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NONASYMPTOTIC MODELLING OF THIN PLATES REINFORCED BY A SYSTEM OF STIFFENERS

Wiesław Nagórko[1], Czesław Woźniak[2]

[1] *Department of Civil Engineering and Geodesy, Warsaw Agricultural University, Poland*
[2] *Institute of Mathematics and Computer Sciences, Technological University of Częstochowa, Poland*

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ABSTRACT

The subject of considerations is a thin elastic plate reinforced by a large number of periodically spaced elastic stiffeners.

The purpose of this contribution is to propose a certain new averaged 2D – model of the periodic structure under consideration. This purpose could be also attained by applying the known asymptotic homogenization approach i. e. by taking into account the results given in [1]. However, the homogenized 2D – model derived in [1], represented by the Kirchhoff's plate equations with constant (effective) coefficients, cannot be applied to the analysis of problems in which the effect of a period length on the dynamic plate behaviour plays an important role. In the present contribution in order to remove this drawback we apply an alternative nonasymptotic approach to the modelling of periodic structures which is based on the tolerance averaging technique. All details concerning this technique as well as the full list of references can be found in [2]; nevertheless to make this paper self consistent, we outline in Section 3 fundamental concepts of the tolerance averaging.

It is assumed that the plate under consideration is thin and can be described in the framework of the Kirchhoff's plate theory. At the same time we assume that the torsional rigidity of stiffeners can be neglected and that their deflection is governed

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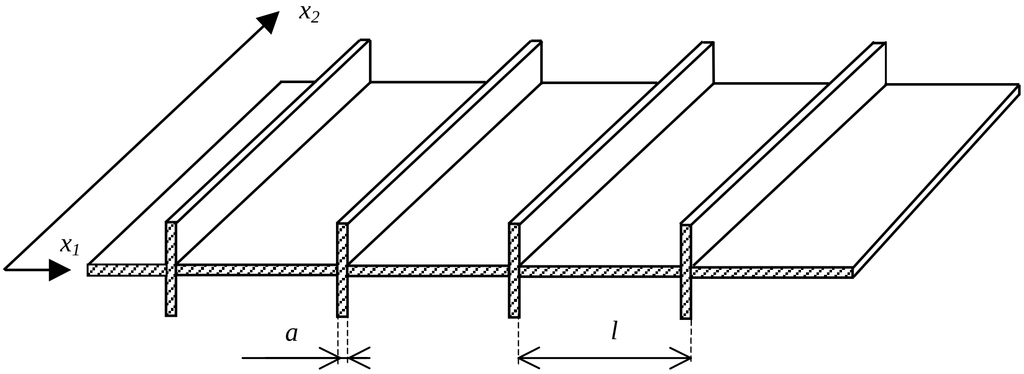
by the Euler – Bernoulli beam theory equation. For the sake of simplicity we shall also neglect the rotational inertia effect on a dynamic structure behaviour.

Key words: plates, nonasymptotic modelling, composite structures, periodic structures

SETTING OF THE MODELLING PROBLEM

Let $\Omega = (0, L_1) \times (0, L_2)$ be a region on the Ox_1x_2-plane occupied by the plate midplane in its undeformed configuration (Figure 1). We assume that the axes of undeformed stiffeners are situated on the parametric lines $x_1 = nl$, $n = 1,2, \ldots, M$, where $L_1 = (M+1)l$. Hence Ω is the plane of symmetry for the structure.

Figure 1.



We have stated above that the number *M* of stiffeners is large, i.e., condition $l << L_1$ and $a << l$, where *a* and *l* are shown in Figure 1, has to be satisfied in every problem under consideration. It is assumed that both the plate and stiffeners are homogeneous and isotropic with constant bending stiffnesses *D* and *B*, respectively. At the same time ρ_0 and *m* stand for the constant mass densities of the plate and stiffeners, respectively. The plate is subjected to the transversal loading $p_0 = p_0(x_1, x_2, t)$, $(x_1, x_2) \in \Omega$; here and subsequently t is a time coordinate. The transversal loading applied to the stiffeners will be denoted by $s_0 = s_0(nl, x_2, t)$, $n \in \{1,2, \ldots, M\}$, $x_2 \in (0, L_2)$. The mutual interactions between the plate and the stiffeners are realised by a system of internal forces $r = r(nl, x_2, t)$, $n \in \{1,2, \ldots, M\}$, $x_2 \in (0, L_2)$. All considerations will be carried out in the framework of the linearized theory.

