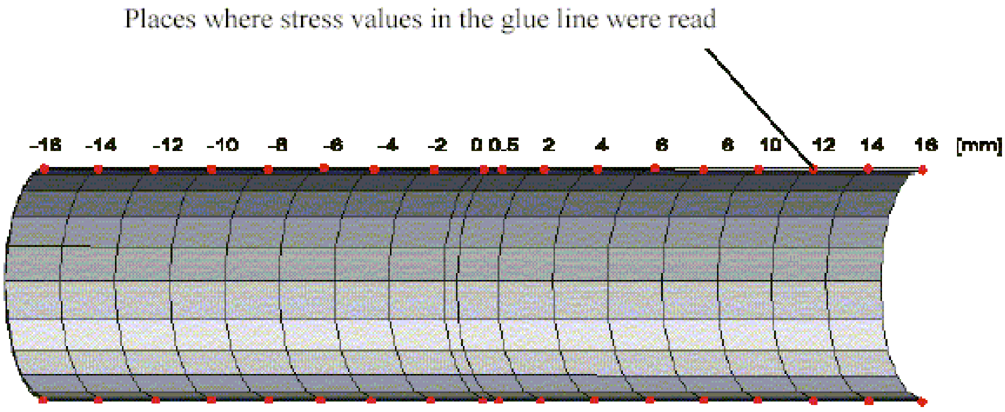



**Fig. 10. The way stress values in the dowel were collected**

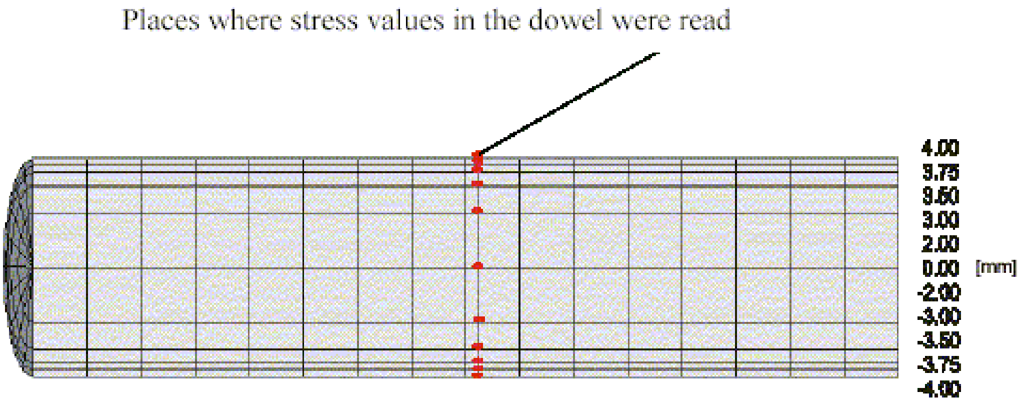
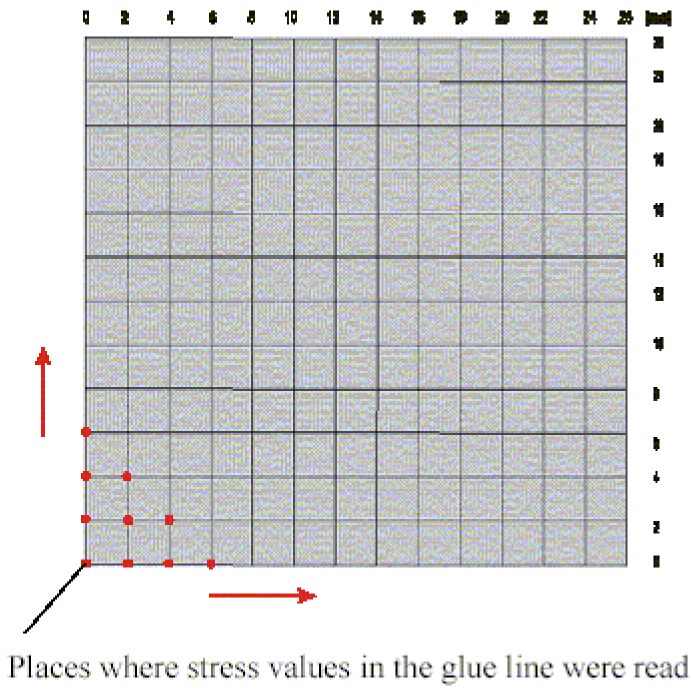



