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A TAPER MODEL FOR NORWAY SPRUCE (*PICEA ABIES* (L.) KARST.)

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ABSTRACT

A method of estimation of the stem form and volume of forest trees is developed. It is based on multiple regression equations used to determine the stem diameter at any relative height. Four variants of equations are developed. In the simplest one the diameter at breast height and tree height are the explanatory variables, while in the remaining equations the number of required variables increases. This method permits to estimate the volume of stems, as well as the volume of their portions, and it is free of systematic errors. The models developed in this study may be used in forest inventory, quality assessment of standing trees, and stand pricing.

Key words: stem form, taper model

INTRODUCTION

The volume estimation is one of the basic tasks of forest inventory. Usually the volume tables for standing trees or empirical formulas are used for this purpose [1, 3, 4]. However, in some cases, the volume estimation is not enough. In the case of stands assigned for the final felling the quality assessment of standing trees is necessary in order to estimate how much wood of desired dimensions there is in the stand. Also the estimation of the volume participation of individual wood assortments is necessary in stand pricing. This requires utilization of labour-consuming methods, sample trees, assortment tables, or taper tables. In the age of computers it is more rational to use for this purpose the taper models permitting to compute the morphological curve of the tree stem, and then to estimate the volume of wood of any given dimensions. Much consideration has been given to this problem in the dendrometric literature, especially during recent decades when new possibilities in the form of numerical modelling have appeared. As the result of studies concerning the tree form estimation several model solutions have been developed. The models used for description of the stem profile are usually called the taper models.

There may be three basic groups of such models distinguished. They differ in the method of the stem profile description.

The first group includes the taper models describing the morphological curve by means of a single equation where the diameter at different stem heights is the dependent variable, while the diameter at breast height, height, and other characteristics which may additionally explain the variation of the tree form, are the independent variables. Methods elaborated by Kozak [11]; Max and Burkhart; Newnham [11]; Mc Tague and Stansfield; Stadelman, Wensel, and Krumland [15]; Ormerod [12]; and Newbery and Perez [14] belong to this group.

The second group includes the taper models developed by Bruchwald [2], Siekierski [13], or Dudzińska [6]. They are based on the percentage participation of volume of 15 sections in the total stem volume, which was made dependent on the form or the diameter at breast height index, the tree height and form factor, and the stand mean diameter at breast height, mean height, and also the stand form factor. The diameter in the section's middle is computed on the basis of the volume percentage of a given section and total tree volume. The mid-section diameters determine the stem morphological curve which permits to calculate the volume of any stem portion.

The third group of taper models is composed of the models in which the stem profile is described on the basis of a certain number of diameters determined at relative stem heights. Separate equations are used to compute diameters at individual relative heights. Such a method of the stem form estimation was used by Kilkki, Sarmäki and Varmola [10]; Varmola [8]; and Böckmann [14].

The methods in which the morphological stem curve is described by a single equation are generally quite complicated, and their parameters are difficult to estimate. According to Van Laar and Akcy [14] such models are little exact.

The application of methods from groups two and three is connected with inconvenience lying in the fact that only diameters at relative heights may be estimated. The remaining diameters required, e.g. for computation of volume of a stem portion, must be estimated by interpolation. However, the fact that they are free of systematic errors at any stem section seems to make them very useful in estimation of a stem portion volume.

The purpose of this study was to develop a method of construction of taper models for stands of Norway spruce (*Picea abies* (L.) Karst.)

MATERIAL AND METHODS

Taper data for this study came from section measurements of 1142 trees from five over 100 years old Norway spruce stands growing in the Wisła and Ujsoly Forest Districts ([Table 1](#)).

Table 1. Characteristics of analysed stands

| No. | Locality | | | | Sample plots | | | Taxation characteristics | | | | | |
|-----|-----------------|----------------|-------------|------------------|--------------|--------------|-----------|--------------------------|--------|-------|------------|-----------------------------------|----------------|
| | Forest District | Forest Section | Compartment | Forest site type | Plot name | Altitude (m) | Area (ha) | Age | D (cm) | H (m) | Site class | Stand volume (m ³ /ha) | Stocking index |
| 1 | Ujsoly | Laliki | 26b | LMG* | L600 | 600 | 0.75 | 111 | 42.4 | 35.02 | I.0 | 478 | 0.60 |
| 2 | Wisła | Beskidek | 64c | LMG | B650 | 650 | 1.61 | 120 | 44.4 | 36.68 | Ia.8 | 586 | 0.71 |
| 3 | Wisła | Olecki | 34c | LMG | O800 | 800 | 1.19 | 103 | 40.8 | 30.11 | I.9 | 413 | 0.62 |
| 4 | Ujsoly | Petkówka | 249c | LG** | P830 | 830 | 1.08 | 113 | 50.1 | 38.15 | Ia.3 | 590 | 0.66 |
| 5 | Ujsoly | Petkówka | 246c | LMG | P1000 | 1000 | 1.36 | 122 | 48.8 | 36.13 | Ia.9 | 494 | 0.62 |

*Mixed mountain forest, ** Mountain forest

The taper model presented in this study is based on the equations developed to estimate relative diameters (d_{wj}) at 20 relative stem heights (h_j). Relative diameters were estimated according to the formula:

$$d_{wj} = \frac{d_j}{d} \quad (1)$$

where:

d_{wj} – relative diameter

d_j – diameter outside bark at relative heights h_j ($h_j = 0.0125; 0.05; 0.10; 0.15; \dots; 0.95$)

d - diameter at breast height outside bark.

