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TESTING OF MATHEMATICAL MODELS FOR ASTRONOMICAL REFRACTION ELIMINATION

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[ABSTRACT](#)
[INTRODUCTION](#)
[THE PROCESS OF MEASUREMENT](#)
[THEORETICAL BASES OF REFRACTION MODELS](#)
[DATA PROCESSING](#)
[CONCLUSIONS](#)
[REFERENCES](#)

ABSTRACT

The effect of refraction on the results of astronomical measurement by using the discussed method of the deflection of the vertical determination is assessed in the article. Values obtained by using formulas for astronomical refraction modelling by means of meteorological data were compared with those obtained directly by processing of measured angles. Various models for calculation of astronomical refraction were also compared. Statistical analysis was used for the comparison. The testing was aimed at the influence of daytime, season, temperature, atmospheric pressure and point position on the value of refraction. The acquired results give evidence that astronomical refraction can be eliminated not to impair the results of measurement.

Key words: astronomical measurement, refraction,

INTRODUCTION

At the Department of Surveying (Faculty of Civil Engineering, University of Technology in Brno, Czech Republic) the method of deflection of vertical determination taken from combination of the astronomical measurements executed by total station and measurements obtained from GPS has been developing since 1998. More details are described in [1]. The principle of the method of astronomical measurements is the stars zenith angles measurement at given time moments. With assumption that the stars coordinates are known, it is possible to compute astronomical station coordinates. The mentioned method of astronomical measurement is in stage of testing; it is based on repeated observation at the points with known geographic coordinates. It has enabled to gather gross amounts of data used in other applications and experiments as well. One of them is the opportunity of comparing the values of refraction angles (obtained from astronomical measurements) with the values gained from the various models for calculation of astronomical refraction (mathematical models, if you prefer). In general, the astronomical refraction is considered to be better simulatable than the terrestrial one.

THE PROCESS OF MEASUREMENT

The measurements took place in the year 2002 at stations on the terrace of building B in the area of Faculty of Civil Engineering (in spring and in winter) and in the experimental network Kralický Sněžník as well (the spring and the summer 2002). The observations were performed in different parts of night and in different climatic and microclimatic conditions. Each time the instruments were stationed on pillars. Mostly the Topcon GTS 6A instrument was used, or Topcon GTS 300 with the same indicated angle accuracy was used. The data were gathered to portable PC and PC-time rectified to the world time UTC by recording the radio time signal DCF 77.

The principle of the astronomic measurement

Our measuring scheme uses the method of elevation position lines specially adapted for precise measuring [1]. As compared with special astronomical-geodetic instruments it is necessary to take account of impossibility of targeting of the moving star more times than once with unchanged position of the telescope. It means that we are able to obtain just one result of measuring from the star crossing over the hair cross. That was the reason for us to invent a new method of measuring, where mentioned confinement is respected. In this way of measuring zenith angles and times are logged to the PC. Each sight line inputs to the process of computation one independent value.

Stars are measured in pairs. The stars in one pair have to be under near the same zenith angle, but in different course bearing of 180 degrees and time interval from 4 to 12 minutes. Common processing of the pairs, chosen by the way mentioned above, removes the astronomical refraction effect on the result of measuring (astronomical coordinates). We need not simulate the values of refraction, but we are able to compute them directly from geodetic observations [1].

Within the testing of the hypothesis of eliminating the effect of refraction on resulting coordinates the meteorological data were measured to be used as entrance values for mathematical models of refraction. It allows the computation of the coordinates by using the postulates about the elimination of the effect of refraction just by special choice of measuring configuration and processing method as well. The results of both accesses appeared nearly the same.

The registration of meteorological data

The input parameters for refraction angles computation were the values of temperature and pressure measured near the station of observation. For calculation of values of refraction angles gained on terrace were used the meteorological data taken from record of meteorological station, situated at the doorstep of observation pillars. This station is a part of permanent GPS station TUBO. Outputs of meteorological station are accessible on web at the address <http://tubo.fce.vutbr.cz>. In the case of measuring in the area of Kralický Sněžník, meteorological data were taken from portable electronic instruments. Period of one meteorological record at TUBO meteorological station was 1 minute in contrast to 20 minutes at Kralický Sněžník area.

THEORETICAL BASES OF REFRACTION MODELS

For calculation of refraction angles based on measured meteorological data, three models were used. All ways of computation are based just on the temperatures and pressures measuring near the instrument and the star zenith angle.

1) refraction scheme of Radau. The value of refraction angle R is given by

$$R = R_0 \cdot G \cdot K, \quad (1)$$

where the values of R_0 , G and K are interpolated from schedule. R_0 is called normal refraction; it is a function of measured zenith angle z' . G and K are correction terms for effect of pressure and temperature. For example see [4].

2) the second method stems from the formula (1) mentioned above, but it permits analytical computation by using hand calculator without any tables, [4]. The calculus of each form is following:

$$R_0 = \text{tg } z'(57.586'' - 0.0668'' \text{tg}^2 z'), \quad (2)$$

$$G = \frac{B}{1001,931}, \quad (3)$$

$$K = \left(\frac{1,03417}{1 + 0,00367 \cdot T_v} \right)^{1,0003 + 0,00132 \text{tg}^2 z'}. \quad (4)$$

The values B and T_v are the values of pressure [mbars] and temperature [C].

It is possible to apply the formulas assigned at points 1) and 2) for zenith angles smaller than 60. For discussed reasons, these are fully sufficient, because zenith angles in range of 25-30 were used in our case.

