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SOME GENERALISATION OF THE EQUATION OF VIRTUAL WORK IN THERMOELASTICITY

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ABSTRACT

The paper presents a global formulation of the initial – boundary problem of coupled thermoelasticity. The principle of virtual work was used. The equation for virtual work in thermoelasticity has been formulated for various types of thermal boundary conditions as well as internal sources of heat. The problem of coupled thermoelasticity in a plane stress state, especially in a rigid disk, is considered in a further part of the paper. In the formulation of the virtual work principle, some changes of the boundary thermal conditions have been considered on a plane bordering the rigid disk, on the assumption that the temperature distribution is symmetrical with respect to the central surface of the rigid disk.

Key words: coupled thermoelasticity, virtual work, rigid disk.

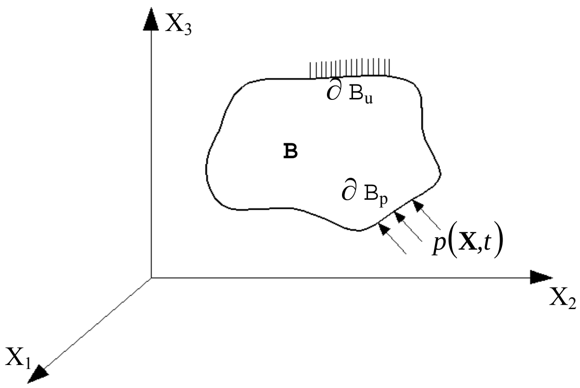
INTRODUCTION

The subject of this paper represents some generalisation of equations for the virtual work principle, coupled with the problem of outset – edge thermoelasticity. The bilateral coupling between the field of strain coincides with the field of temperature. Strains, displacements and stresses are not only a result of the interaction of temperature changes, but also deformation of the body related to these changes. The field of temperature takes place also as a result of changes in the phase or internal friction that exist in the material. In such a case, the coupling between equations of motion and equation of thermal conduction occurs, where a term connected with the deformation of the body appears. The evaluation of the temperature distribution is not possible without simultaneous analysis of the state of deformation. As a rule, the influence of elastic deformation on the field of temperature is, from the quantitative point of view, slight and practically negligible. The coupling of these fields can be very important for great and rapid changes of temperature and very intensive processes of heat conduction. In such a case, the parameters of material, both elastic and thermal, ought to be described as temperature-dependent functions, which consequently leads to formulation of nonlinear, physical equations with variable material functions. In such a case, the coupling effect of the field of deformation with the field of temperature cannot be neglected. The presented in the paper generalisation of virtual work equations of thermoelasticity is based upon simultaneous consideration of different kinds of thermal boundary conditions. The formulation of virtual work equations of thermoelasticity in rigid disks is presented in a further part of the paper. The generalisation of these equations rest upon a consideration – apart from all kinds of thermal boundary conditions at the edge of a rigid disk – also possibilities to change these conditions at the planes limiting the rigid disk.

LOCAL FORMULATION OF EQUATIONS OF THERMOELASTICITY

The body in a region $\overline{\mathrm{B}}$ shown in Fig. 1, which is a subset of an Euclidean three – dimensional space R^3, is examined. B indicates the inside of this region, $\partial\mathrm{B}$ its boundary surface, on one part of which the loading - $\partial\mathrm{B}_p$ and on another part the displacements - $\partial\mathrm{B}_u$ are known.

Fig. 1. Examined solid body



The presented analysis refer to an isotropic, homogeneous medium, geometrically and physically linear. The change of temperature of the considered body is so small, that it does not cause any essential changes in the material coefficients of thermo-elasticity. The initial – boundary problem of thermoelasticity in a local approach is characterized by the following set of differential equations:

1. Equations of equilibrium

$$\sigma_{ji,j} + \rho f_i = \rho \ddot{u}_i \tag{2.1}$$

where: $i,j = 1,2,3$, $\sigma_{ij} = \sigma_{ij}(X,T,t)$, $\boldsymbol{X},T,t \in \mathrm{B} \times \langle T_0, T_1 \rangle \times \langle 0, \infty)$ – components of the stress tensor, and $\sigma_{ji,j} = \dfrac{\partial \sigma_{ji}}{\partial X_j}$, ρ - density, f_i – intensity of body forces per mass unit, $\ddot{u}_i = \dfrac{\partial^2 u_i}{\partial t^2}$ – component of the acceleration vector, T_0 - initial temperature.