**Fig. 11. The way stress values in the glue line of the tenon and mortise joint were collected**

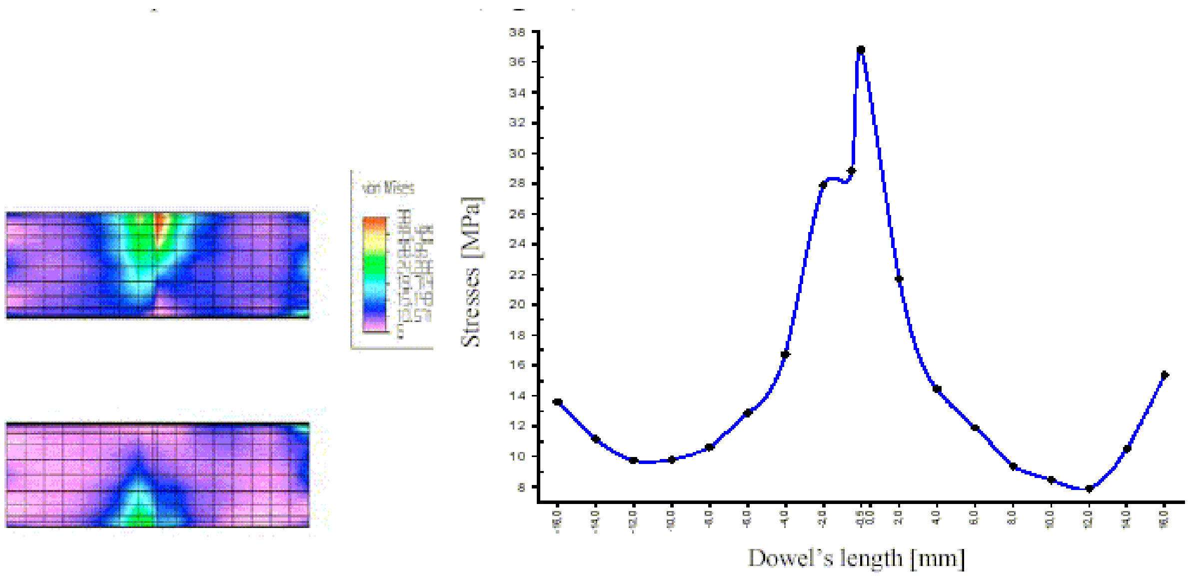


## ANALYSIS OF RESULTS

The obtained results of numerical calculations indicated places in the analysed joints with the highest values of stresses and constituted the basis for considering these points as essential for further analyses of research results.

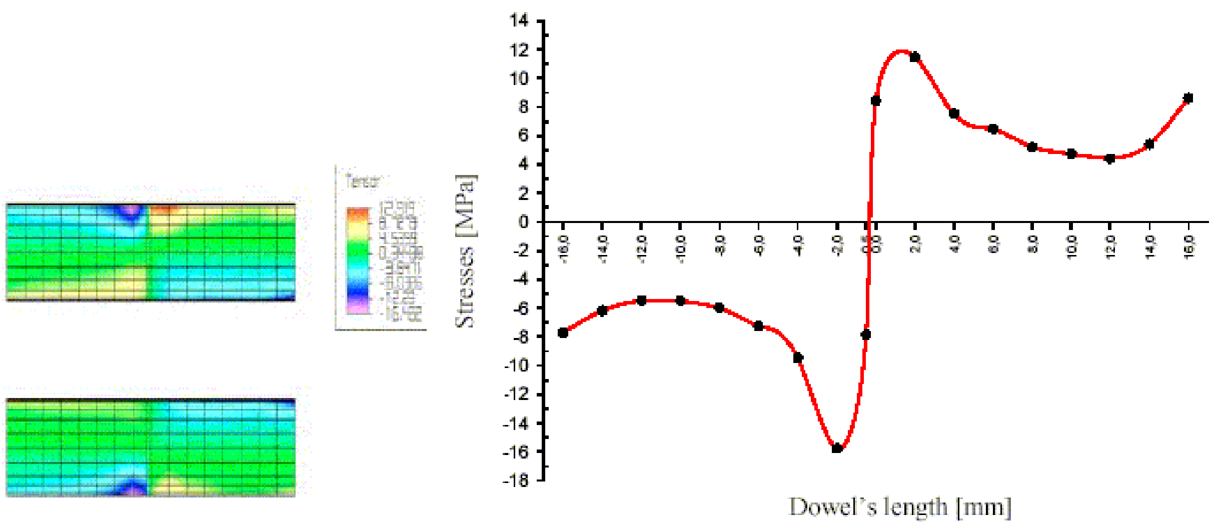
The analysis of the distribution of reduced stresses in glue lines of dowel joints revealed that the biggest of those stresses were generated in joints of the horizontal underframe and the rear leg. They were characterised by the extreme increase of their value in the centre of the glue line, i.e. in the place of osculation of the planes of arms of the joint. Another regularity was the occurrence of minimal stresses usually on the side where the dowel was set parallel to wood fibres (Fig. 12).

