Electronic Journal of Polish Agricultural Universities is the very first Polish scientific journal published exclusively on the Internet, founded on January 1, 1998 by the following agricultural universities and higher schools of agriculture: University of Technology and Agriculture of Bydgoszcz, Agricultural University of Cracow, Agricultural University of Lublin, Agricultural University of Poznan, Higher School of Agriculture and Teacher Training Siedlce, Agricultural University of Szczecin, and Agricultural University of Wroclaw.

ELECTRONIC
JOURNAL
OF POLISH
AGRICULTURAL
UNIVERSITIES

2001
Volume 4
Issue 2
Series
WOOD TECHNOLOGY

SMARDZEWSKI J., GAWROŃSKI T. 2001. FEM ALGORITHM FOR CHAIR OPTIMISATION **Electronic Journal of Polish Agricultural Universities**, Wood Technology, Volume 4, Issue 2.
Available Online http://www.ejpau.media.pl

FEM ALGORITHM FOR CHAIR OPTIMISATION

Jerzy Smardzewski, Tomasz Gawroński
Department of Furniture Design, August Cieszkowski Agricultural University of Poznań, Poland

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ABSTRACT

Current trends to improve the quality and reliability of manufactured finished products cause that more and more attention is paid to the optimisation of rigidity-strength dimensioning of furniture elements constructional nodes. The objective of this study was to estimate the effectiveness of static optimisation methods determining the minimum of material volume in time function and sampling number. Moreover, the performed investigations aimed at establishing minimal dimensions of component elements of the construction at maximum strength

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parameters. The experiments carried out revealed that static optimisation of the construction of the chair with the aid of the Monte-Carlo method integrated with the FEM environment allowed to reduce material consumption to 53% of the initial volume within 17 seconds of work of the application. A sub-optimal solution, sufficient for engineering practice, is reached, on average, in time 18 times shorter than that reached using the random walk method.

Key words: furniture, numeric optimisation, strength, rigidity, material, time

INTRODUCTION

Both scientific experience and engineering practice indicate that decision making processes in the course of solving complex designing problems require an analysis of a great number of different construction variants. These types of decision-making processes take much time and do not always result in the selection of an optimal solution. That is why deterministic or numerical optimisation methods are applied in a wide range of technical areas, which assist in the selection of the best solution. Computer software, which have already taken roots in machine and construction industries, still find a limited application in furniture industry. A majority of manufactured furniture continues to be designed exclusively on the basis of craftsmanship and engineering practice. However, pressures resulting from quality and reliability requirements of manufactured final products make it more and more necessary to pay increasingly more attention to rigidity-strength optimisation of dimensioning of furniture elements and constructional joints.

The rigidity-strength procedure of furniture design comprises the determination of values of outside loads affecting the construction, the establishment of the distribution of inner forces and then the calculation of dimensions of elements and constructional joints [2]. Analytical methods of solving spatial systems of statically indeterminable furniture structures, because of their complexity and time consumption, are not applicable in industrial practice. Therefore, it is advisable to introduce numerical methods, which shorten the cycle of calculations and spare the engineer a great deal of arduous mathematical investigations. Eckelman and Suddarth [3] elaborated, on the basis of their own algorithms of matrix calculations of flat frame deformations occurring in furniture constructions, a Fortran computer program in which dowel joints were used. Joints of wooden constructions were treated in this program as semi-rigid with a non-linear dependence between the bending moment and angle deformation. It should be said that a satisfactory convergence was found between the calculated values of deformations and results of laboratory tests.