2. Equations of geometry

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \tag{2.2}$$

where: ε_{ij} – components of the strain tensor, u_i – component of the displacement vector.

3. Physical equations (Duhamel-Neuman relations) [1,2,3]:

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + (\lambda e - \gamma\Theta)\delta_{ij} \tag{2.3}$$

where: μ, λ - Lame's material constants, e – dilatation, relative change of volume, $\Theta = T - T_0$ – change of temperature, γ - coefficient proportional to specific heat at constant deformation, [J/Km^3].

4. Equation of heat conductivity [1,2,3]:

$$\lambda_0 \Theta,_{ii} - c_\varepsilon \dot{\Theta} - \gamma T_0 \dot{e} + W = 0 \tag{2.4}$$

where: λ_0 – coefficient of thermal conductivity, [W/Km^3], c_ε - specific heat at constant deformation, [J/Km^3], W – output of the internal source of heat, [W/m^3].

5. Boundary conditions
- Statical boundary conditions

$$p_i - \sigma_{ij} n_j = 0 \tag{2.5}$$

where: $p_i = p_i(X, T, t)$, $\boldsymbol{X}, T, t \in \partial \mathrm{B}_\mathrm{p} \times \langle T_0, T_1 \rangle \times \langle 0, \infty)$ – components of the load's vector, n_j – directional cosine of the angle between the vector normal to the boundary area v and X_j axis.

- Geometrical boundary conditions

$$u_i = \hat{u}_i \tag{2.6}$$

$\hat{u}_i = \hat{u}_i(X, T, t)$, $\boldsymbol{X}, T, t \in \partial \mathrm{B}_\mathrm{u} \times \langle T_0, T_1 \rangle \times \langle 0, \infty)$.

- Thermal boundary conditions

a. The first kind of boundary conditions determines at any moment **t** the temperature distribution over the surface of the body, defined by the formula:

$$g_i - \Theta n_i = 0 \tag{2.8}$$

where: $g_i = \hat{\Theta} n_i$, $\hat{\Theta} = \hat{T} - T_0$, $\hat{T}$ –temperature of the fluid surrounding the body, $\partial \mathrm{B}_1$ - part of the boundary area with the thermal boundary of the first kind, $\boldsymbol{X}, t \in \partial \mathrm{B}_1 \times \langle 0, \infty)$.

b. The second type of boundary conditions determines at any moment **t** the distribution of the stream of heat density at a surface of the body, defined by the formula:

$$\frac{\lambda_0}{\alpha} \Theta,_i + k_i = 0 \tag{2.9}$$

where: $k_i = \frac{1}{\alpha} \hat{q}_i$ is a particular function, $\hat{q}_i$ – component of the density vector of the heat flux upon a surface of the body, [W/m^2], α - surface film conductance, [W/m^2], $\partial \mathrm{B}_2$ - part of the boundary surface, where thermal conditions of the second kind are given, $\boldsymbol{X}, t \in \partial \mathrm{B}_2 \times \langle 0, \infty)$.

c. The third type of boundary conditions determines at any time **t** the ambient medium temperature as well as a relationship, which describes the conversion of heat between the body and medium.

$$m_i - \frac{\lambda_0}{\alpha}\Theta,_i - \Theta n_i = 0 \tag{2.10}$$

where: $m_i = \hat{\Theta} n_i$ is a particular function, $\partial \mathrm{B}_3$ - a part of the boundary area, where thermal conditions of the third kind are given, $\boldsymbol{X}, t \in \partial \mathrm{B}_3 \times \langle 0, \infty)$.

6. Initial conditions

- for the displacement

$$\begin{aligned} u_i &= u_{oi} \\ \dot{u}_i &= \dot{u}_{oi} \end{aligned} \tag{2.11}$$

- for the temperature

$$T = T_0 \tag{2.12}$$

The presented above differential equations constitute a set of equations of thermoelasticity. These equations are directly coupled and can not be solved separately. In the case of the problem of decoupling, in the equation of heat conductivity the term connected with deformation of the body vanishes. Then, for the known boundary and initial condition, the classical (simplified) equation of thermal conductivity is solved, which yields the function of temperature changes Θ. For the known temperature, the remaining differential equations of thermoelasticity can be solved.

