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GRADIENT OPTIMISATION OF SKELETON FURNITURE WITH DIFFERENT CONNECTIONS

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ABSTRACT

The objective of this study was to determine the impact of joint rigidity on results of skeleton furniture construction optimisation expressed by the function of minimum material volume. The performed experiments proved that joint rigidity exerts a significant effect on optimisation results both with regard to the difference of individual dimensions of element cross-sections and total construction volume. Gradient optimisation allowed reducing the volume of skeleton constructions by 72 – 76% of their initial volume within the time of less than 1.4 hours.

Key words: chair, joints, optimisation

INTRODUCTION

In a typical algorithm of numerical determination of stresses using the finite element method (FEM), it is assumed that the constructional node is completely rigid or constitutes an articulated joint. The first case constitutes an approximation of a furniture joint made with the use of glue, while the other occurs, among others, in a three-element articulated joint of side frames of garden furniture [8].

It is possible to find in literature suggestions concerning methods that allow taking into account joint properties in rigidity-strength calculations of furniture and wooden constructions. Eckelman and Suddarth [3] came forward with a computer program in the Fortran lV language employing their own matrix algorithm for calculating 2D frames in furniture constructions. In their experiments they used dowel joints which were modelled as semi-rigid nodes with a non-linear correlation between their bending moments and angle deformations. The paper puts forth the division of the non-linear domain of the angle node deformation function into a series of intervals and then the determination, in each of the intervals, a linear correlation which would approximate the initial deformation function. On the other hand, Hrčka [5] carried out an analysis of strength and rigidity of a wood construction connected by means of Unimot joints and employed two different approaches to the theoretical analysis of joint rigidity. The first approach assumed the introduction, into the static scheme, of semi-rigid bar elements to replace semi-rigid joints, while the second approach consisted in assigning constructional nodes appropriate reaction coefficients which expressed the relationship between the deformation angle and bending moment operating in the node. Dzięgielewski and Smardzewski [2] carried out laboratory experiments on wall angle joints in skeleton furniture and, for each type of the examined joints, determined equivalent modulus of joint elasticity. The obtained results provided, on the one hand, an estimation of their rigidity and, on the other, served as data for numerical calculations of rigidity of furniture bodies. Elements with equivalent moduli of elasticity were then introduced into the static scheme of the analysed construction to substitute real joints.

In investigations on optimisation of wood constructions carried out so far researchers did not take into account characteristic traits of furniture joints assuming that their rigidity corresponded to the rigidity of the adjacent material. Smardzewski [8] carried out a numerical dimension optimisation of constructional nodes in a side frame of a chair employing the method of random walk and systematic search, while Smardzewski and Dzięgielewski [9] performed construction optimisation of cabinet furniture employing the method of random walk. On the other hand, Smardzewski and Gawroński [10] succeeded in integrating numerical methods of static optimisation in FEM environment and developed an optimisation algorithm of skeleton furniture in which values of internal forces were determined after each step of optimisation. This allowed taking into consideration the phenomenon of the alteration in the distribution of values of internal forces in a construction statically indeterminable in the result of the change of the cross section of component elements. Furthermore, the authors also reduced joints to rigid constructional nodes assuming that the rigidity of joints corresponded to the rigidity of the adjacent material.

Continuous quality and reliability improvement of computers as well as their widespread application in design offices of furniture factories encourage elaboration of algorithms of optimisation of furniture constructions in which rigidity characteristics of the applied joints would be taken under consideration. It is, therefore, reasonable to replace statistical methods of furniture optimisation, which have been applied so far, by gradient methods which allow obtaining much more precise results.

OBJECTIVE OF THE RESEARCH PROJECT

The aim of this study was to determine the influence of joint rigidity on results of skeleton furniture construction optimisation as expressed by the function of minimum material volume. In addition, in the course of the performed investigations assessment of the effectiveness of the gradient optimisation method was also carried out.

RESEARCH METHODOLOGY

The object of optimisation was a chair frame with a crosspiece (Fig. 1) modelled in accordance with the assumptions of the finite element method (FEM). The permanent factors of the performed optimisations included: locations of individual constructional nodes, value of the operating outside load, kind of the applied material, type of joint as well as minimal and maximal sectional dimensions of beam elements. On the other hand, variable factors comprised: dimensions of the cross-section of beam elements, values of internal forces and stresses in elements. The authors observed volume changes of the optimised constructions and time required to carry out calculations on the basis of which effectiveness of the applied gradient optimisation algorithm was assessed. The comparison of optimisation results of individual furniture pieces with different joints was used to estimate the influence of joint rigidity on the result of construction optimisation.