Many later studies commonly employed a finite element method (FEM), which is based on the concept of rigidity matrix. This method was also used by Gustafsson [4] in the optimisation process of the height position of a chair-connecting member. The author compared successive variants of the optimised construction assuming the maximum bending moment affecting the frame as the optimisation criterion. The elaborated model was verified qualitatively in the next study [5] in which the problem of buckling of the compressed constructional elements was analysed in detail. Moreover, Gustafsson [6] compared results of numerical calculations with results of laboratory measurements of the deformed piece of furniture the model of which was prepared on the basis of results of optimisation calculations and arrived at the conclusion that, in many cases, the solution obtained with the aid of the applied computer program portrayed the true state of deformations and stresses. The observed numerical discrepancies were attributed to, on the one hand, inexactitudes of laboratory tests resulting from technical problems and, on the other, to differences in wood strength features during compression and tension, which were not taken under consideration in a typical FEM algorithm. The method of finite elements was also applied by Smardzewski [7] in his computer application of Panda-1 utilised to analyse the construction of carcass furniture with symmetrical side frames. These problems were further continued in studies, which discussed in detail principles for the calculation of connection dimensions in dowel and tenon joints on the basis of a recognised distribution of internal forces [9].

In industrial conditions, apart from the need to determine the reliability of a given construction, a natural objective of the designer is to minimise manufacturing costs of given articles which, in turn, is connected with minimisation of material consumption. This stresses the need to elaborate computer programs capable of applying numerical optimisation of furniture constructions. In the case of carcass furniture, significant savings can be achieved by minimising dimensions of the cross section of wood elements. Typical computer applications analysing force distribution and internal stresses in construction members allow to verify the strength of a system which was designed and dimensioned earlier or to compare several variants of the same design. Therefore, their application, in the case when a simultaneous decision concerning several dimensions of the same construction must be taken, may be difficult or even impossible. The selection of the best – from the point of view of a definite criterion – construction parameters can be achieved applying numerical optimisation algorithms [1] working on mathematical models of the projected system. Investigations in this field were undertaken by Smardzewski [8] who presented algorithms and results of optimisation of a chair side frame using the method of systematic search and random walk. The objective of those calculations was to determine cross-sectional dimensions of scantling elements and connection dimensions, while maintaining appropriate strength parameters and minimal volume of the applied material. In the above-mentioned study, values of internal forces, calculated by means of a separate FEM processor, were used as input data for the application, which realised the optimisation process.

One of the basic difficulties encountered during the optimisation process of furniture constructions, which, in their majority, constitute statically indeterminable systems, is a variation in the distribution of internal forces affected by changes in cross-sectional dimensions of component elements. This indicates a need to develop optimisation algorithms in which values of internal forces will be calculated after each change of geometrical conditions.

OBJECTIVE OF THE STUDY

The objective of this study was to determine the efficiency of methods of static optimisation, which allow to determine minimal material volume in time function and number of samplings. Moreover, the performed investigations aimed at determining minimal dimensions of component elements of the structure at maximum strength parameters. The cognitive aspect of the study is to demonstrate possibilities of integration of the static optimisation method in FEM environment, while its practical feature demonstrates the possibility of utilisation of the program by furniture engineers and designers.

METHODS EMPLOYED IN THE STUDY

The object of the performed experiments was a wooden chair with a crosspiece designed as shown in Figure 1 [8]. Because of the axial symmetry of the system, its static and strength analyses were confined only to one side frame of the chair. The applied solution made it necessary to assume loads equalling half of the loads recommended by the Polish standard BN-83/7140-12.11 p. 1.4.2.1.

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The study employs statistical methods of static optimisation: the Monte-Carlo method and random walk. On the basis of algorithms of these methods, a computer program in the C^{++} language was developed consisting of an optimisation module built into the algorithm of the method of finite elements. This solution allowed a continuous correction of the optimisation process on the basis of strength conditions.

The application comprises optimisation of flat systems, including statically indeterminable ones. Because of the skeletal character of the construction, the finite element is a beam with a permanent cross-section, two nodes and three degrees of node freedom.

Optimisation was performed at constant conditions of support and load of the structure as well as at constant material parameters. The variable factors included cross-sectional dimensions of constructional elements, which were simultaneously treated as decision variables of the optimisation. The observed values were: the construction volume as the target function as well as distribution values of internal forces. The efficiency evaluation criterion of the applied test algorithm included: time of optimisation and the number of samplings.