PRINCIPLE OF VIRTUAL WORK IN THERMOELASTICITY

The starting point for derivation of the virtual work equations is the equation of motion as well as static boundary conditions. If both these equations are multiplied by the virtual displacement δu_i and then properly integrated by volume and boundary surface, one can obtain the following formulae:

$$\int_{\mathrm{B}} \rho f_i \, \delta u_i d\mathrm{B} + \int_{\partial \mathrm{B}_p} p_i \, \delta u_i \, d(\partial \mathrm{B}) - \int_{\mathrm{B}} \rho \ddot{u}_i \, \delta u_i d\mathrm{B} = \int_{\mathrm{B}} \sigma_{ij} \, \delta \varepsilon_{ij} d\mathrm{B} \tag{3.1}$$

The above equation represents a generalisation of the principle of Lagrange's virtual work for thermoelasticity problems. The stress tensor, which appears in the above equation, is a function of temperature. Using Duhamel-Neumann's relations, one may obtain:

$$\begin{aligned} &\int_{\mathrm{B}} \rho f_i \, \delta u_i d\mathrm{B} + \int_{\partial \mathrm{B}_p} p_i \, \delta u_i \, d(\partial \mathrm{B}) - \int_{\mathrm{B}} \rho \ddot{u}_i \, \delta u_i d\mathrm{B} = \\ &= \int_{\mathrm{B}} 2\mu \varepsilon_{ij} \, \delta \varepsilon_{ij} d\mathrm{B} + \int_{\mathrm{B}} e \delta_{ij} \delta \varepsilon_{ij} \, d\mathrm{B} - \gamma \int_{\mathrm{B}} \Theta \delta e d\mathrm{B} \end{aligned} \tag{3.2}$$

In the case of coupled thermoelasticity, it is indispensable to introduce an additional relationship, which takes the phenomena of thermoconductivity into consideration. In further analysis, Biot's vector H , connected with the heat flux density, is introduced by the following relations [1,2,3]:

$$q_i = T_0 \dot{H}_i \tag{3.3}$$

Substituting Fourier's equations to the above expression

$$q_i = -\lambda_0 \Theta,_i \tag{3.4}$$

then multiplying by the virtual increase of Biot's vector and integrating by volume, one may obtain a relationship as follows:

$$\int_{B}\left(\Theta,_i + \frac{T_0}{\lambda_0}\dot{H}_i\right)\delta H_i \, d\mathrm{B} = 0 \tag{3.5}$$

After appropriate transformations of the above relationship, by making use of extended equations of heat conductivity and considering thermal boundary conditions, which have also been multiplied by the virtual increase of Biot's vector and integrated along a proper part of the boundary area, and using the equation of virtual work for thermal conductivity or the second variation equation connected with the process of heat transfer, the following shape was obtained:

$$\begin{aligned}
&\frac{c_\varepsilon}{T_0}\int_{B}\Theta\delta\Theta\, d\mathrm{B} + \gamma\int_{B}\Theta\delta e\, d\mathrm{B} - \frac{1}{T_0}\int_{B}\Theta\delta\omega\, d\mathrm{B} + \frac{T_0}{\lambda_0}\int_{B}\dot{H}_i\,\delta H_i\, d\mathrm{B} + \\
&+\int_{\partial B_1} g_i\delta H_i\, d(\partial \mathrm{B}) + \int_{\partial B_2}\left(\Theta n_i + \frac{\lambda_0}{\alpha}\Theta,_i\right)\delta H_i\, d(\partial\mathrm{B}) + \int_{\partial B_2} k_i\,\delta H_i\, d(\partial\mathrm{B}) + \\
&-\int_{\partial B_3}\frac{\lambda_0}{\alpha}\Theta,_i\,\delta H_i\, d(\partial\mathrm{B}) + \int_{\partial B_3} m_i\delta H_i\, d(\partial\mathrm{B}) = 0
\end{aligned} \tag{3.6}$$

where $\omega = \int_0^t W\,dt + \omega_0$ denotes the global quantity of heat emitted per unit of volume from the time instant t=0 to time t, and ω_0 – determines the initial quantity of heat.

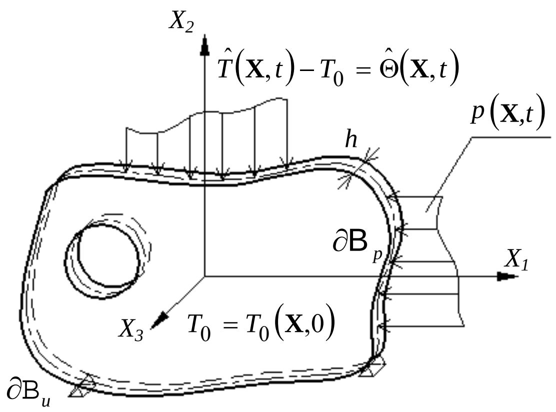
Equations (3.2) and (3.6) are mutually coupled. The expression responsible for coupling is an integral $\gamma\int_{B}\Theta\delta e\, d\mathrm{B}$, which decouples the field of deformation and temperature if nullified in the equation of virtual work of thermoelasticity. Equations of virtual work were also presented in papers [1-3], however internal sources of heat and thermal boundary conditions were neglected there. Equations of virtual work of thermoelasticity presented in [4] were derived by using the variance of temperature increase $\delta\Theta$. In this equation, the boundary conditions of thermal type have not been considered, either.