The diameters at relative heights were defined as a function of some biometric characteristics of trees selected by correlation analyses. The index of position in the stand height structure was used, among others, in the equations. It was computed as a value of standardised characteristic according to the formula:

$$W_{h_i} = \frac{h_i - H}{S_h} \quad (2)$$

where:

H – mean stand height

S_h – standard deviation of height

h_i – height of tree i

Relationship between diameters at individual relative heights and explanatory variables was expressed by the equation:

$$d_{wj} = b_{0j} + b_{1j} \cdot x_1 + b_{2j} \cdot x_2 + b_{3j} \cdot x_3 + \dots + b_{mj} \cdot x_m \quad (3)$$

where:

d_{wj} – relative diameter at relative height (h_j)

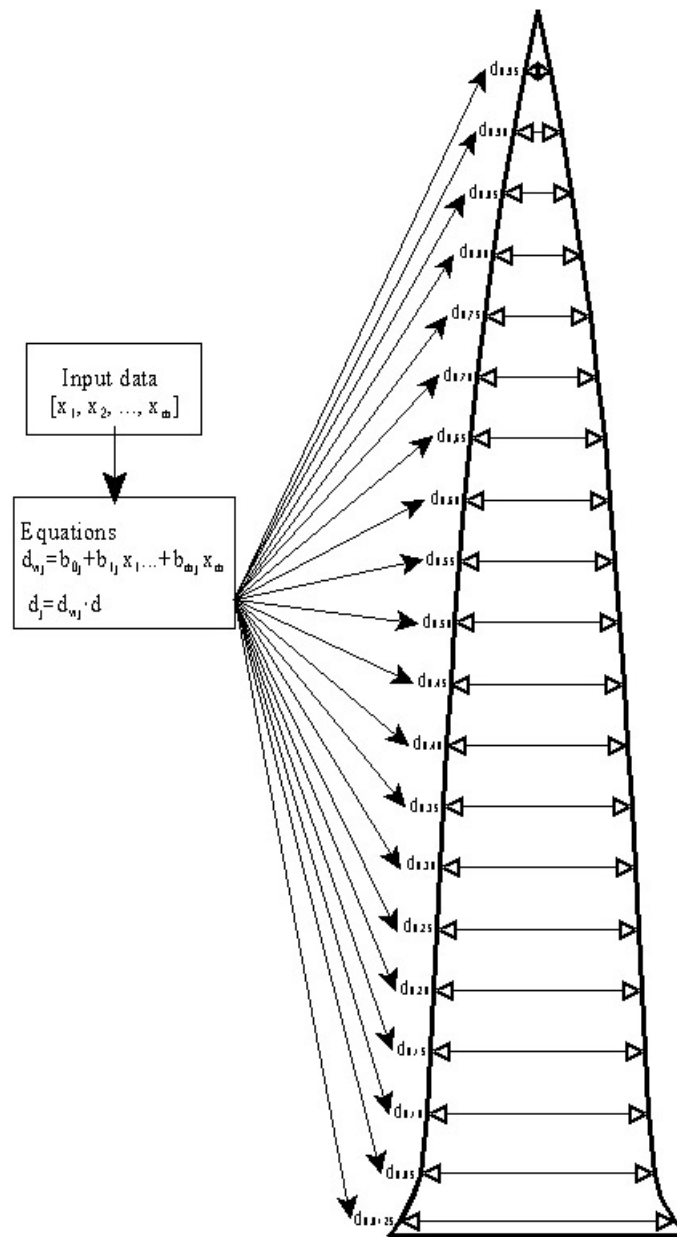
$b_{0j} \dots b_{mj}$ – equation parameters for estimation of diameter at relative height (h_j)

$x_1, x_2, x_3 \dots x_m$ - independent variables correlated with estimated diameters

m – number of independent variables.

A method of determination of the stem morphological curve on the basis of taper equations is given in [Fig.1](#).

Fig. 1. Diagram of development of the stem morphological curve on the basis of taper equations (d_{vj} - relative diameter at height j , d_j - diameter at height j , d - diameter at breast height)



The usefulness of the models was evaluated by determination of their morphological accuracy, understood as the compatibility of the diameters estimated from the model with actual diameters [7], and their dendrometric accuracy, i.e. the compatibility of volume of stems or their portions computed by any method with the actual volume.