Let $w = w(x_1, x_2, t)$, $(x_1, x_2) \in \Omega$, be the deflection of the plate midplane at time t which for $x_1 = nl$, $n \in \{1,2, \ldots, M\}$, coincides with the deflection of the stiffener axes. Under aforementioned denotations the well known equation of the Kirchhoff's plate theory yields

$$D[w_{,1111}(x_1, x_2, t) + 2w_{,1122}(x_1, x_2, t) + w_{,2222}(x_1, x_2, t)] + \rho_0 \ddot{w}(x_1, x_2, t) = \\ = p_0(x_1, x_2, t) + r(nl, x_2, t)\delta(x_1) \qquad (1)$$

where $(x_1, x_2) \in \Omega$, $n \in \{1,2, \ldots, M\}$ and

$$\delta(x_1) = \begin{cases} 0, & if\ x_1 \neq nl, \quad n = 1,\ldots,M \\ \infty, & if\ x_1 = nl, \quad n = 1,\ldots,M \end{cases}$$

is the Dirac function. At the same time the Euler – Bernoulli beam equation related to the deflection of stiffeners takes the form

$$Bw_{,2222}(nl, x_2, t) + m\ddot{w}(nl, x_2, t) = s(nl, x_2, t) - r(nl, x_2, t)\delta(x_1)$$

where $x_2 \in (0, L_2)$ and $n \in \{1,2, \ldots, M\}$. Setting $\widetilde{f}(x_1) = f(x_1)\delta(x_1)$ $x_1 \in (0, L_1)$ for an arbitrary integrable function $f(\cdot)$ (which may be constant or may also depend on x_2, t), the above equation can be written in the form

$$\widetilde{B}(x_1)w_{,2222}(x_1, x_2, t) + \widetilde{m}(x_1)\ddot{w}(x_1, x_2, t) = \widetilde{s}(x_1, x_2, t) - r(nl, x_2, t)\delta(x_1) \qquad (2)$$

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Eq. (2) is assumed to hold for every $(x_1, x_2) \in \Omega$, every $n \in \{1,2, \dots, M\}$ and every time *t*.
Introducing functions

$$C(x_1) = D + \widetilde{B}(x_1), \quad \rho(x_1) = \rho_0 + \widetilde{m}(x_1), \quad p(x_1, x_2, t) = p_0(x_1, x_2, t) + \widetilde{s}(x_1, x_2, t)$$

and eliminating from (1), (2) the plate – stiffener interactions *r*, we arrive at the equation

$$Dw_{,1111}(x_1, x_2, t) + 2Dw_{,1122}(x_1, x_2, t) + C(x_1)w_{,2222}(x_1, x_2, t) + \rho(x_1)\ddot{w}(x_1, x_2, t) = p(x_1, x_2, t) \qquad (3)$$

which is assumed to hold for every $(x_1, x_2) \in \Omega$ and every instant *t*. Equation (3) describes the behaviour of the periodically stiffened plate under consideration provided that the torsional rigidity of stiffeners and rotational inertia effect can be neglected, cf. Section 1. The characteristic feature of Eq. (3) is the highly oscillating non – continuous form of the functional coefficients $C(x_1), \rho(x_1)$ which for $l / L_1 << 1$ (i.e. for a large number *N* of stiffeners) makes finding solutions to BVP for this equation very complicated. Thus, the problem arises how to obtain an approximate model of the stiffened plate under consideration in which instead of Eq. (3) we shall deal with a certain system of equations with constant coefficients. In order to solve this modelling problem we shall apply to Eq. (3) the tolerance averaging method. As we have stated in Abstract, to make this paper self – consistent, in the subsequent section we outline the general concepts and assumptions related to the tolerance averaging of differential equations with periodic coefficients.