CONSTRUCTION OF THE MATHEMATICAL MODEL

In order to develop a mathematical model of optimisation, the selected chair side frame was subjected to discretisation as shown in Figure 1 isolating individual elements with a rectangular cross-section. In accordance with the principles of division of wooden frame constructions into finite elements [9], construction nodes were placed in points of support, points of application of concentrated external loads, collapse of axis, termination of scantling elements and in places of change of physical or geometrical properties. Additionally, each component of the construction was divided into sections of constant length in order to allow simulation of the non-linear variation of cross-sectional dimensions. For the described system, the cube of decision variables assumed the following form:

$$K_z = \{b, h_i : b_{\min} \le b \le b_{\max}, h_{\min} \le h_i \le h_{\max}, i = 1 \ldots n\}$$

where:

b – width of cross-section equal for all finite elements,
h_i – height of cross-section of finite element i,
n – number of finite elements.

Permanent optimisation parameters included values characterising material:

E – modulus of linear elasticity,
k_g – wood maximum bending strength,
k_t – wood maximum shearing strength,
k – coefficient of safety.

Variable parameters comprised values of internal forces in elements:

M_i – maximum bending moment in element i,
T_i – maximum shearing strength in element i.

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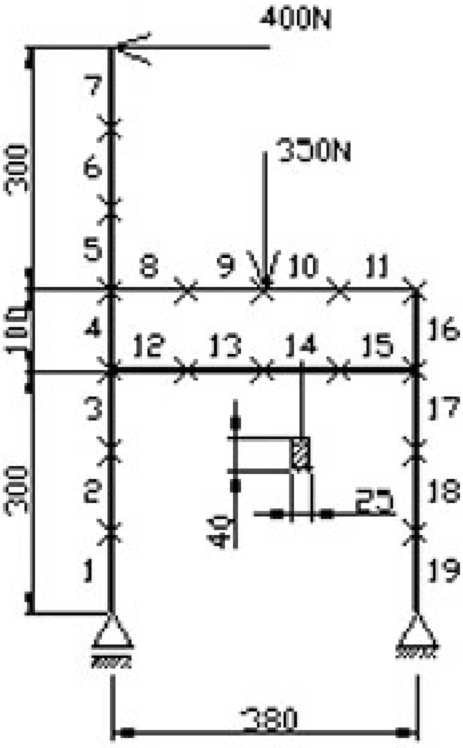
Fig.1. Static diagram and the method of the chair side frame discretisation



The acceptable area was delineated by a set of limiting conditions in the form of the following inequality:

$$\Phi = \{b, h_i : \varphi_j (b, h_i) \geq 0, i = 1..n\}$$

where, consecutively, the following are taken into account:

- the condition of the resistance of the element to bending:

$$k \frac{6M_i}{bh_i^2} \leq k_g$$

- the condition of the resistance of the element to shearing:

$$k \frac{T_i}{bh_i} \leq k_t$$

Additionally, the mathematical model comprises a number of limiting conditions directly influencing mutual relationships between dimensions of the construction. These conditions were introduced, on the one hand, for aesthetic purposes and, on the other, in order to reduce

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the acceptable areas by eliminating solutions, which, on the basis of an initial heuristic evaluation, can be assessed as very distant from optimal ones.

$$h_1 \le h_2 \le h_3$$

$$h_5 \le h_6 \le h_7$$

$$h_{19} \le h_{18} \le h_{17}$$

The minimisation of the construction volume was adopted as the target function, which was expressed as the following formula:

$$V = \sum_{i=1}^{n} b h_i l_i \rightarrow \min$$

in which:

n – number of finite elements,
l_i – length of element.