Because of the symmetry of the spatial configuration, the calculation model was restricted only to the side frame of the examined piece of furniture (Fig. 2) retaining all its material and load characteristics. Simultaneously, it was decided that the frame would be made from beech and pinewood, while joints will include stop-housed joints and dowel joints. In order to compare optimisation effects on models similar to real ones, the authors carried out construction optimisation also on nodes whose rigidity corresponded to the rigidity of the adjacent material.

Fig. 1. Object of optimisation

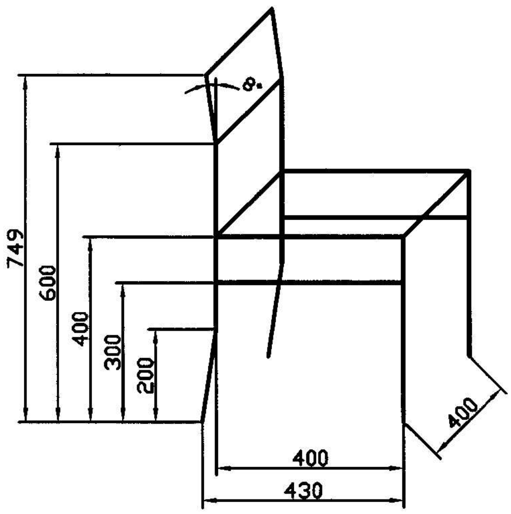
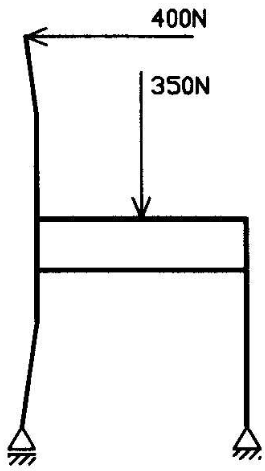
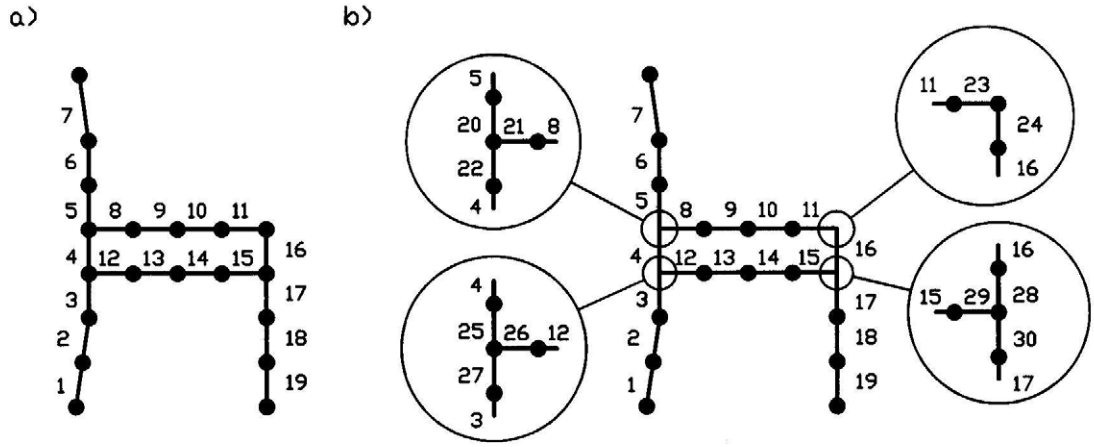


Fig. 2. Static scheme of the optimised construction



Bearing in mind assumptions adopted in earlier experiments concerned with construction optimisation [8,10] and requirements concerning chair strength, forces of 700N operating on the seat and 800N affecting the back support were assumed as outside loads of the considered construction. Limiting strength calculations to the side frame of the examined piece of furniture, only half of the value of the outside operating load was taken under consideration (Fig. 2). The way of discretisation of the construction containing joints with rigidity corresponding to that of the adjacent material is presented in Fig. 3a. On the other hand, in the case of constructions with stop-housed and dowel joints in the neighbourhood of constructional nodes, the authors applied additional beam elements of 10 mm length which were characterised by equivalent moduli of elasticity E_z of the joint (Fig. 3b). The deflection of beams modelled in this way simulated true deformations of furniture joints.