TOLERANCE AVERAGING TECHNIQUE

The tolerance averaging technique is based on the fact that every observation and/or measurement as well as any numerical calculation can be carried out only to within a certain accuracy. Henceforth an ambiguous idea of "accuracy" will be replaced by the well-defined mathematical notion of a tolerance relation or, for short, a tolerance. From a purely formal point of view it is a binary relation, defined on a certain nonempty set *S*, that is reflexive and symmetric. In all considerations it will be assumed that this relation is not transitive. An arbitrary tolerance relation will be denoted by $\cong$. Hence, for every $s \in S$ we have $s \cong s$, and for every $(s_1, s_2) \in S \times S$ condition $s_1 \cong s_2$ implies $s_2 \cong s_1$. The concept of *tolerance space*, proposed by Zeeman in [3], in connection with his topology of the brain and applied in [4] to some problems in mechanics, is defined as a pair $(S, \cong)$, where *S* is referred to as the underlying set and $\cong$ is a tolerance on *S*. In many special cases *S* can be interpreted as a certain linear normed space and tolerances is defined by a condition: $s_1 \cong s_2$ if and only if $\|s_1 - s_2\| \le \varepsilon$ where ε is a positive real number, which is called the *tolerance parameter*, and $\| \cdot \|$ is a norm in *S*. Reals ε can be regarded as admissible accuracies related to the computations of elements belonging to *S* or to measurements of physical objects represented by elements of *S*. Following Fichera, [5], we say that if an experimenter measures some physical quantity in the fixed system of units, then the statement "the value of this quantity is *s*" means that to the quantity under consideration can be attributed any value of the interval $[s - \varepsilon, s + \varepsilon]$, where ε is some positive constant that depends on the degree of refinement of the instruments used to perform the measurement. Hence, the concept of tolerance has some features common with interval analysis [6]. Roughly speaking, the tolerance relation $s_1 \cong s_2$ can be treated as a certain indiscernibility relation between elements s_1, s_2 of *S*. From a computational viewpoint, the relation $s_1 \cong s_2$ means that s_1 can be approximated with a required accuracy by s_2 and vice versa. The reader is referred to [2] for a more detailed discussion of this concept. The foundations of the tolerance approach are closely related to the general ideas of what is called mathematics in the alternative set theory [7]. In this theory the notion of indiscernibility is crucial. Throughout this article, the tolerance relation will be based on the simplest situation in which the underlying set *S* is a set of real numbers R equipped with a pertinent unit measure and where the tolerance parameter ε is a positive number, related to this unit measure, that determines the admissible accuracy related to computations of elements of *S* (or measurements of physical objects represented by elements of *S*) in the problem under consideration. This means that in the subsequent considerations $s_1 \cong s_2$ if and only if $\|s_1 - s_2\| \le \varepsilon$. We shall also write $s \cong 0$ if $|s| \le \varepsilon$. For the sake of simplicity, all tolerances will be denoted by the common symbol $\cong$ with the specification of ε because every tolerance defined on R is uniquely defined by a certain tolerance parameter ε. It can be seen that the relation $s_1 \cong s_2$ for an arbitrary real number *k*, $k \ne 0$, implies a new tolerance relation $ks_1 \cong ks_2$ provided that the tolerance parameter in the latter relation is equal to $|k|\varepsilon$.

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Let $F(\cdot)$ be a real-valued bounded function defined in $<0, L>$ the values of which are endowed with a certain unit measure and have to be determined within the known tolerance. This means that we know the tolerance parameter ε_F (which depends on the choice of $F(\cdot)$ and on the unit measure assigned to the range of $F(\cdot)$) such that $F(x) \cong F(y)$ if and only if $|F(x) - F(y)| \le \varepsilon_F$; it means that, from the point of view of the performed investigations, we shall not discern between the values of $F(\cdot)$ at points *x* and *y*. Here the range of function $F(\cdot)$ represents the underlying set for the tolerance relation determined by the tolerance parameter ε_F. The values of all real-valued functions $F(\cdot)$ and their derivatives defined in $<0, L>$ which are unknowns in the problem under consideration, are assumed to satisfy the aforementioned indiscernibility condition; the set of these functions will be denoted by F. The set F constitutes the domain of the mapping $\mathsf{F} \ni F \mapsto \varepsilon_F$, which will be denoted by $\varepsilon(\cdot)$. A pair (F, $\varepsilon(\cdot)$) will be referred to as the tolerance system [2]. This is the leading concept of the tolerance-averaging technique discussed throughout this article, which means that the values of all unknown fields in every problem under consideration are specified only to within a certain tolerance.