COMPUTER PROGRAM

On the basis of the above-presented mathematical model, an algorithm for conducting calculations was developed which was implemented in the C++ language in the form of a computer program. At the beginning of work, the application allows the user to select an optimisation method after which data about the construction are fed from the graphic pre-processor, which forms part of the CAD system. The consecutive stages of application work depend on the selected optimisation method.

The approach adopted to combine the algorithm of the Monte-Carlo method with that of the finite element method is illustrated in Figure 2. Each sampling of point *X* found in the cube of decision variables, is followed by checking of technological-construction conditions. If they are fulfilled, the value of the target function *f'* for each *X* point is calculated and compared with the current minimal value of the target function f. If $f' < f$ is fulfilled, verification of strength conditions follows. If the construction shows an appropriate strength, point X is considered as the best one of all the sampled points so far.

Because of static indeterminability of the construction, each verification of strength conditions after the change of element dimensions requires a fresh activation of the processor of the finite element method in order to determine the value of internal forces. Because of the complexity of the FEM algorithm, it is expected that the time of work of the computer needed to calculate internal forces will be many times longer than the time required to calculate the value of the target function or control of the compatibility of decision variables with technological-constructional assumptions. Therefore, in order to shorten the working time of the application, the strength conditions in the Monte-Carlo method are checked as the last ones and, then, only for those points of the cube of decision variables which fulfil the $f' < f$ condition.

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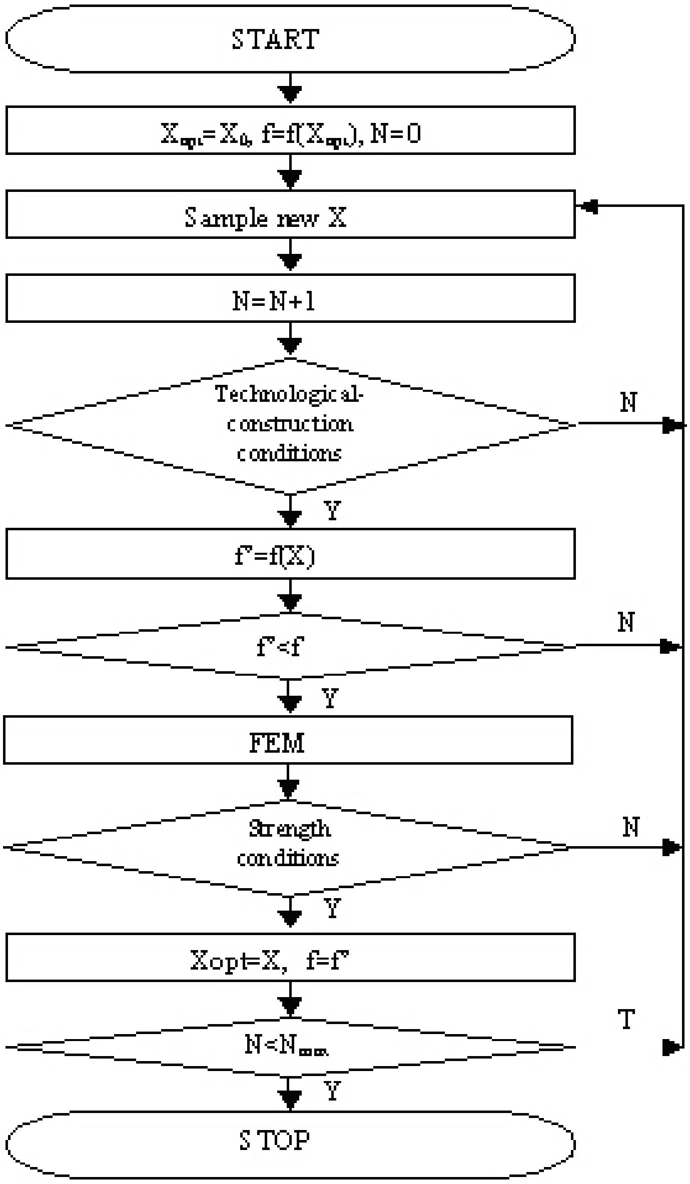
Fig. 2. Optimisation algorithm by means of the Monte-Carlo method



In the case of the method of random walk, this type of algorithm optimisation of calculations is considerably restricted. Verification of strength conditions is, in this case, necessary in order to classify the point sampled from the space of decision variables. However, the advantage of this method is that it explores primarily the edge of the acceptable area where the optimal point is usually located. The technique of incorporation of the optimisation module into the FEM algorithm is presented in Figure 3.