Fig. 3. Way of discretisation of the chair construction:
a/ construction with theoretical nodes b/ construction with elements of equivalent modulus of elasticity



Decision variables of the optimisation process comprised the height of the rectangular cross-section of individual beam elements. The vector of decision variables for such a system adopted the following form:

$$\bar{x} = \{h_i : i = 1 \ldots n\} \quad (1)$$

where:
h_i – section height of the finite element i,
n – number of finite elements.

Breadths of rectangular cross-sections of furniture elements were calculated on the basis of a constant, experimentally assessed cross-sectional ratio of breadth to height (b/h) of beam elements equalling 5/7.

The acceptable area comprised a set of inequality constraints $\varphi_j(\bar{x}) \geq 0$ where j is the number of constraint conditions. Because of a negligible proportion of stresses derived from normal and shear forces occurring in the side frame of a chair, the condition of maintaining the appropriate strength of the construction was achieved by limiting the value of the bending moment operating in individual elements of the construction:

$$\frac{6M_i}{b_i h_i^2} \leq k_{g(i)} \quad (2)$$

where:
M_i – maximal bending moment in element i,
$k_{g(i)}$ – acceptable bending stress in element i.

In the algorithm for determining the size of the acceptable area, for constructional, technological and aesthetic reasons, it was necessary to take into consideration the condition of maintaining values of individual decision variables in an arbitrarily determined interval:

$$h_{\min} \leq h_i \leq h_{\max} \quad (3)$$

A natural optimisation criterion constituting minimum volume of the consumed material was assumed as the objective function:

$$V(\bar{x}) = \sum_{i=1}^{n} b_i h_i l_i \rightarrow \min \quad (4)$$

where:
l_i – length of the finite element i.

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In order to avoid problems associated with taking into account requirements of the acceptable area, the task of optimisation was reduced to the minimisation of the objective function without constraints. For this reason, a method of outside punishment function was applied which allowed fulfilling limiting conditions indirectly. The new, modified objective function was written down as follows:

$$Q(\bar{x}) = V(\bar{x}) + B(\bar{x}) \rightarrow \min, \tag{5}$$

On the other hand, the general form of the barrier function was expressed using the following formula:

$$B(\bar{x}) = r\sum_{j=1}^{m} \frac{1}{\varphi_j(\bar{x})} \tag{6}$$

where r designates a constant coefficient, which assures the maintenance of appropriate proportions between values of the initial objective function and barrier function.

Assuming as a function of constraints for expressions (2) and (3) and by introducing expressions (4) and (6) into formula (5), the following final version of the objective function was obtained:

$$Q(\bar{x}) = \sum_{i=1}^{n} b_i h_i l_i + r\sum_{i=1}^{n}\left(\frac{1}{k_{g(i)} - \frac{6M_i}{b_i h_i^2}} + \frac{1}{h_i - h_{i(\min)}} + \frac{1}{h_{i(\max)} - h_i} \right) \tag{7}$$

In order to determine equivalent moduli of elasticity E_z of the applied furniture joints, an analytical model of a constructional node presented in Fig. 5 was elaborated. The described joint, under the influence of a known force P, undergoes a deformation in the result of which point A moves by the distance of δ. It was assumed that the deflections of beams in sections $\overline{AB}$ and $\overline{DE}$ correspond to deflections of wood constructional elements, while those observed in sections $\overline{BC}$ and $\overline{CD}$, to deformations of the furniture joint. The length a of $\overline{BC}$ and $\overline{CD}$ sections is 1 cm, which corresponds to the length of beam elements simulating deformations of constructional nodes in the discretised static scheme of the optimised construction. The elasticity modulus of E_d beams in $\overline{AB}$ and $\overline{DE}$ sections corresponded to Young's modulus of the applied wood, while the E_w elasticity modulus ascribed to beams in sections $\overline{BC}$ and $\overline{CD}$ was equal to the sought equivalent modulus of elasticity of the E_z constructional node.