**Fig. 12. Distribution of reduced stresses along the glue line (accord. to von Mises)**



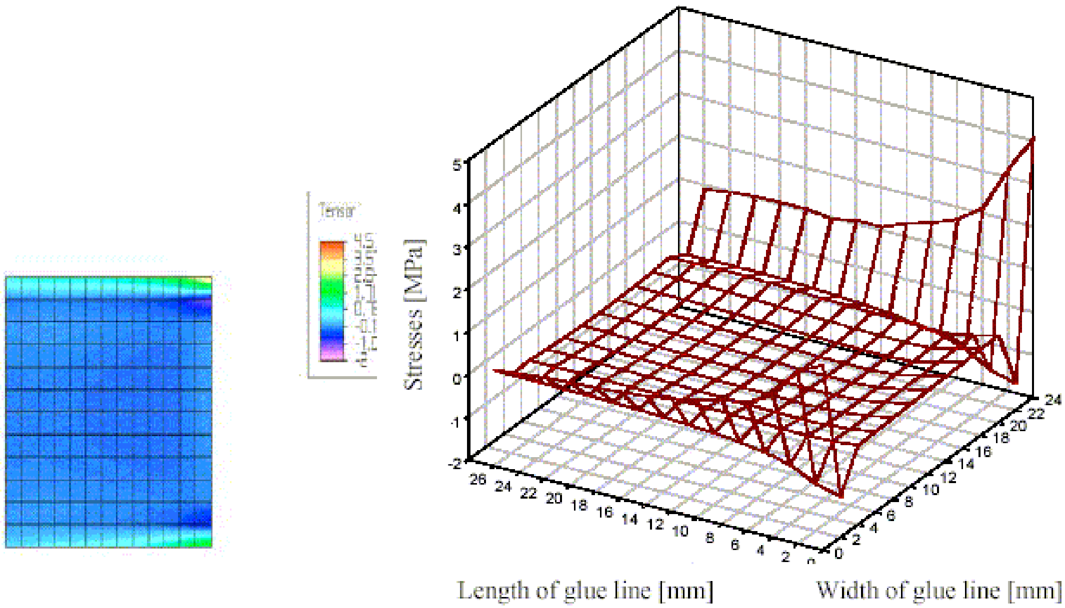

Tangential stresses in glue lines analysed along the length of the discussed dowel joint distributed inversely symmetrically for the right and left sides of the bond (Fig.13). There was a parabolic increase of values towards the centre of the glue line followed by identical values but with the opposite sign on the other side of the dowel. The lowest tangential stresses in the joint glue line began from the value of |6| MPa in the central part of the glue line length formed on the left half of the dowel and increased to the value of |17| and |12| MPa approaching the limiting level of wood and glue spin shear strength.