Due to the condition $l << L = L_1$, the parameter *l* will be referred to as the microstructure size. Combining the concept of tolerance system and that of the microstructure size *l*, we define $T = (\mathsf{F}, \varepsilon(\cdot), l)$ and formulate some important definitions. To this end define $\Delta = (-l/2, l/2)$, $\Delta(x) = \Delta + x$ and $L_\Delta =< l/2, L - l/2)$.

Definition 1. The differentiable function $F \in \mathsf{F}$ will be called *slowly varying* (in the known tolerance system $T = (\mathsf{F}, \varepsilon(\cdot), l)$ and with respect to a microstructure size *l*), $F \in SV(T)$, if for every $y_1, y_2 \in Dom\ F$ condition $|y_1 - y_2| < l$ implies $|F(y_1) - F(y_2)| \le \varepsilon_F$ and if similar conditions hold also for the derivatives $\partial F(\cdot), \dot{F}(\cdot)$ of $F(\cdot)$ (with pertinent tolerance parameters $\varepsilon_{\partial F}, \varepsilon_{\dot{F}}$, respectively).

Definition 2. The differentiable function φ defined in $<0, L>$ will be called periodic-like, $\varphi \in PL(T)$, if for every $x \in L_\Delta$ there exists the Δ-periodic function φ_x such that for every $x \in Dom\ \varphi$ the condition $|y - x| < l$ implies $|\varphi(y) - \varphi_x(x)| \le \varepsilon_\varphi$ and similar conditions hold for all derivatives of φ. The function φ_x will be called the *l-periodic approximation* of φ in $\Delta(x)$.

Definition 3. The differentiable function ψ will be called oscillating, $\psi \in PL^*(T)$, if $\psi \in PL(T)$ and $<\psi>=0$ in L_Δ.

Independently of the tolerance we introduce the notion of averaging. For an arbitrary integrable function *g* defined in $<0, L>$ we introduce the averaging operation by means of the formula

$$<g>(x) = \frac{1}{l} \int_{\Delta(x)} g(y)dy \qquad x \in L_\Delta$$

obviously, if $g(\cdot)$ is an *l*-periodic function, then $<g>$ is constant.

Using these definitions we shall formulate the basic assertions and lemmas that will be used in the proposed method of modelling.

Assertions. If $F \in SV(T)$, $\varphi \in PL(T)$, and φ_x is an *l*-periodic approximation of φ in $\Delta(x)$, than for every $f \in L^\infty_{\text{per}}(\Delta)$ and $h \in C^1_{\text{per}}(\overline{\Delta})$ such that $\max\{|h(y)| : y \in \overline{\Delta}\} \le l$, the following propositions hold for every $x \in L_\Delta$:

(A1)	$\langle fF\rangle(x) \cong <f> F(x)$	for	$\varepsilon = \langle \lvert f \rvert \rangle \varepsilon_F$
(A2)	$\langle f\varphi\rangle(x) \cong <f\varphi_x>(x)$	for	$\varepsilon = \langle \lvert f \rvert \rangle \varepsilon_\varphi$
(A3)	$\langle f\partial(hF)\rangle(x) \cong <fF\partial h>(x)$	for	$\varepsilon = \langle \lvert f \rvert \rangle (\varepsilon_F + \varepsilon_{\partial F})$
(A4)	$\langle h\partial(f\varphi)\rangle(x) \cong -<f\varphi\partial h>(x)$	for	$\varepsilon = \varepsilon_G + \varepsilon_{\partial G},\quad G =< hf\varphi > l^{-1}$

where ε is a tolerance parameter that defines the tolerance $\cong$.

Lemmas. Using the preceding notation, the following propositions hold:

(L1) If $\varphi \in PL(T)$ and $f \in C_{\text{per}}(\overline{\Delta})$ then $fF(\cdot) \in PL(T)$.

(L2) If $g \in PL(T)$ then for some $g^0 \in SV(T)$ and $g^* \in PL(T)$ the decomposition $g = g^0 + g^*$ exists.