Therefore, the δ displacement is the apparent sum of displacements resulting from deflections of wood elements δ_d and the constructional node δ_w:

$$\delta = \delta_d + \delta_w \tag{8}$$

Solving the assumed structure using the force method

$$\delta = \frac{1}{E_d J}\int_{A_d} M\overline{M}dA_d + \frac{1}{E_w J}\int_{A_w} M\overline{M}dA_w \tag{9}$$

where:
M – true bending moment in beam elements,
$\overline{M}$ – virtual bending moment in beam elements,
J – axial moment of inertia of section,

and performing graphic integration, the displacement of the load point A was determined assuming the following form:

$$\delta = \frac{Pb^3}{3E_d J} + \frac{P}{E_w J}\left(\frac{1}{3}a^3 + b^2 a\right) \tag{10}$$

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Fig. 4. Model for the determination of the equivalent modulus of elasticity of the constructional node

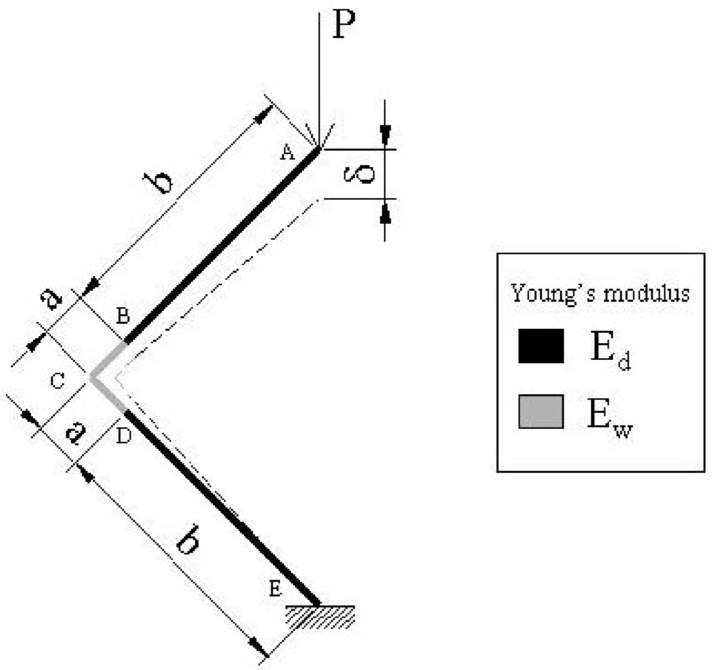
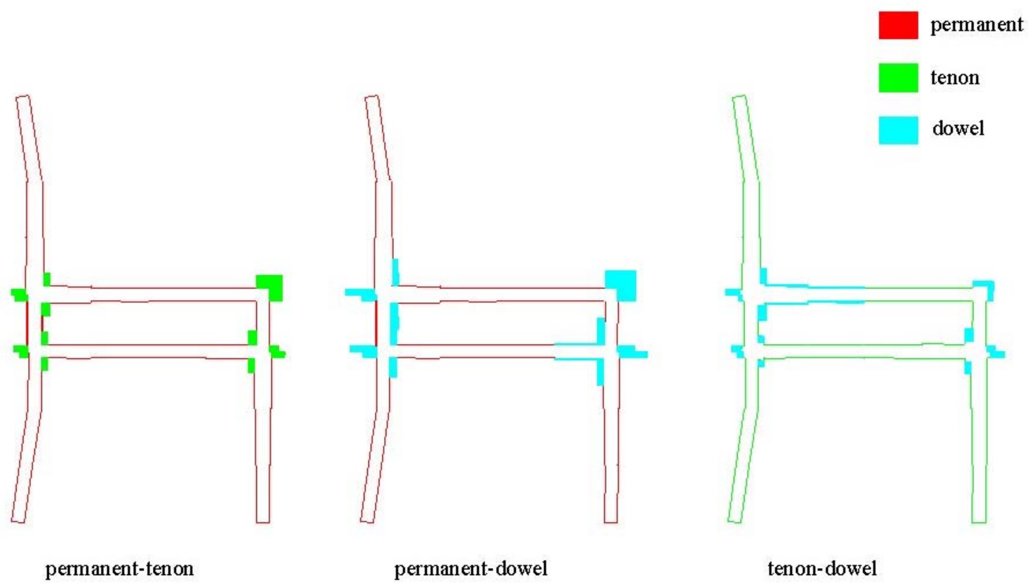