**Fig. 13. Distribution of tangential stresses $\tau_{XZ}$ along the glue line**



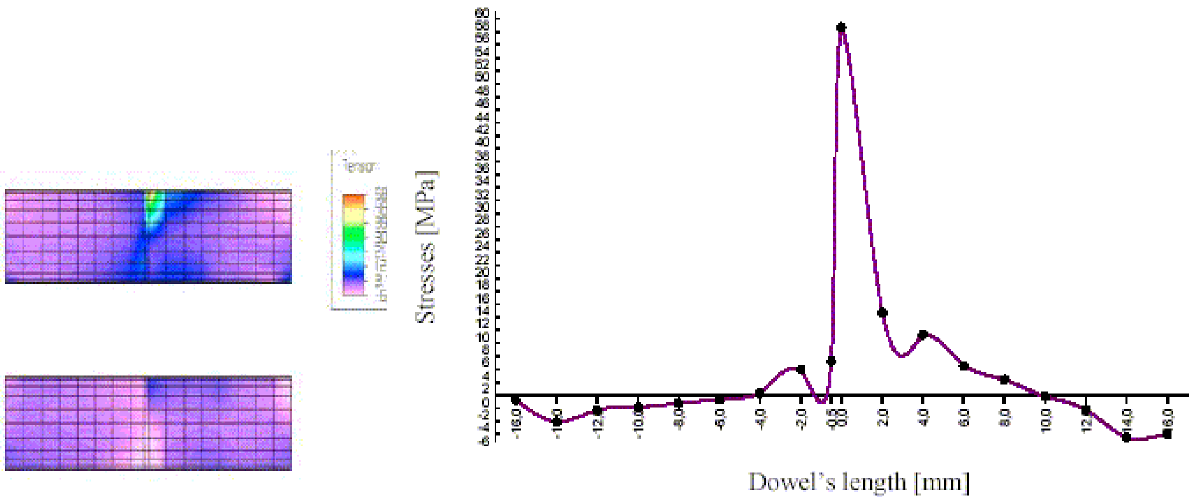
In the case of mortise joints, the highest values of tangential stresses occurred at edges and corners of glue bonds. As evident from the enclosed diagrams (Fig. 14), in the case of connections with mortise joints, we can observe not only shearing of the bond caused by the bending moment but also shearing caused by normal forces visible as permanent stresses along the length of the glue line. Therefore, what is seen is the shearing of the bond caused by the pulling of the tenon from the mortise. It is quite evident from the inventory of maximal stress values shown in Figure 14 that allowable values of shear strength coefficients for glue bonds and wood were not exceeded here.

**Fig. 14. Tangential stresses $\tau_{XZ}$ in the glue line of tenon and mortise joint**



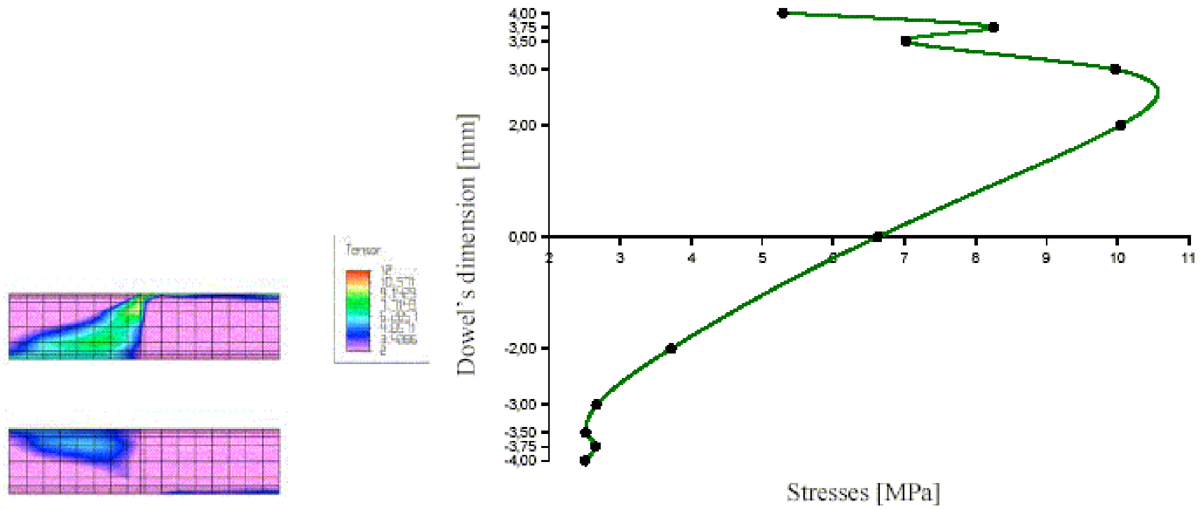

The described tangential stresses occurred in glue bonds only during shear or torsion loads. However, in the course of useful loads, construction nodes were exposed to complex internal loads which generated normal stresses (Fig. 15). A characteristic feature for this type of distribution was the oscillation of $\sigma_Z$ values in the neighbourhood of zero along a considerable length of the glue line set on the side surface of the dowel and their increase in the part corresponding to the half of the dowel length, i.e. at the point of contact of two constituent elements of the chair frame. The value of these stresses reached |60| MPa exceeding considerably the shear and tear strength limits of glue lines.