RESULTS

The analyses of the dependence of individual relative diameters on the chosen biometric characteristics of trees led to selection of variables which explain their dispersion to a highest degree. They are as follows: diameter at breast height (d), height (h), relative crown length (l_{kw}), position in height structure (W_h), diameter at height 0.1h ($d_{0.1}$), and 0.5h ($d_{0.5}$). The selected explanatory variables were used in development of four variants of the taper model (designated with letters from A to D).

In the model A the tree diameter at breast height and tree height are the independent variables in individual regression equations (4):

$$d_{wj} = b_{0j} + b_{1j} \cdot d + b_{2j} \cdot h \quad (4)$$

where:

d_{wj} = relative diameter at height h_j $\{j=0.0125h, 0.05h, 0.10h, 0.15h, \dots, 0.95h\}$
 b_{0j}, b_{1j}, b_{2j} – equation parameters for the diameter at height (h_j).

Values of the equation (4) parameters are shown in [Table 2](#).

Table 2. Equation parameters of the model A

| Relative diameter | Parameters | | |
|-------------------|------------|---------|----------|
| | b_0 | b_1 | b_2 |
| $d_{w0.0125}$ | 1.2166 | -0.0013 | -0.0015 |
| $d_{w0.05}$ | 1.0691 | -0.0012 | -0.0018 |
| $d_{w0.10}$ | 0.9867 | -0.0023 | 0.0004** |
| $d_{w0.15}$ | 0.9441 | -0.0026 | 0.0012 |
| $d_{w0.20}$ | 0.9094 | -0.0026 | 0.0015 |
| $d_{w0.25}$ | 0.8796 | -0.0026 | 0.0016 |
| $d_{w0.30}$ | 0.8508 | -0.0025 | 0.0016 |
| $d_{w0.35}$ | 0.8223 | -0.0024 | 0.0015 |
| $d_{w0.40}$ | 0.7955 | -0.0022 | 0.0012 |
| $d_{w0.45}$ | 0.7706 | -0.0021 | 0.0009* |
| $d_{w0.50}$ | 0.7394 | -0.0021 | 0.0007** |
| $d_{w0.55}$ | 0.6985 | -0.0021 | 0.0009* |
| $d_{w0.60}$ | 0.6575 | -0.0023 | 0.0011 |
| $d_{w0.65}$ | 0.6113 | -0.0028 | 0.0017 |
| $d_{w0.70}$ | 0.5530 | -0.0031 | 0.0022 |
| $d_{w0.75}$ | 0.4965 | -0.0036 | 0.0027 |
| $d_{w0.80}$ | 0.4298 | -0.0037 | 0.0027 |
| $d_{w0.85}$ | 0.3406 | -0.0035 | 0.0028 |
| $d_{w0.90}$ | 0.2317 | -0.0027 | 0.0025 |
| $d_{w0.95}$ | 0.1192 | -0.0015 | 0.0016 |

* parameter insignificant for $\alpha = 0.01$

** parameter insignificant for $\alpha = 0.05$

In the model B, besides the tree diameter at breast height and tree height also the relative crown length (l_{kw}) and position in the height structure (W_h) are the independent variables:

$$d_{wj} = b_{0j} + b_{1j} \cdot d + b_{2j} \cdot h + b_{3j} \cdot l_{kw} + b_{4j} \cdot W_h \quad (5)$$

Using the procedure of variance analysis in the regression analysis it was demonstrated that the coefficient b_2 is insignificant ($\alpha = 0.05$) for the equation 5 in the diameter prediction from the heights 0.10h and 0.15h, and from 0.40h to 0.60h ([Table 3](#)).

Table 3. Equation parameters of the model B

| Relative diameter | Parameters | | | | |
|----------------------|----------------|----------------|----------------|----------------|----------------|
| | b ₀ | b ₁ | b ₂ | b ₃ | b ₄ |
| d _{w0.0125} | 1.1935 | -0.0011 | -0.0012* | 0.0118** | -0.0032** |
| d _{w0.05} | 1.0725 | -0.0016 | -0.0018 | 0.0328 | 0.0031 |
| d _{w0.10} | 1.0091 | -0.0029 | 0.0001** | 0.0395 | 0.0068 |
| d _{w0.15} | 0.9717 | -0.0033 | 0.0007** | 0.0445 | 0.0080 |
| d _{w0.20} | 0.9331 | -0.0034 | 0.0011 | 0.0552 | 0.0081 |
| d _{w0.25} | 0.9027 | -0.0034 | 0.0012 | 0.0565 | 0.0081 |
| d _{w0.30} | 0.8711 | -0.0032 | 0.0012 | 0.0527 | 0.0072 |
| d _{w0.35} | 0.8491 | -0.0031 | 0.0010 | 0.0452 | 0.0078 |
| d _{w0.40} | 0.8240 | -0.0029 | 0.0007** | 0.0398 | 0.0077 |
| d _{w0.45} | 0.8004 | -0.0027 | 0.0003** | 0.0297* | 0.0071 |
| d _{w0.50} | 0.7739 | -0.0025 | 0.0001** | 0.0141** | 0.0069 |
| d _{w0.55} | 0.7275 | -0.0024 | 0.0003** | -0.0037** | 0.0046 |
| d _{w0.60} | 0.6861 | -0.0023 | 0.0006** | -0.0267** | 0.0029** |
| d _{w0.65} | 0.6317 | -0.0024 | 0.0013 | -0.0566 | -0.0007** |
| d _{w0.70} | 0.5694 | -0.0023 | 0.0019 | -0.0867 | -0.0037* |
| d _{w0.75} | 0.5073 | -0.0025 | 0.0024 | -0.1063 | -0.0060 |
| d _{w0.80} | 0.4417 | -0.0026 | 0.0025 | -0.1111 | -0.0061 |
| d _{w0.85} | 0.3481 | -0.0025 | 0.0027 | -0.1006 | -0.0060 |
| d _{w0.90} | 0.2276 | -0.0020 | 0.0026 | -0.0655 | -0.0055 |
| d _{w0.95} | 0.1158 | -0.0013 | 0.0017 | -0.0213 | -0.0021 |