Fig. 5. Comparison of optimisation results of beech wood constructions for various joints (dimensional differences were magnified 5x)



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A further transformation of the above dependence, because of E_w, the authors obtained the formula for the equivalent modulus of elasticity of the discussed constructional node:

$$E_z = \frac{ab^2 + \frac{1}{3}a^3}{\frac{\delta J}{P} - \frac{b^3}{3E_d}} \tag{11}$$

The values of the equivalent modulus of elasticity for stop-housed and dowel joints connected together with a polioctanevinyl glue for beech and pine wood were calculated on the basis of results of laboratory investigations reported in a study by Frieske [4].

Values of the observed factors applied during calculations served for the estimation of the effectiveness of the optimisation process. An index of volume reduction R was employed as the main indicator of the effectiveness of the optimisation process:

$$R = 1 - \frac{V_{opt}}{V_p} \tag{12}$$

where:
V_{opt} – volume of the construction after optimisation,
V_p – volume of the construction before optimisation.

Taking under consideration the fact that, in practical applications, a relatively short time of operation of the computer program is important, the authors decided to adopt index of the dynamics of the optimisation process D as an additional index of optimisation effectiveness:

$$D = \frac{R}{T} \tag{13}$$

where:
T – duration time of the optimisation process.

METHOD OF CARRYING OUT CALCULATIONS

On the basis of the mathematical optimisation model adopted in the methodology, the authors developed a computer program in the C++ language in which they utilised the concept of the integration of the FEM environment with the module of static optimisation as presented in their earlier study [10]. The developed program was later estimated for various frame constructions. The purpose of the assessment was, on the one hand, verification of the correctness of the program operation and, on the other, selection of an appropriate value of the coefficient of the barrier function r. According to Oswald [7] the process of optimisation of a construction should be carried out by repeating the process of minimisation of the objective function for consecutive values of r_j, constituting a decreasing sequence, until the difference of results of successive function minimisations is smaller than the assumed accuracy of calculations. On the other hand, Dietrich [1] allows the use of a constant value of coefficient r, which would be selected in such a way to insure appropriately small values of the punishment function inside the acceptable area. Integration of the gradient method in the FEM environment caused that the minimisation process of the objective function using a standard PC computer took so long that the first concept turned out inapplicable in the present study. On the other hand, the second assumption presented problems with the selection of an appropriate value of the barrier function. It turned out that, with the increase of the r value, the barrier function B tended to be at variance with its ideal form and, consequently, the obtained result of optimisation was less accurate. Conversely, low r values resulted in the termination of calculations after finding a local extreme unacceptable for the decision-making process. For this reason, the authors decided to conduct the optimisation process in two stages. In the first step, the objective function was minimised at r equalling 10^7 allowing to obtain an intermediate, sub-optimal solution, which served as a starting point for the second stage of optimisation during which the value of r was 10^4. In this situation, the optimisation time T was assumed as the total time of the duration of both the first and second stages of optimisation.

In the next step, input data for the optimisation program were prepared using the following:

- The scheme of distribution of elements and constructional nodes in accordance with Fig. 2 and the arrangement of loads and support of the construction in accordance with Fig. 2,
- Minimum width of the tenon cross-section b_{min} corresponding to 15 mm and, hence h_{min} equalling 21 mm,
- Maximal dimensions of the cross-section height h_{max} for the entire construction equalling 90 mm,
- Initial dimensions of the cross-sectional height of wood elements $h_{20(0)}$ … $h_{30(0)}$ equalling 50 mm, while for elements simulating furniture connections $h_{20(0)}$ … $h_{30(0)}$ equalling 75 mm,
- Young's moduli [6]: 16 000 MPa for beech wood and 12 000 MPa for pine,
- Equivalent moduli of elasticity of constructional elements calculated on the basis of strength tests of furniture joints [4] equalling: 1271.99 MPa for tenon joints and 774.94 MPa for dowel joints of beech wood and 1046.80 MPa and 501.13 MPa for respective joints of pine wood,
- Acceptable bending stresses for wood elements [6] divided by a safety coefficient equalling 2.5 in values for beech wood – 42 MPa and for pine wood – 34 MPa,
- Acceptable bending stresses for elements simulating constructional elements at the value of 21.4 MPa for tenon joints from beech wood, 11.2 MPa for dowel joints of beech wood, 19.6 MPa for tenon pine wood joints and 7.5 MPa for dowel pine wood joints.