(L3) If $F \in SV(T)$ and $f \in C_{\text{per}}(\overline{\Delta})$ then $fF(\cdot) \in PL(T)$.

(L4) If $F \in SV(T)$, $G \in SV(T)$ and $kF + mG \in F(\overline{\Omega})$ for some reals *k* and *m* then $kF + mG \in SV(T)$.

The proofs of these propositions can be found in [3].
The proposed method of modelling is based on two assumptions.

Conformability Assumption. In the modelling of dynamic problems for the periodic plates under consideration, the displacement $w(\cdot, x_2, t)$ must satisfy the condition $w(\cdot, x_2, t) \in PL(T)$, which holds for every x_2, t. This condition may be violated only near the boundary of a plate.

Tolerance Averaging Assumption. In the averaging of equations involving slowly varying and/or periodic – like functions, the left-hand sides of formulae (A1)-(A4) will be approximated respectively by their right-hand sides.

These assumptions will constitute the foundations of the tolerance-averaging technique applied to Eq. (3).

MODELLING APPROACH

From the conformability assumption and (L2) we obtain the decomposition

$$w(\cdot, x_2, t) = w^0(\cdot, x_2, t) + u(\cdot, x_2, t) \tag{4}$$

where $w^0(\cdot, x_2, t) \in \mathrm{SV(T)}$ and $u(\cdot, x_2, t) \in \mathrm{PL(T)}$ are referred to as the averaged and fluctuation displacements, respectively.

Let us restrict the domain $< 0, L_1 >$ of a function $w(\cdot, x_2, t)$ in (3) to $\Delta(x) =< x - \frac{l}{2}, x + \frac{l}{2} >$ for some $x \in L_\Delta$. The tolerance averaging approach to Eq. (3) will be realised in four steps.

1^0 We formulate the variational equation for the periodic approximation $u_x(\cdot, x_2, t)$, $x \in L_\Delta$, of fluctuations $u(\cdot, x_2, t)$ in $\Delta(x)$. To this end we substitute $w(\cdot, x_2, t) \cong w^0(\cdot, x_2, t) + u_x(\cdot, x_2, t)$ into (3) and multiply both sides of (3) by an *l*-periodic test function $v(x)$ satisfying condition $< v >= 0$. After averaging the resulting equation over $\Delta(x)$ and simple manipulation we get

$$\begin{aligned} &D < u_x,_{11} v,_{11} > (x, x_2, t) + 2D < u_x,_{22} v,_{11} > (x, x_2, t) + < Cu_x,_{2222} v > (x, x_2, t) + \\ &+ < \tilde{B} v > (x, x_2, t) w^0,_{2222} (x, x_2, t) + < \tilde{m} v > (x, x_2, t) \ddot{w}^0 (x, x_2, t) + < \rho \ddot{u}_x v > (x, x_2, t) = \\ &=< pv > (x, x_2, t), \end{aligned} \tag{5}$$

for every $x \in L_\Delta$, $x_2 \in < 0, L_2 >$ and every time *t*.

2^0 Using the Galerkin method we look for the approximate solution $u_x(y, x_2, t)$, $y \in \Delta(x)$ to (5) in the form

$$u_x(y, x_2, t) = h^A(y) W^A(x, x_2, t) \tag{6}$$

where $W^A(\cdot, x_2, t)$ are new unknown functions and $h^A(y)$ are postulated a priori *l*-periodic mode-shape functions satisfying conditions $< h^A >= 0$, $h^A(y) \in O(l^2)$, $lh^A,_1(y) \in O(l^2)$, $l^2 h^A,_{11}(y) \in O(l^2)$; here and

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in the subsequent analysis superscripts A, B run over 1, ... , N and the summation convention holds. Substituting the right-hand side of (6) into (5) and setting $v = h^B W^B$ we arrive at the system of N differential equations for N unknown fields W^B :

$$
\begin{aligned}
& D < h^A{}_{,11} h^B{}_{,11} > W^B (x_1, x_2, t) + 2D < h^A{}_{,11} h^B > W^B{}_{,22} (x_1, x_2, t) + \\
& \qquad\qquad < Ch^A h^B > W^B{}_{,2222} (x_1, x_2, t) + < \tilde{m} h^A > \ddot{w}^0 (x_1, x_2, t) + \\
& < \rho h^A h^B > \ddot{W}^B (x_1, x_2, t) + < \tilde{B} h^A > w^0{}_{,2222} (x_1, x_2, t) = < p h^A > (x_1, x_2, t)
\end{aligned} \tag{7}
$$