* parameter insignificant for $\alpha = 0.01$

** parameter insignificant for $\alpha = 0.05$

In the model C, besides d , h , l_{kw} , W_h , also the diameter from the height 0.10 ($d_{0.1}$) was taken into account for the estimation of relative diameters:

$$d_{wj} = b_{0j} + b_{1j} \cdot d + b_{2j} \cdot h + b_{3j} \cdot l_{kw} + b_{4j} \cdot W_h + b_{5j} \cdot d_{0.1} \quad (6)$$

Parameters of the equation (6) for individual relative diameters are included in [Table 4](#).

Table 4. Equation parameters of the model C

| Relative diameter | Parameters | | | | | |
|----------------------|----------------|----------------|----------------|----------------|----------------|----------------|
| | b ₀ | b ₁ | b ₂ | b ₃ | b ₄ | b ₅ |
| d _{w0.0125} | 1.1870 | -0.0021 | -0.0012 | 0.0093 | ** -0.0037** | 0.0013** |
| d _{w0.05} | 1.0083 | -0.0116 | -0.0020 | 0.0080** | -0.0017 | 0.0131 |
| d _{w0.10} | 0.9072 | -0.0187 | -0.0002 | 0.0002** | -0.0008 | 0.0208 |
| d _{w0.15} | 0.8700 | -0.0190 | 0.0004 | 0.0052** | 0.0004** | 0.0208 |
| d _{w0.20} | 0.8340 | -0.0187 | 0.0008 | 0.0169 | 0.0006** | 0.0203 |
| d _{w0.25} | 0.8069 | -0.0182 | 0.0010 | 0.0194 | 0.0009** | 0.0196 |
| d _{w0.30} | 0.7783 | -0.0176 | 0.0009 | 0.0168* | 0.0003** | 0.0190 |
| d _{w0.35} | 0.7585 | -0.0171 | 0.0008 | 0.0102** | 0.0010** | 0.0185 |
| d _{w0.40} | 0.7365 | -0.0165 | 0.0005** | 0.0060** | 0.0012** | 0.0179 |
| d _{w0.45} | 0.7173 | -0.0156 | 0.0001** | -0.0024** | 0.0009** | 0.0170 |
| d _{w0.50} | 0.6952 | -0.0147 | -0.0002** | -0.0163** | 0.0010** | 0.0161 |
| d _{w0.55} | 0.6540 | -0.0138 | 0.0001** | -0.0321 | -0.0009** | 0.0151 |
| d _{w0.60} | 0.6160 | -0.0132 | 0.0004** | -0.0538 | -0.0024** | 0.0143 |
| d _{w0.65} | 0.5677 | -0.0123 | 0.0011 | -0.0813 | -0.0055 | 0.0131 |
| d _{w0.70} | 0.5127 | -0.0111 | 0.0017 | -0.1086 | -0.0079 | 0.0116 |
| d _{w0.75} | 0.4590 | -0.0100 | 0.0023 | -0.1249 | -0.0096 | 0.0099 |
| d _{w0.80} | 0.4023 | -0.0087 | 0.0024 | -0.1264 | -0.0090 | 0.0081 |
| d _{w0.85} | 0.3170 | -0.0073 | 0.0026 | -0.1126 | -0.0084 | 0.0064 |
| d _{w0.90} | 0.2054 | -0.0054 | 0.0025 | -0.0741 | -0.0072 | 0.0045 |
| d _{w0.95} | 0.1031 | -0.0033 | 0.0016 | -0.0262 | -0.0031 | 0.0026 |

* parameter insignificant for $\alpha = 0.01$

** parameter insignificant for $\alpha = 0.05$

In the fourth variant of taper equations (model D), besides variables used in the model C, also the diameter determined at the mid-length ($d_{0.5}$) was considered. The choice of the diameter from the height 0.50h was connected with the fact that it is most frequently positioned outside the crown's reach, and this decides on the possibility of its indirect measurement. A general form of the multiple regression equations for this model variant is as follows:

$$d_{wj} = b_{0j} + b_{1j} \cdot d + b_{2j} \cdot h + b_{3j} \cdot l_{kw} + b_{4j} \cdot W_h + b_{5j} \cdot d_{0.1} + b_{6j} \cdot d_{0.5} \quad (7)$$

Parameters of the equation (7) for individual relative diameters are included in [Table 5](#).