PROBLEM OF THERMOELASTICITY IN RIGID DISKS

A rigid disk is in a plane stress state in a flat area X_1, X_2 (Fig. 2). Thus, the principal stresses are: σ_{11}, σ_{22} and $\sigma_{12} = \sigma_{21}$. The remaining stresses σ_{33}, $\sigma_{13} = \sigma_{31}$, $\sigma_{23} = \sigma_{32}$, are considered as negligible. Using the condition $\sigma_{33} = 0$, the unit strain ε_{33} can be described as follows:

$$\varepsilon_{33} = -\frac{\lambda}{2\mu+\lambda}\left(\varepsilon_{11}+\varepsilon_{22}\right) + \frac{\gamma\Theta}{2\mu+\lambda}. \tag{4.1}$$

Fig. 2. Considered rigid disk



After substitution of the above to the constitutive equation, the following relationship for the plane stress has been obtained:

$$\sigma_{ij} = 2\mu\left[\varepsilon_{ij} + \frac{1}{2\mu+\lambda}(\lambda e_* - \gamma\Theta)\delta_{ij}\right] \tag{4.2}$$

where: $e_* = \varepsilon_{11} + \varepsilon_{22} = const$.

Using dependence (4.1), one may present the extended equation of thermal conductivity for the a plane stress state as follows:

$$\Theta,_{ii} - \left(\frac{1}{\chi} + \frac{1+\nu}{1-\nu}\alpha_t\eta\right)\dot{\Theta} - \frac{1-2\nu}{1-\nu}\eta\dot{e}_* = -\frac{W}{\lambda_0} \tag{4.3}$$

where: $\chi = \dfrac{\lambda_0}{c_\varepsilon}$, $\eta = \dfrac{\gamma T_0}{\lambda_0}$.

The extended equation of thermal conductivity was also presented in paper [1], but the term $\dfrac{1+\nu}{1-\nu}\alpha_t\eta$ was neglected there. Equation (4.3) is valid under condition that the influence of temperature takes place on the surface of the rigid disk, otherwise there is no exchange of heat in the direction perpendicular to the rigid disk when the disk is thermally insulated by lagging on the areas $X_3 = \pm h/2$. On the assumption of allowable variation of thermal boundary conditions on the planes limiting the rigid disk ($X_3 = \pm h/2$), two cases are possible:

1) symmetrical, with respect to the middle surface of the rigid disk, distribution of temperature,
2) arbitrary distribution of temperature.

In the first case, the plane state of stress will be preserved, but admission of any arbitrary distribution of temperature towards the middle surface of the rigid disk brings an additional bending state (rigid disk – slab). In the following considerations, it is assumed that the temperature distribution $\Theta(X_1, X_2, X_3)$ is symmetrical with respect to the middle surface of the rigid disk [5,6]. After integration of thermal conductivity equation (2.4) along the thickness of the rigid disk, and multiplication by $1/h$, one obtains:

$$\lambda_0(\partial_1 + \partial_2)\frac{1}{h}\int_{-h/2}^{h/2}\Theta\, dX_3 + \frac{\lambda_0}{h}\left[\frac{\partial\Theta}{\partial X_3}\right]_{-h/2}^{h/2} - c_\varepsilon \frac{\partial}{\partial t}\frac{1}{h}\int_{-h/2}^{h/2}\Theta\, dX_3 +$$

$$-\gamma T_0 \frac{\partial}{\partial t}\frac{1}{h}\int_{-h/2}^{h/2} e\, dX_3 + \frac{1}{h}\int_{-h/2}^{h/2} W\, dX_3 = 0 \tag{4.4}$$