On the basis of the above data, using the developed computer program, the authors carried out the optimisation of a chair side frame with a crosspiece made of beech wood with permanent joints - with the elasticity module corresponding to that of wood, with tenon and dowel joints as well as of a chair side frame with a crosspiece made of pine wood with the above-mentioned types of joints.

ANALYSIS OF RESULTS

Optimal values of cross-sectional heights of chair constructional elements are presented in Table 1. It is evident from the analysis of these data that, for the two examined wood species, both in the case of tenon and dowel joints, dimensions of cross-sectional dimensions of elements simulating the joints exceeded considerably cross-sectional dimensions of wood elements. This phenomenon was in keeping with expectations because, for all the cases of the optimised construction, both the elasticity module and bending strength of constructional nodes were significantly lower than in the case of the remaining elements. The obtained results are also corroborated by engineering practice, which confirms that the construction of skeleton furniture is usually damaged in the result of a failure of furniture connections. Many furniture models are characterised by thickened side underframes and back legs of chairs in the neighbourhood of these two elements. Both in the case of chairs with tenon and dowel joints and for constructions of both wood species, the biggest dimensions of constructional nodes were obtained in places where the side underframe joined the back leg and the front leg joined the crosspiece. These findings are in agreement with results of a study by Smardzewski [8] in which he presented dimensional changes of a chair side frame in the result of dimensional optimisation of individual tenon joints.

In order to assess the influence of the type of joint on optimisation results, dimensions of optimal cross sections of beam elements were compared in relation to the applied type of joint by superimposing the consecutive schemes of optimised constructions indicating the difference of the h dimension for a given element in a fivefold magnification (Fig. 5, 6). Dimensional differences equal to or smaller than 0.5 mm were treated as insignificant and were not superimposed on figures.

Comparing constructions with ideally rigid nodes with constructions having nodes of real rigidity (Fig. 5a,b; 6a,b) it was found that there was a considerable difference between optimal dimensions of elements simulating real joints and wood elements adjacent to nodes in a construction with permanent joints. This means that the design of a piece of furniture on the basis of a simplified model of optimisation, which assumes that the rigidity and strength of a joint is identical with the rigidity of the surrounding material, will result in a construction characterised by too small dimensions of constructional nodes and, consequently, of too poor strength. From among the analysed materials, bigger variations were obtained for pine wood, while, within a single construction, the biggest node dimensional differences were found in the place where the back leg joined the side underframe. On the other hand, in the case of wooden beam elements, only single, small deviations of cross-sectional dimensions were observed. This means that a model simplified in this way can only be applied to determine cross-sectional dimensions of wood elements and only in a definite distance from the constructional node. It is not useful to select dimensions of furniture joints. Therefore, a comprehensive construction optimisation requires determination of the mechanical characteristics of the applied furniture joints.

Table 1. Dimensions of the heights of cross sections of elements before and after optimisation