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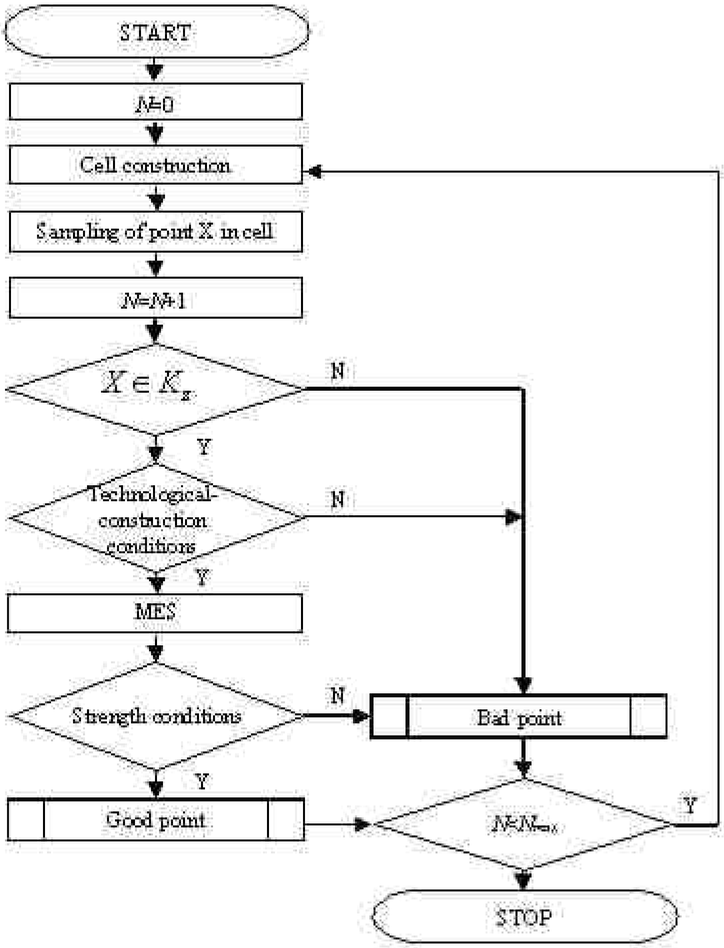
Fig. 3. Optimisation algorithm by means of the random walk method



OPTIMISATION OF THE CONSTRUCTION OF THE CHAIR SIDE FRAME

Prior to the initiation of the optimisation process, values of internal forces in constructional elements were determined (Fig. 4) and on their basis it was established that the examined object fulfils strength conditions before optimisation. Next, using the above-described computer application, optimisation of the construction of the chair side frame was performed adopting the following parameters:

- extreme cross-sectional dimensions of elements: $b_{min}= h_{min} = 15$ mm, $b_{min}= h_{min} = 60$ mm,
- material characteristics (beech wood): Young modulus $E = 18\ 000$ MPa, bending strength along fibres $k_g = 100$ MPa, shearing strength across fibres $k_t = 8$ MPa,
- safety coefficient $k = 2.5$,
- parameters of cell formation in the method of random walk: initial cell dimensions – 9 mm, length of good series $S_d = 10$, length of bad series $S_z = 10$, coefficient of cell dimension change $k = 1.5$.