Introducing the mean value of temperature, dilatation and internal sources of heat along the thickness of the rigid disk:

$$\begin{aligned}
\overline{\Theta}(X_1, X_2, t) &= \frac{1}{h}\int_{-h/2}^{h/2}\Theta(X_1, X_2, X_3, t)\, dX_3 \,, \\
\overline{e}(X_1, X_2, t) &= \frac{1}{h}\int_{-h/2}^{h/2} e(X_1, X_2, X_3, t)\, dX_3 \,, \\
\overline{W}(X_1, X_2, t) &= \frac{1}{h}\int_{-h/2}^{h/2} W(X_1, X_2, X_3, t)\, dX_3
\end{aligned} \tag{4.5}$$

and taking into account the conditions of unbounded heat exchange in the plane $X_3 = \pm h/2$

$$\left.\frac{\partial\Theta}{\partial X_3}\right|_{X_3=\frac{h}{2}} = \frac{\alpha}{\lambda_0}\widetilde{\Theta}, \qquad \left.\frac{\partial\Theta}{\partial X_3}\right|_{X_3=-\frac{h}{2}} = -\frac{\alpha}{\lambda_0}\widetilde{\Theta}, \tag{4.6}$$

where $\widetilde{\Theta} = \hat{T} - T$ is a difference of temperatures between the surrounding medium and the surface of the rigid disk, leads to:

$$\overline{\Theta},_{ii} + \varepsilon_0\widetilde{\Theta} - \frac{1}{\chi}\dot{\overline{\Theta}} - \eta\dot{\overline{e}} + \frac{\overline{W}}{\lambda_0} = 0, \tag{4.7}$$

where $\varepsilon_0 = \dfrac{2\alpha}{h\lambda_0}$, $\dot{\overline{e}} = \dot{\overline{\varepsilon}}_{11} + \dot{\overline{\varepsilon}}_{22} + \dot{\overline{\varepsilon}}_{33}$.

Using relation (4.1) and denoting the average values of temperature, dilatation and internal heat sources in the following way:

$$\begin{aligned}\overline{\Theta}(X_1, X_2, t) &= \Theta(X_1, X_2, t),\\ \overline{e}(X_1, X_2, t) &= e(X_1, X_2, t),\\ \overline{W}(X_1, X_2, t) &= W(X_1, X_2, t)\end{aligned} \tag{4.8}$$

it was finally obtained:

$$\Theta,_{ii} + \varepsilon_0\widetilde{\Theta} - \left(\frac{1}{\chi} + \frac{1+\nu}{1-\nu}\alpha_t\eta\right)\dot{\Theta} - \frac{1-2\nu}{1-\nu}\eta\dot{e}_* + \frac{\overline{W}}{\lambda_0} = 0, \tag{4.9}$$

where $\dot{e}_* = \dot{\varepsilon}_{11} + \dot{\varepsilon}_{22}$.

The above relationship represents an approximate form of the heat conductivity equations. In the case when the planes limiting the rigid disk $X_3 = \pm h/2$ are insulated, the parameter $\varepsilon_0 = 0$.

PRINCIPLE OF VIRTUAL WORK OF THERMOELASTICITY IN RIGID DISKS

The equation of virtual work and the second equation of variation, connected with heat conduction, have been presented in section 3 of this paper. After using constitutive equations (4.2), generalised Lagrange's principle of virtual work of thermoelasticity in rigid disks can be presented in the following from:

$$\begin{aligned}&\int_{\mathrm{B}} \rho f_i\,\delta u_i\,d\mathrm{B} + \int_{\partial\mathrm{B}_p} p_i\,\delta u_i\,d(\partial\mathrm{B}) - \int_{\mathrm{B}} \rho\ddot{u}_i\,\delta u_i\,d\mathrm{B} =\\ &= \int_{\mathrm{B}}\left(2\mu\varepsilon_{ij}\,\delta\varepsilon_{ij} + \frac{2\mu\nu}{1-\nu}e_*\delta\varepsilon_{kk}\right)d\mathrm{B} + \frac{1-2\nu}{1-\nu}\gamma\int_{\mathrm{B}}\Theta\delta\varepsilon_{kk}\,d\mathrm{B} = 0.\end{aligned} \tag{5.1}$$

Adding the thermal boundary conditions, which were integrated along the proper part of the boundary surface to (3.5), and multiplying by the variation of Biot's vector δH_i, one obtaines for the stress state:

$$\int_{\mathrm{B}}\left(\Theta,_i + \frac{T_0}{\lambda_0}\dot{H}_i\right)\delta H_i\,d\mathrm{B} + \int_{\partial\mathrm{B}_1}(g_i - \Theta n_i)\delta H_i\,d(\partial\mathrm{B}) + \tag{5.2}$$

$$+\int_{\partial B_2}\left(\frac{\lambda_0}{\alpha}\Theta,_i+k_i\right)\delta H_i\,d(\partial B)+\int_{\partial B_3}\left(m_i-\frac{\lambda_0}{\alpha}\Theta,_i-\Theta n_i\right)\delta H_i\,d(\partial B)=0\,,\qquad i=1,2.$$