**Fig. 15. Normal stresses $\sigma_Z$ along the glue line of dowel joint**



While evaluating the strength of furniture constructions, it is also essential to examine the state of tangential stresses in the joint material; this refers, in particular, to wood shear strength. Wood shear strength along its fibres was determined in the central part which is most exposed to damage (Fig.16).
Distance from dowel axis.

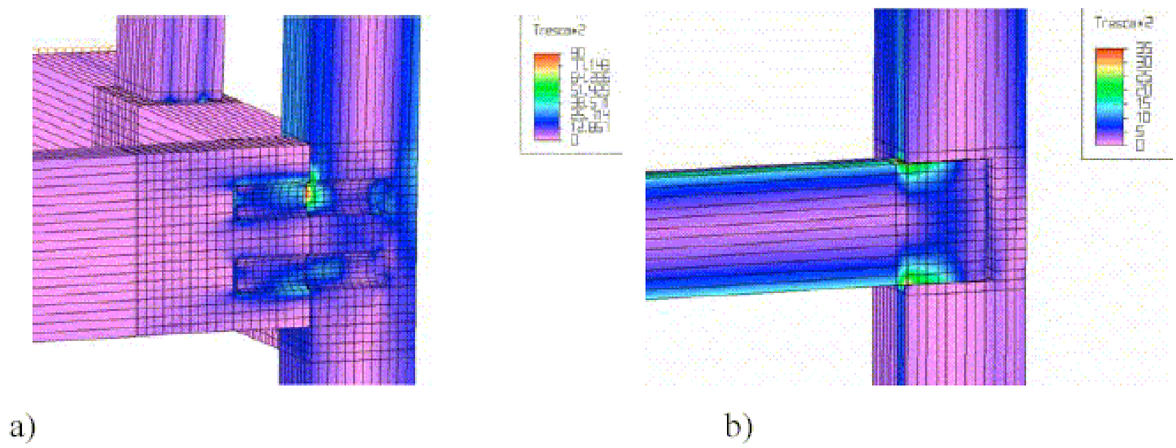
**Fig. 16. Tangential stresses $\tau_{XZ}$ in dowels**




Although the distribution of tangential stresses along vertical cross sectional diameters of dowels shown in Figure 16 was not symmetrical, nevertheless it is possible to notice a certain similarity in the figure with the parabolic distribution transferred to the top or bottom part of the cross section. This transfer is probably associated with either the top or bottom position of dowels in the joint. Another regularity was also visible in the joint dowel, i.e. tangential stresses did not exceed |11| MPa and, therefore, they did not reach values allowable for wood.

When analysing the distribution of stresses in dowel and mortise angle joints or semi-cross furniture constructions, the author decided to present also the distribution of reduced stresses according to Tresci. It is believed that this form of result presentation is more comprehensive and will facilitate the understanding of the problem of stress distribution inside elements of constructions. In order to better illustrate the distribution of reduced stresses, the presented cross section of the joint has glue lines and dowels hidden (Fig.17).

**Fig. 17. The picture of reduced stresses for: a/ dowel joint; b/ tenon and mortise joint**




It is evident from the enclosed drawings that, in places showing the highest reduced stresses in cross sections of construction elements, the highest combined stresses in glue lines and joint elements were also generated.

## CONCLUSIONS

On the basis of the performed calculations, obtained results and their analysis, the following conclusions can be drawn:

1. The strength of construction nodes of skeleton furniture depended primarily on values of normal stresses, which were generated in glue bonds of angle joints.
2. At standard loads (accord. to PN-ISO 7173: 1994), tangential stresses in glue lines did not exceed allowable values for shear strength of wood and glue bonds.
3. Because of considerable dimensions of glue lines in mortise joints, tangential stresses in these points of constructions reached 30% of the value of tangential stresses occurring in glue lines of dowel joints.
4. The level of tangential stresses in the dowel cross section of joints of the side underframe connection with the backrest leg reached maximum values which did not exceed 11 MPa. In this case, these stresses did not exceed values acceptable of shear stresses for wood.
5. The place with the greatest effort in the structure of skeleton furniture was the connection of the side underframe with the backrest leg.

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Jerzy Smardzewski, Tomasz Papuga
Department of Furniture Design
The August Cieszkowski Agricultural University of Poznań
Wojska Polskiego 28, 60-637 Poznań, Poland
e-mail: JSmardzewski@woodcock.au.poznan.pl