Element	Wood species											
	Beech						Pine					
	Type of joint											
	Permanent		Stop-housed		Dowel		Permanent		Stop-housed		Dowel	
	State before/after optimisation											
	Before	After	Before	After	Before	After	Before	After	Before	After	Before	After
	Dimension h [mm]											
1	50.0	22.7	50.0	22.7	50.0	22.7	50.0	22.7	50.0	22.7	50.0	22.7
2	50.0	22.7	50.0	22.7	50.0	22.7	50.0	22.7	50.0	22.7	50.0	22.7
3	50.0	22.7	50.0	22.8	50.0	22.8	50.0	22.7	50.0	22.8	50.0	22.8
4	50.0	25.2	50.0	24.3	50.0	24.5	50.0	26.6	50.0	25.5	50.0	25.4
5	50.0	31.0	50.0	30.8	50.0	31.3	50.0	33.3	50.0	33.0	50.0	33.0
6	50.0	27.8	50.0	27.8	50.0	28.3	50.0	29.9	50.0	29.9	50.0	29.9
7	50.0	23.5	50.0	23.5	50.0	23.8	50.0	25.1	50.0	25.2	50.0	25.2
8	50.0	29.1	50.0	28.8	50.0	29.7	50.0	31.6	50.0	31.6	50.0	31.7
9	50.0	23.6	50.0	23.8	50.0	24.6	50.0	25.8	50.0	26.3	50.0	26.5
10	50.0	22.8	50.0	22.7	50.0	22.7	50.0	22.7	50.0	22.7	50.0	22.7
11	50.0	23.0	50.0	23.0	50.0	23.0	50.0	23.3	50.0	23.2	50.0	23.0
12	50.0	22.9	50.0	22.9	50.0	22.9	50.0	23.2	50.0	23.0	50.0	22.9
13	50.0	22.7	50.0	22.7	50.0	22.7	50.0	22.7	50.0	22.7	50.0	22.7
14	50.0	22.9	50.0	23.0	50.0	23.3	50.0	23.8	50.0	23.9	50.0	24.2
15	50.0	25.7	50.0	25.9	50.0	26.4	50.0	27.6	50.0	27.3	50.0	27.6
16	50.0	22.9	50.0	23.1	50.0	23.0	50.0	23.5	50.0	23.6	50.0	23.3
17	50.0	29.5	50.0	29.3	50.0	29.8	50.0	31.7	50.0	31.4	50.0	31.4
18	50.0	25.9	50.0	25.9	50.0	26.4	50.0	27.8	50.0	27.8	50.0	27.8
19	50.0	22.8	50.0	22.8	50.0	22.8	50.0	23.0	50.0	23.0	50.0	23.0
20			75.0	41.1	75.0	51.4			75.0	42.4	75.0	59.1
21			75.0	38.1	75.0	48.6			75.0	40.2	75.0	56.8
22			75.0	32.5	75.0	35.4			75.0	32.4	75.0	37.5
23			75.0	32.1	75.0	35.6			75.0	33.7	75.0	39.9
24			75.0	32.1	75.0	35.6			75.0	33.7	75.0	40.1
25			75.0	34.5	75.0	41.4			75.0	35.3	75.0	46.9
26			75.0	31.7	75.0	36.3			75.0	32.5	75.0	40.6
27			75.0	28.8	75.0	32.4			75.0	29.6	75.0	37.2
28			75.0	29.4	75.0	34.6			75.0	32.4	75.0	40.7
29			75.0	35.1	75.0	44.1			75.0	35.1	75.0	50.1
30			75.0	39.3	75.0	49.1			75.0	40.5	75.0	56.5

In Figures 5c and 6c, the authors compared optimisation results of constructions with tenon and dowel joints. It was found that in the case of both wood species optimal dimensions of dowel joints were greater than those of tenon joints. This was justified by a higher bending strength and greater equivalent modulus of elasticity of tenon joints.

Table 2 tabulates initial and final wood material volumes, optimisation times, the volume reduction index R and the index of optimisation dynamics D. The analysis of the data allowed to conclude that differences in initial volumes of the examined constructions occurred only between furniture with idealised and real joints and that these differences could be attributed to greater dimensions of elements simulating joints. In the case of both experimental wood species, the smallest final volumes of constructions and, simultaneously, the biggest index of volume reduction were obtained in the case of constructions with ideally rigid joints. On the other hand, from among construction with true joints, the most advantageous values of these parameters were obtained for constructions with tenon joints.

Fig. 6. Comparison of optimisation results of pine wood constructions for various joints (dimensional differences were magnified 5x)