When evaluating the accuracy of developed procedures it was found that in the case of the model A the mean error in estimation of diameter at individual relative heights was 0.00 every time ([Table 6](#)). The standard deviations varied from 0.75 cm at the height 0.95h to 1.86 cm at the height 0.0125h. A considerable increase of accuracy of the determination of the stem morphological curve, as compared with the variant based on the tree diameter at breast height and height (model A), was only possible by the measurement of additional diameters on a standing tree. When they were taken into account the prediction accuracy of individual relative diameters increased considerably. The standard deviations of individual diameters considerably decreased, with the exception of the diameter at the height 0.0125h. In the case of the model C this was particularly evident in the lower part of the stem. The value of the standard deviation of the diameter estimation error dropped there by about 0.5 cm. After utilisation of the model D the standard deviation of errors in the diameter estimation especially decreased for the diameters situated in the range from 0.20h to 0.85h. A considerable increase of

accuracy in the diameter estimation, especially in the upper part of the stem took place after including in equations the diameter from the height 0.50h (model D). Standard deviations of the diameters estimated in such a way were not greater than 1.16 cm, with the exception of those situated at the height of 0.0125h. in the case of the model D also the range of the extreme errors considerably decreased as compared with other model variants (especially A and B).

Table 5. Equation parameters of the model D

| Relative diameter | Parameters | | | | | | |
|----------------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| | b ₀ | b ₁ | b ₂ | b ₃ | b ₄ | b ₅ | b ₆ |
| d _{w0.0125} | 1.1886 | -0.0022 | -0.0012* | 0.0077** | -0.0036** | 0.0030* | -0.0022** |
| d _{w0.05} | 1.0097 | -0.0117 | -0.0020 | 0.0066** | -0.0016 | 0.0146 | -0.0019 |
| d _{w0.10} | 0.9075 | -0.0187 | -0.0002 | -0.0001** | -0.0008 | 0.0211 | -0.0004* |
| d _{w0.15} | 0.8678 | -0.0189 | 0.0004 | 0.0075** | 0.0002** | 0.0184 | 0.0031 |
| d _{w0.20} | 0.8296 | -0.0185 | 0.0008 | 0.0215 | 0.0003** | 0.0154 | 0.0062 |
| d _{w0.25} | 0.8007 | -0.0178 | 0.0009 | 0.0259 | 0.0005** | 0.0129 | 0.0087 |
| d _{w0.30} | 0.7704 | -0.0171 | 0.0009 | 0.0251 | -0.0003** | 0.0104 | 0.0111 |
| d _{w0.35} | 0.7486 | -0.0165 | 0.0007 | 0.0206 | 0.0003** | 0.0077 | 0.0139 |
| d _{w0.40} | 0.7249 | -0.0157 | 0.0003 | 0.0181 | 0.0003** | 0.0052 | 0.0163 |
| d _{w0.45} | 0.7038 | -0.0147 | 0.0000** | 0.0117* | -0.0001** | 0.0022 | 0.0190 |
| d _{w0.50} | 0.6796 | -0.0137 | -0.0003 | 0.0000** | -0.0001** | -0.0010 | 0.0219 |
| d _{w0.55} | 0.6386 | -0.0128 | 0.0000** | -0.0160 | -0.0020 | -0.0018 | 0.0216 |
| d _{w0.60} | 0.6010 | -0.0123 | 0.0003** | -0.0381 | -0.0034 | -0.0020 | 0.0210 |
| d _{w0.65} | 0.5533 | -0.0114 | 0.0010 | -0.0662 | -0.0065 | -0.0027 | 0.0203 |
| d _{w0.70} | 0.4995 | -0.0103 | 0.0016 | -0.0948 | -0.0089 | -0.0028 | 0.0185 |
| d _{w0.75} | 0.4472 | -0.0093 | 0.0022 | -0.1126 | -0.0105 | -0.0030 | 0.0165 |
| d _{w0.80} | 0.3925 | -0.0081 | 0.0023 | -0.1161 | -0.0098 | -0.0027 | 0.0138 |
| d _{w0.85} | 0.3094 | -0.0069 | 0.0025 | -0.1048 | -0.0089 | -0.0018* | 0.0106 |
| d _{w0.90} | 0.2003 | -0.0051 | 0.0024 | -0.0688 | -0.0076 | -0.0010** | 0.0071 |
| d _{w0.95} | 0.1009 | -0.0031 | 0.0016 | -0.0239 | -0.0032 | 0.0002** | 0.0031 |