Taking advantage of the approximate form of extended equation of heat conductivity (4.9) as well as relationships (3.3) and (3.4), one writes down:

$$\dot{H}_{i,i}=\frac{\lambda_0}{T_0}\left[\varepsilon_0\widetilde{\Theta}-\left(\frac{1}{\chi}+\frac{1+\nu}{1-\nu}\alpha_t\eta\right)\dot{\Theta}-\frac{1-2\nu}{1-\nu}\eta\,\dot{e}_*+\frac{W}{\lambda_0}\right], \tag{5.3}$$

$$H_{i,i}=\frac{\lambda_0}{T_0}\int_0^t\left[\varepsilon_0\widetilde{\Theta}-\left(\frac{1}{\chi}+\frac{1+\nu}{1-\nu}\alpha_t\eta\right)\dot{\Theta}-\frac{1-2\nu}{1-\nu}\eta\,\dot{e}_*+\frac{W}{\lambda_0}\right]dt=$$

$$=\tau-\frac{\lambda_0}{T_0}\left(\frac{1}{\chi}+\frac{1+\nu}{1-\nu}\alpha_t\eta\right)\Theta-\frac{\lambda_0}{T_0}\frac{1-2\nu}{1-\nu}\eta\,e_*+\frac{\omega}{\lambda_0}, \tag{5.4}$$

where

$$\tau=\frac{\lambda_0}{T_0}\varepsilon_0\int_0^t\widetilde{\Theta}\,dt+\tau_0\,. \tag{5.5}$$

The variation of Biot’s vector can be described as follows:

$$\delta H_{i,i}=\delta\tau-\frac{\lambda_0}{T_0}\left(\frac{1}{\chi}+\frac{1+\nu}{1-\nu}\alpha_t\eta\right)\delta\Theta-\frac{\lambda_0}{T_0}\frac{1-2\nu}{1-\nu}\eta\,\delta e_*+\frac{1}{\lambda_0}\delta\omega\,. \tag{5.6}$$

After proper transformation and use of the Gauss-Ostrogradski’s theorem as well as relationships (5.6), equation (5.2) can be presented as:

$$\left(\frac{c_\varepsilon}{T_0}+\frac{1+\nu}{1-\nu}\gamma\alpha_t\right)\int_B\Theta\delta\Theta\,dB+\frac{1-2\nu}{1-\nu}\gamma\int_B\Theta\delta e_*\,dB-\int_B\Theta\delta\tau\,dB+\frac{1}{T_0}\int_B\Theta\delta\omega\,dB+$$

$$\frac{T_0}{\lambda_0}\int_B\dot{H}_i\,\delta H_i\,dB+\int_{\partial B_1}g_i\,\delta H_i\,d(\partial B)+\int_{\partial B_2}\left(\Theta n_i+\frac{\lambda_0}{\alpha}\Theta,_i\right)\delta H_i\,d(\partial B)+ \tag{5.7}$$

$$\int_{\partial B_2}k_i\,\delta H_i\,d(\partial B)-\frac{\lambda_0}{\alpha}\int_{\partial B_3}\Theta,_i\,\delta H_i\,d(\partial B)+\int_{\partial B_3}m_i\,\delta H_i\,d(\partial B)=0\,.$$

The above relationship presents the second variation equation connected with heat conductivity for a plane stress.

CONCLUSIONS

In the paper, a generalised form of virtual work in thermoelasticity coupled with different types of thermal boundary conditions and internal sources of heat is presented. Equations for the virtual work principle related referred a rigid disk by different scenarios of temperature changes along the thickness of the rigid disk as well as change of boundary conditions are also formulated. Such a case was presented by Nowacki [1,2,3], but the problem of internal source of heat and different types of boundary conditions in equations of virtual work in thermoelasticity was neglected in his research. The second variation equation of thermoelasticity connected with heat conductivity, presented in this paper, was introduced by using the variation of Biot’s vector δH_i. Somewhat different approach was presented by Kączkowski [4], where the variation of temperature increase $\delta\Theta$ was used to the formulation of virtual work principles.

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