3^0 From the decomposition (4) and (L2), (7), by means of the approximation (6), we conclude that $W^A(\cdot, x_2, t) \in SV(T)$ and we arrive at the following approximation formula for the displacement field

$$
w(x_1, x_2, t) = w^0 (x_1, x_2, t) + h^A (x_1) W^A (x_1, x_2, t) \tag{8}
$$

4^0 We substitute the right-hand sides of (8) into (3) and average this result over $\Delta(x)$. After simple manipulations we obtain

$$
\begin{aligned}
& Dw^0{}_{,1111} (x_1, x_2, t) + 2Dw^0{}_{,1122} (x_1, x_2, t) + < C > w^0{}_{,2222} (x_1, x_2, t) + \\
& \qquad + < \tilde{B} h^A > W^A{}_{,2222} (x_1, x_2, t) + < \rho > \ddot{w}^0 (x_1, x_2, t) + < \tilde{m} h^A > \ddot{W}^A (x_1, x_2, t) = \\
& \qquad\qquad = < p > (x_1, x_2, t)
\end{aligned} \tag{9}
$$

It can be seen that the above modelling approach leads from equation (3) for the total displacement field $w(\cdot)$, which has highly oscillating coefficients, to the system of equations (7), (9) for the averaged displacement field $w^0(\cdot)$ and N extra unknowns $W^A(\cdot)$. Equations (7), (9) have constant coefficients and hence constitute the proper mathematical tool for the analysis of special problems. It has to be emphasised that solutions to (7), (9) have a physical sense only if $w^0(\cdot, x_2, t)$, $W^A(\cdot, x_2, t)$ are slowly varying functions, i.e., if conditions

$$
w^0(\cdot, x_2, t) \in \mathrm{SV(T)} \qquad W^A(\cdot, x_2, t) \in SV(T) \tag{10}
$$

hold for every $x_2 \in < 0, L_2 >$ and every time t.

Summarizing the obtained results we can state that the derived nonasymptotic model of a stiffened plate under consideration is governed by the system of equations (7), (9) for the basis unknown $w^0(\cdot)$ and $W^A(\cdot)$, by the physical reliability conditions (10) and by the approximation formula (8) for the total displacements. The new unknown fields $W^A(\cdot)$ will be referred to as the fluctuational variables because together with the mode-shape functions $h^A(\cdot)$ determine the fluctuations $u(\cdot)$ of the displacements $w(\cdot)$, cf. formula (4). It can be observed that the constant coefficients in (7), (9) which depend on $h^A(\cdot)$ describe the effect of the microstructure size l on the averaged behaviour of the stiffened plate. It follows that the derived nonasymptotic model, in contrast to the homogenized asymptotic model, [1], can be used to the analysis of phenomena related to the existence of what can be called the microstructure length-scale effect. A simple example is the occurrence of higher order free vibration frequencies caused by a periodic structure of the stiffened plate.

In order to determine the form of coefficients in the general model equations (7), (9) we have to introduce *a priori* a system of the mode-shape functions $h^A(\cdot)$ leading to the Galerkin approximation (6) of the variational equation (5). Generally speaking, functions $h^A(\cdot)$ should approximate the expected principal modes of plate free vibrations which are l-periodic and their mean values in every interval $\Delta(x)$ are equal to zero. Using the Galerkin method, the choice of mode-shape functions is based on knowledge of the principal modes of free vibrations in similar system, cf. [9], p.309. That is why in the problems under consideration we shall postulate mode-shape functions in the form of trigonometric functions. The special case of the model equations (7), (9) will be given in the subsequent section.