* parameter insignificant for $\alpha = 0.01$

** parameter insignificant for $\alpha = 0.05$

Table 6. Characteristics of accuracy in estimation of diameter at different stem heights according to individual variants of taper equations

| Diameter | Model A | | | | Model B | | | | Model C | | | | Model D | | | |
|---------------------|-----------|--------------------|--------------------|----------|-----------|--------------------|--------------------|----------|-----------|--------------------|--------------------|----------|-----------|--------------------|--------------------|----------|
| | Mean (cm) | Standard dev. (cm) | Extreme error (cm) | | Mean (cm) | Standard dev. (cm) | Extreme error (cm) | | Mean (cm) | Standard dev. (cm) | Extreme error (cm) | | Mean (cm) | Standard dev. (cm) | Extreme error (cm) | |
| | | | negative | positive | | | negative | positive | | | negative | positive | | | negative | positive |
| d _{0.0125} | 0.00 | 1.86 | -10.28 | 6.97 | -0.01 | 1.86 | -10.36 | 7.20 | -0.01 | 1.85 | -10.53 | 7.21 | -0.01 | 1.85 | -10.37 | 7.10 |
| d _{0.05} | 0.00 | 1.04 | -3.81 | 4.52 | -0.01 | 1.03 | -3.89 | 4.41 | -0.01 | 0.62 | -3.15 | 2.76 | -0.01 | 0.61 | -2.97 | 2.66 |
| d _{0.10} | 0.00 | 1.35 | -5.64 | 5.09 | -0.01 | 1.32 | -5.71 | 4.88 | 0.00 | 0.29 | -2.31 | 1.08 | 0.00 | 0.28 | -2.26 | 1.11 |
| d _{0.15} | 0.00 | 1.43 | -5.29 | 4.82 | -0.01 | 1.40 | -5.61 | 4.57 | 0.00 | 0.54 | -2.87 | 1.97 | 0.00 | 0.51 | -2.88 | 1.64 |
| d _{0.20} | 0.00 | 1.47 | -5.28 | 5.53 | -0.02 | 1.43 | -6.46 | 5.27 | -0.01 | 0.65 | -3.27 | 2.07 | -0.01 | 0.57 | -3.45 | 1.73 |
| d _{0.25} | 0.00 | 1.48 | -5.07 | 5.77 | -0.02 | 1.45 | -6.89 | 5.62 | -0.01 | 0.75 | -3.61 | 2.27 | -0.01 | 0.60 | -2.33 | 2.19 |
| d _{0.30} | 0.00 | 1.49 | -4.87 | 6.43 | -0.02 | 1.46 | -6.03 | 6.31 | -0.01 | 0.83 | -3.86 | 3.38 | -0.01 | 0.62 | -2.98 | 1.87 |
| d _{0.35} | 0.00 | 1.51 | -4.84 | 6.41 | -0.01 | 1.49 | -5.40 | 6.25 | 0.00 | 0.92 | -4.46 | 3.95 | -0.01 | 0.59 | -3.05 | 2.00 |
| d _{0.40} | 0.00 | 1.52 | -5.40 | 6.79 | -0.01 | 1.50 | -5.69 | 6.63 | 0.00 | 0.98 | -4.09 | 4.09 | -0.01 | 0.57 | -2.42 | 1.70 |
| d _{0.45} | 0.00 | 1.52 | -4.78 | 5.97 | -0.01 | 1.50 | -4.69 | 5.80 | 0.00 | 1.05 | -3.45 | 5.17 | 0.00 | 0.49 | -2.44 | 1.36 |
| d _{0.50} | 0.00 | 1.51 | -4.66 | 6.41 | 0.00 | 1.50 | -4.63 | 6.22 | 0.01 | 1.11 | -3.91 | 5.22 | 0.00 | 0.30 | -3.04 | 1.06 |
| d _{0.55} | 0.00 | 1.51 | -4.71 | 6.42 | 0.01 | 1.51 | -4.72 | 6.22 | 0.02 | 1.17 | -3.84 | 4.98 | 0.01 | 0.53 | -3.50 | 2.39 |
| d _{0.60} | 0.00 | 1.50 | -5.17 | 6.22 | 0.02 | 1.50 | -5.11 | 6.06 | 0.03 | 1.22 | -3.83 | 4.49 | 0.02 | 0.68 | -3.06 | 3.11 |
| d _{0.65} | 0.00 | 1.50 | -5.04 | 6.16 | 0.03 | 1.51 | -4.71 | 6.07 | 0.04 | 1.28 | -5.23 | 4.43 | 0.03 | 0.84 | -4.65 | 4.29 |
| d _{0.70} | 0.00 | 1.48 | -4.96 | 5.58 | 0.04 | 1.47 | -4.55 | 5.35 | 0.05 | 1.30 | -4.52 | 4.94 | 0.04 | 0.97 | -3.98 | 5.05 |
| d _{0.75} | 0.00 | 1.47 | -5.21 | 5.11 | 0.05 | 1.46 | -4.61 | 5.69 | 0.06 | 1.34 | -5.35 | 4.92 | 0.05 | 1.10 | -3.74 | 5.70 |
| d _{0.80} | 0.00 | 1.40 | -4.34 | 4.15 | 0.05 | 1.39 | -3.97 | 4.83 | 0.06 | 1.31 | -4.18 | 4.92 | 0.05 | 1.16 | -4.49 | 5.34 |
| d _{0.85} | 0.00 | 1.28 | -3.90 | 4.60 | 0.05 | 1.27 | -3.90 | 4.44 | 0.05 | 1.22 | -3.89 | 4.94 | 0.05 | 1.13 | -3.65 | 4.87 |
| d _{0.90} | 0.00 | 1.08 | -4.46 | 5.01 | 0.03 | 1.06 | -4.20 | 4.83 | 0.03 | 1.03 | -4.42 | 5.11 | 0.03 | 0.98 | -3.86 | 4.26 |
| d _{0.95} | 0.00 | 0.75 | -4.95 | 2.76 | 0.01 | 0.75 | -4.89 | 2.67 | 0.01 | 0.73 | -4.87 | 2.94 | 0.01 | 0.72 | -4.82 | 2.75 |