3) the third formula was taken from "astronomical almanach" (published in Russia).

$$\log R = m_i + \log \text{tg } z' + \lambda + \gamma + B, \quad (5)$$

where m_i , λ , γ and B are factors interpolated in tables [5]. First two factors are the function of zenith angle z , γ is a function of temperature and B is the function parameter of pressure. This formula can be applied till zenith angle of 80°.

DATA PROCESSING

Relative comparison of the refraction angle models

At [table 1](#) there are shown values of refraction angles counted by formulas mentioned above. The numeric differences between each method are less than 0.1°.

In respect to the accuracy of measured zenith angles the differences in calculated refraction angles are irrelevant. The standard error is larger than 1, [2], it is significantly larger than ascertained variances in each theorem. That's the reason why there was tested just theorem number 2).

Table 1. Comparison the models for calculation the refraction angles

Entrance data			Counted values of refraction angles according to formulas ["]		
zenith angle [°]	Temperature [°C]	Pressure [mbar]	ad1)	ad2)	ad3)
30.0	11.0	972	32.0	32.1	32.1
30.0	6.8	979	32.7	32.8	32.8
20.0	12.1	897	18.6	18.6	18.7
30.0	3.8	979	33.0	33.1	33.1
30.0	20.0	976	31.1	31.1	31.1
35.0	9.7	944	37.9	37.8	37.9

Comparison of the mathematically simulated refraction angles with refraction angles determined from measured data directly

For sampling usability and accuracy of chosen statistical analyses theorem of measured data was applied. At first we had computed the refraction angle of each sight line (zenith angle computed from coordinates minus measured angle). This angle was then named refraction angle computed. Both refraction angles would be in range of measuring errors the same their difference would be near to zero. This difference is the subject of debates described below. The differences, values of zenith angles, temperatures and pressure make the sets for statistical analyses. It is possible to determinate the effects of zenith angle, temperature, pressure or the other exterior influences on the value of difference. The main hypothesis can be stated as:

$$\text{difference} = 0.$$

The hypothesis presupposes that the measured refraction angle and the computed one are consistent. If we refuse this hypothesis in any chosen set, it will be indication of no consistency. It is indication of some effects we dont include in our model. It could be atmosphere influence (different configuration of air spaces compared with the theory presumptions), local influences (abnormality of station surroundings), instrumental influences (errors in outputs from instruments thermometer, barometer, etc.) or other errors.

When we refuse the fundamental hypothesis at one selective set, then we can test other hypothesis. We can test an effect of zenith angles, temperatures, pressure, daytime (alternate air space arrangement in different night-times), season, stations option or technique of meteorological data gathering on the difference.

All measurement of astronomical-geographic coordinates, made by one instrument, by one observer during one night upon just one station, was selected as a fundamental unit, called observe-night. During one night there is, of course, possible to observe upon more than just one station, such measurings were processed separately. These are different observe-nights.

The homogeneity of data files

By reason of large time span between the first measurement and last one (6 months) there came out the problem of instrument time-stability. Data gathered from winter campaign had showed indispensable differences in comparison with the autumn campaign. Therefore the sets were divided into two parts (winter + spring) and (autumn). At the first part, there was determined index error 14.7for used instrument Topcon GTS 6A, in the second one 13.5.

The first part of measuring (winter + spring) served mainly for testing the effect of temperature and general calculability of theorem and the second part (autumn) of measuring for testing the effect of zenith angle, time of measuring and microclimate of station.

Measurement characteristics of both parts was very different. First part (winter + spring) contained nearly pure the measuring upon just one station, situated high above the terrain, in so far without the effect of station microclimate. In the second part (autumn), on the contrary, measurements were carried out on different stations. There were stations with varied and bad microclimate surroundings (stations situated on slope, at the high growed forest, sights near the terrain surface).

The testing of partial sets

The sets were tested for equality of mean value to zero, with presumption of unknown standard error. The criterion

$$R = \frac{\bar{X} - mi_0}{S} \sqrt{n} \quad (6)$$

was tested by Student distribution on significance level $\alpha = 5\%$. The values \bar{X} and S are unbiased estimators of the mean value and variance, mi_0 is estimated mean value of the set. In this case $mi_0 = 0$.

The hypotheses “difference = 0” was rejected by statistical test just in two cases from the part (winter + spring). In the other sets, there wasn't certified deviation of the measured zenith angles from computed ones. So, there wasn't proved the effect of different day-time (beginning of night, midnight, early morning), effect of the zenith angle, temperature or pressure. In the second part of the set (autumn) there wasn't proved the effect of microclimate of surroundings the station either. These findings are necessary to be considered in connection to power of the test. It is defined by standard error of measurement $3''$, the size of sets from 14 till 30 refraction angles and the significance level $\alpha = 5\%$.

Two sets with systematic influence proved, were the output of field measurements (the area of Králický Sněžník). Strong overrunning of critical value of the test was caused (with the highest probability) by wrong placement of instruments for measuring meteorological data (they were laying on the ground and way out of station). It means, that there hadn't been measured the temperature at the instrument on the pillar, but the temperature at the ground level. The difference could be more than several degrees centigrade.

CONCLUSIONS

- the tested models used for calculation of astronomic refraction differ each other in range of $1''$. This finding is valid for any night-time, zenith angles less then 35° and the temperature in the range $0 - 20^\circ\text{C}$.
- if the station is suitably chosen (distance between the sights and the blocks are ample), the effect of microclimate doesn't prove.
- at the field measurement it is necessary to respect advisable position of thermometer. It is unsuitable place it on the high tempered roof of the car, on the rock or on the ground.
- all instruments for meteorological data measuring have to be calibrated and regularly checked
- measurement of conditions of atmosphere every 20 minutes is possible to consider as a sufficient.

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