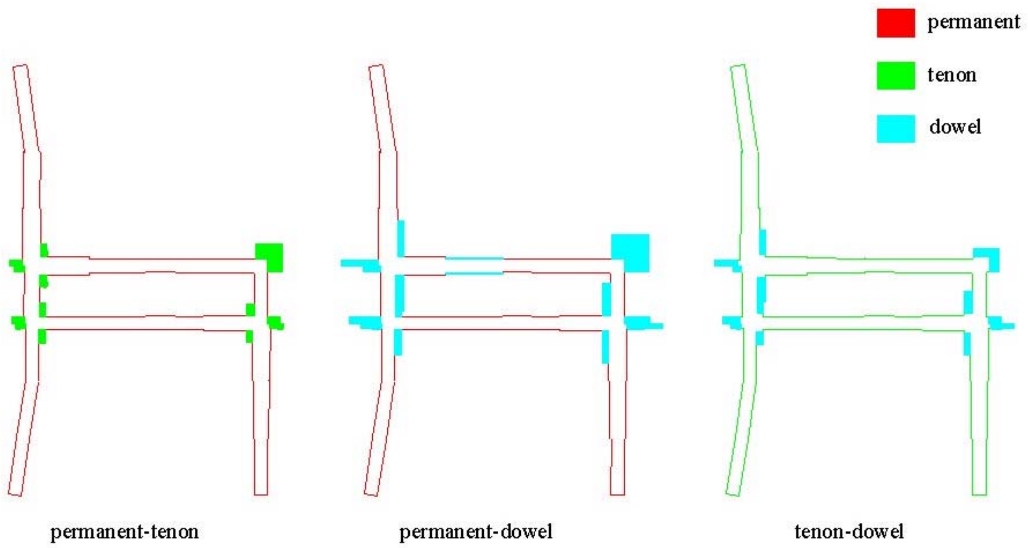

Table 2. Optimisation effectiveness

Examined trait			Joints		
			Permanent	Stop-housed	Dowel
Type	Beech	Initial volume V_p [cm^3]	3476	3730	3730
		Volume after optimisation V_{opt} [cm^3]	858	896	953
		Optimisation time T [h]	0.13	1.32	1.23
		Volume reduction index R	0.75	0.76	0.74
		Optimisation dynamics index D [h^{-1}]	5.70	0.58	0.61
	Pine	Initial volume V_p [cm^3]	3476	3730	3730
		Volume after optimisation V_{opt} [cm^3]	941	978	1052
		Optimisation time T [h]	0.14	1.29	1.15
		Volume reduction index R	0.73	0.74	0.72
		Optimisation dynamics index D [h^{-1}]	5.39	0.57	0.62

Taking into consideration values of the index of optimisation dynamics, the best effectiveness was obtained in the case of a system with permanent joints. A short time of optimisation of this construction resulted from a smaller number of finite elements, which exerted an impact on both the time of gradient calculation of the objective function and the time of strength calculations. However, due to the above-indicated poor usefulness of the model with idealised nodes to prepare a design of a piece of furniture, this option was not taken under consideration in further studies on the effectiveness of the optimisation process. As to the remaining constructions, a shorter time of optimisation and a better process dynamics was obtained in the case of the structure with dowel joints. As mentioned above, the value of the index of volume reduction for these structures was the least advantageous.

Because of the smallest final volume accompanied by appropriate optimisation of the obtained results in relation to real conditions, of all the examined constructions, the chair manufactured from beech wood with tenon joints should be considered as the most optimal construction. In the course of optimisation of this type of chair the authors succeeded in reducing by 75% the initial volume of the applied material. At the same time, from among structures with real joints, it was this one in which the highest optimisation effectiveness measured by the index of volume reduction was achieved.

Optimised cross-sectional dimensions of construction elements should be used by designers to prepare the final, plastic form of the construction. The design will consist in the replacement of constructional elements characterised by steplike changing cross-sectional dimensions – resulting from a discrete character of the optimise structure – by elements with curvilinear edges.

CONCLUSIONS

Summing up the above considerations, the following conclusions can be drawn:

1. The rigidity of joints exerts a significant influence on optimisation results, both with regard to differences in individual cross-sectional dimensions of elements and the total construction volume. The final volume of the construction having tenon joints, both in the case of beech and pine woods, was by 4% greater in comparison with the volume of the construction with permanent joints.
2. The final volume of the construction with joinery joints was by approximately 12% greater in comparison with volumes of constructions with permanent joints.
3. Gradient optimisation allowed reducing the volume of skeleton constructions by 72% - 76% of their initial volume in a period of time shorter than 1.4 hours.
4. The employment of the concept of equivalent moduli of elasticity allowed simplifying constructional joints in the model of optimised construction to beam elements.
5. The obtained results corroborate the correctness of functioning of the developed program, while the compatibility of the obtained optimal constructions with solutions applied in engineering practice indicate the possibility of the utilisation of the program by furniture designers and constructors.
6. The use of a barrier function allows an indirect realisation of the strength criteria by minimising the value of the objective function.

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