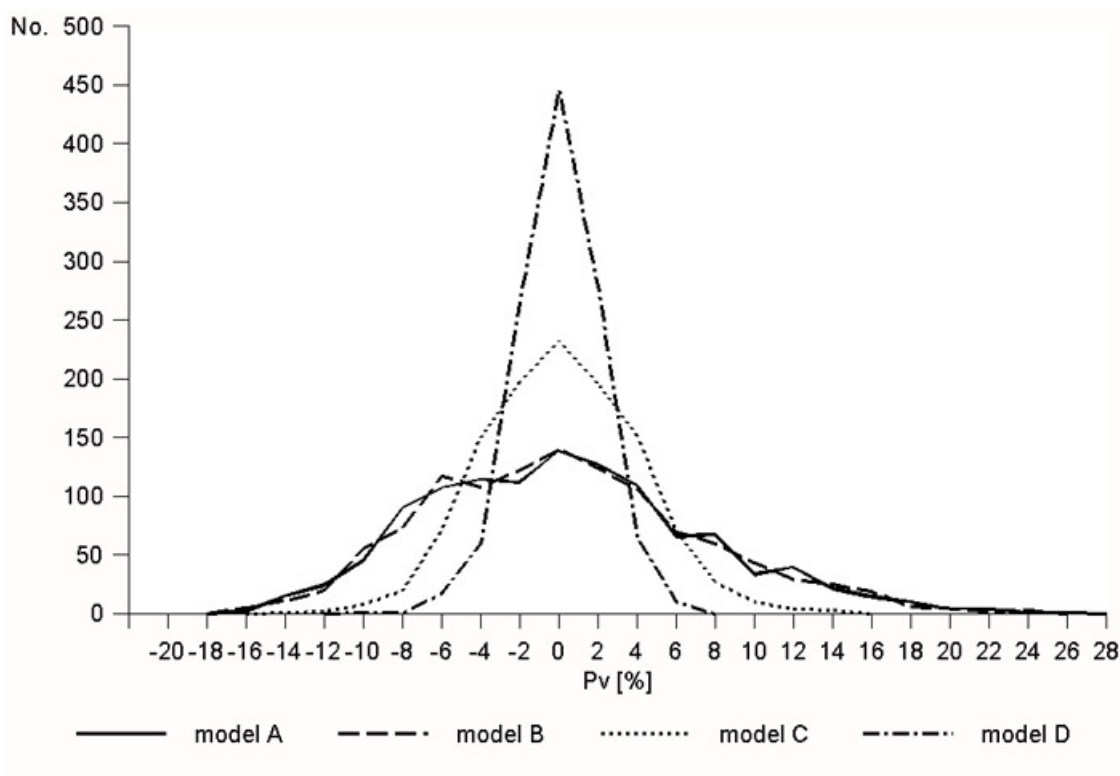
Arithmetic means of the percentage errors of the stem volumes estimated using individual models were in general smaller than 1.00 % (Table 7). Only in one case (P830) the error of the model A was greater than 1.00 %, i.e. 1.48 %.

The greatest range of the percentage errors occurred in the case of the volume estimated according to the model A (Table 7, Fig. 2). The extreme values of the percentage error of a secondary volume estimation of a single tree were from – 16.86 % to 25.10 % for the model A, and from – 10.47 % to 6.84 % for the model D. The standard deviations of the percentage errors varied from 7.02 % for the model A to 2.09 % for the model D.