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MODEL EQUATIONS – SPECIAL CASE

In order to specify the model equations (7), (9) we shall introduce only one mode-shape function given by

$$h(x_1) = l^2 \cos \frac{2\pi x_1}{l} \tag{11}$$

Substituting the right-hand side of (11) into (7), (6) and introducing denotations

$$\overline{B} = \frac{B}{l}, \quad \overline{m} = \frac{m}{l}, \quad \overline{s} = \frac{s}{l}, \quad \overline{D} = D + \overline{B}, \quad \overline{\rho} = \rho_0 + \overline{m}, \quad \overline{p} = p_0 + \overline{s}, \quad E = \frac{1}{2}\left(D + 2\overline{B}\right),$$
$$r = \frac{1}{2}\left(\rho_0 + 2\overline{m}\right), \quad D_2 = 4\pi D, \quad D_0 = 8\pi^2 D$$

we obtain the following system of equation for the averaged displacement field $\mathrm{v} = w^0(x_1, x_2, t)$ and displacement fluctuation field $W = W^1(x_1, x_2, t)$:

$$D\mathrm{v}_{,1111} + 2D\mathrm{v}_{,1122} + \overline{D}\mathrm{v}_{,2222} + \overline{\rho}\mathrm{v} - l^2\overline{B}W_{,2222} - l^2\overline{m}\ddot{W} = \overline{p},$$
$$l^4 EW_{,2222} - l^2 D_2 W_{,22} + D_0 W + l^4 r\ddot{W} - l^2\overline{B}\mathrm{v}_{,2222} - l^2\overline{m}\mathrm{v} = -l^2\overline{s}, \tag{12}$$

Equations (12) have to be satisfied in $\Omega = (0, L_1)\times(0, L_2)$ for every time t and represent a special case of the general model equations (7), (9). The above equations have a physical sense only if conditions

$$\mathrm{v}(\cdot, x_2, t) \in \mathrm{SV(T)}, \qquad W(\cdot, x_2, t) \in SV(T) \tag{13}$$

hold for every $x_2 \in <0, L_2>$ and every time *t*. The total plate deflections can be approximated by a sum $w(x_1, x_2, t) = \mathrm{v}(x_1, x_2, t) + h(x_1)W(x_1, x_2, t)$, with $h(\cdot)$ given by formula (11). It can be seen that the mode – shape function $h(\cdot)$ is responsible for the independent local vibrations of stiffeners in the planes normal to the Ox_2-axis, which are combined with the local vibrations of the plate. Let us observe that neglecting in (12) terms depending on *l* we obtain $W = 0$ and we shall pass to the homogenized model equations

$$D\mathrm{v}_{,1111} + 2D\mathrm{v}_{,1122} + \overline{D}\mathrm{v}_{,2222} + \overline{\rho}\mathrm{v} = \overline{p}$$

of the stiffened plate under consideration.

CONCLUDING REMARKS

In this contribution it was shown that the behaviour of thin elastic plates reinforced by a system of periodically spaced parallel stiffeners can be investigated on the basis of the averaged equations (equations with constant coefficients) derived by the tolerance averaging of the "exact" plate equation (equation with functional highly oscillating non-continuous coefficients). The general form (7), (9) of the obtained averaging plate equations depends on the form of the postulated *a priori* mode-shape functions and for a special choice of these functions leads to equations (12). It can be shown that in contrast to the known homogenized (asymptotic) model of a stiffened plate, the proposed nonasymptotic model describes the effect of the period length on the overall behaviour of the plate. Moreover, using the derived model equations we can satisfy the boundary conditions for $x_1 = 0$ and $x_2 = L_2$ not only for the averaged plate deflections but also for displacement fluctuations. At the same time the applied nonasymptotic modelling approach makes it possible to determine *a posteriori* an accuracy of the obtained solutions to special problem by means of conditions (10). Some applications of the obtained model equations will be given in the forthcoming paper [10].

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Department of Civil Engineering and Geodesy
Warsaw Agricultural University
ul. Nowoursynowska 159
02-787 Warsaw, Poland
e-mail: nagorko@alpha.sggw.waw.pl

Czesław Woźniak
Institute of Mathematics and Computer Sciences
Technological University of Częstochowa
ul. Dąbrowskiego 73
42-200 Częstochowa, Poland
e-mail: wozniak@matinf.pcz.czest.pl