Table 7. Percentage errors of the volume of a single tree estimated according to assumed variants of taper models

| Plot | Model variant | Mean (%) | Extreme errors | | Standard dev. (%) | Skewness |
|-------|---------------|----------|----------------|--------------|-------------------|----------|
| | | | Negative (%) | Positive (%) | | |
| | A | 0.56 | -14.60 | 21.63 | 7.125 | 0.360 |
| L600 | B | 0.72 | -14.00 | 23.22 | 7.199 | 0.401 |
| | C | -0.60 | -12.57 | 13.18 | 4.255 | -0.032 |
| | D | -0.03 | -6.27 | 6.81 | 2.102 | -0.030 |
| | A | 0.25 | -14.85 | 19.43 | 6.781 | 0.351 |
| B650 | B | 0.13 | -15.01 | 18.76 | 6.726 | 0.384 |
| | C | 0.49 | -14.43 | 11.32 | 3.777 | -0.103 |
| | D | -0.26 | -6.79 | 6.84 | 1.979 | -0.118 |
| | A | -0.48 | -13.46 | 19.48 | 6.374 | 0.442 |
| O800 | B | 0.37 | -13.03 | 20.27 | 6.410 | 0.449 |
| | C | 0.12 | -10.56 | 13.44 | 4.070 | 0.193 |
| | D | 0.12 | -6.19 | 6.24 | 2.205 | -0.038 |
| | A | 1.48 | -13.89 | 18.86 | 7.276 | 0.330 |
| P830 | B | 0.84 | -14.72 | 18.74 | 7.162 | 0.329 |
| | C | -0.51 | -8.24 | 9.42 | 3.495 | 0.342 |
| | D | 0.49 | -10.47 | 5.66 | 2.069 | -1.045 |
| | A | -0.14 | -16.86 | 25.10 | 7.789 | 0.673 |
| P1000 | B | -0.49 | -16.84 | 24.14 | 7.690 | 0.630 |
| | C | 0.48 | -8.75 | 14.31 | 4.069 | 0.396 |
| | D | -0.08 | -7.16 | 5.45 | 2.093 | -0.273 |
| | A | 0.27 | -16.86 | 25.10 | 7.018 | 0.445 |
| Total | B | 0.28 | -16.84 | 24.14 | 6.965 | 0.432 |
| | C | 0.09 | -14.43 | 14.31 | 3.933 | 0.126 |
| | D | 0.01 | -10.47 | 6.84 | 2.093 | -0.241 |

Fig. 2. Distribution of percentage errors of secondary stem volume estimated on the basis of diameters determined on the basis of individual taper models



DISCUSSION

The proposed method of determination of the stem form does not cause the occurrence of systematic errors in any stem section. This may decisively affect the accuracy in volume estimation of whole stems, as well as their portions (dimension classes of wood).

The errors in diameter estimations at individual relative heights result from variability of these diameters freed from the effect of explanatory characteristics used in different model variations. Therefore, a hypothesis may be formulated that using this procedure it is impossible to find solutions which at the number of independent variables equal to the number of variables used in the developed taper models will be characterised by a considerably greater accuracy in determination of the stem morphological curve. This is because unexplained variation of the morphological curve is associated with other factors which are not explained by the variables used in the model, especially the diameter at breast height and height (model A).

The taper models developed in this study may be of use in the assessment of assortments (dimension classes) when trees are still standing, in the case of single trees as well as whole stands. Thus they may become a useful tool in the quality assessment of standing trees, and also in the stand pricing. The models C and D may be of a great practical importance. Because of a high accuracy they may be used in fitting of volume tables or empirical formulas to local conditions. Such procedures are followed among others in the forest inventory in Switzerland and Austria.

The equations reported in this paper should be verified on a larger data material representing a wider age range of stands and greater site spectrum before they are used in practice. However, it may be assumed with a considerable probability that for Norway spruce stands designated for final felling in the Wisła and Ujsoły Forest Districts their use now will be free of large errors. It is highly probable that a model of this type may also be used for other conifers, such as pine, fir, larch etc.

The models developed in this study, especially the model A, may be used in all the methods of estimation of stand volume where the diameter at breast height of all trees in a stand, and the height of their certain number, at least such that it would be possible to determine the average stand height, are measured. The simplest version of

the taper model (variant A) may be used to estimate the volume of stems and dimension classes of single trees and whole stands. Having at the disposal a series of diameters at breast height and a stand height curve (also constant height curves [Bruchwald and Wróblewski 1994] may be used) the volume of wood of any given dimensions in a given diameter gradation may be estimated, and then the volumes obtained for dimension classes in individual gradations may be recalculated into the volume of a whole stand.

SUMMARY OF RESULTS AND CONCLUSIONS

1. A quite accurate representation of a stand form may be obtained using multiple regression equations based on two basic dendrometric characteristics of a tree, i.e. diameter at breast height and height. The application of additional characteristics such as crown length and position in the height structure only slightly improves the accuracy of the stem morphological curve.
2. A precise description of the stem form is possible only in the case when the diameters measured at different relative heights are taken into account in taper equations.
3. The diameters determined on the basis of a taper model based on the multiple regression equations permit to estimate accurately the volume of whole stems, as well as their portions. Such a procedure is free of systematic errors.
4. The taper models developed in this study, after verification on an independent empirical material may be of a practical use in forest inventory, quality assessment of standing trees, and stand pricing.

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