In order to determine the effectiveness of both methods, three optimisation trials were carried out for each of them recording several times values of the target function and decision variables after carrying out a definite, set beforehand, number of samplings. These values,

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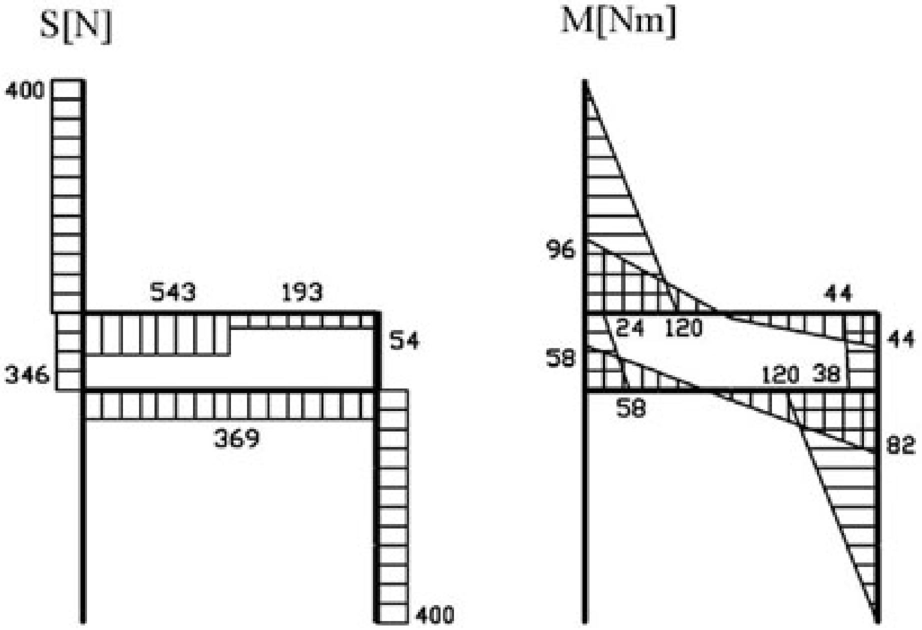
differing for both optimisation methods, were accepted after the initial evaluation of the working time of the application. In the case of the Monte-Carlo method, the result was recorded after: $10 \cdot 10^3$, $50 \cdot 10^3$, $100 \cdot 10^3$, $500 \cdot 10^3$, $1 \cdot 10^6$, $10 \cdot 10^6$, $20 \cdot 10^6$, $30 \cdot 10^6$ and $40 \cdot 10^6$ samplings, while in the case of the method of random walk – after $2 \cdot 10^3$, $5 \cdot 10^3$, $10 \cdot 10^3$, $20 \cdot 10^3$, $30 \cdot 10^3$, $40 \cdot 10^3$, $50 \cdot 10^3$ and $100 \cdot 10^3$ samplings.

Fig. 4. Distribution of shearing forces and bending moments in the chair side frame before optimization



ANALYSIS OF RESULTS

Courses of optimisation processes for both methods are presented in Figures 5 and 6. It is evident from the diagram in Figure 5 presenting changes of the target function in the course of the optimisation process that, when the Monte-Carlo method was used, the application was aiming towards the optimal solution faster. On the other hand, in the case of the random walk method, many more samplings were successful (Fig. 6). It appears that the exploration of the edge of the acceptable area, characteristic for the method of random walk, led to a gradual minimalisation of the target function value, which, in turn, resulted in a more frequent verification of strength conditions on the basis of matrix calculations. Because operations on matrixes constitute a considerable load for the computer processor, the Monte-Carlo method was considered as the more effective method with regard to the minimum time of application work.

In order to determine the number of samplings necessary to obtain a sub-optimal solution considered satisfactory in engineering practice, two intermediate (designated as A and B) and one final (designated as C) result of application work were distinguished in the process of optimisation by means of the Monte-Carlo method (Fig. 6) for which the optimisation time and the volume of the used material are presented in Table 1, and cross-sectional dimensions of elements are shown in Figure 7.

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Fig.5. Material volume in the time function of optimisation

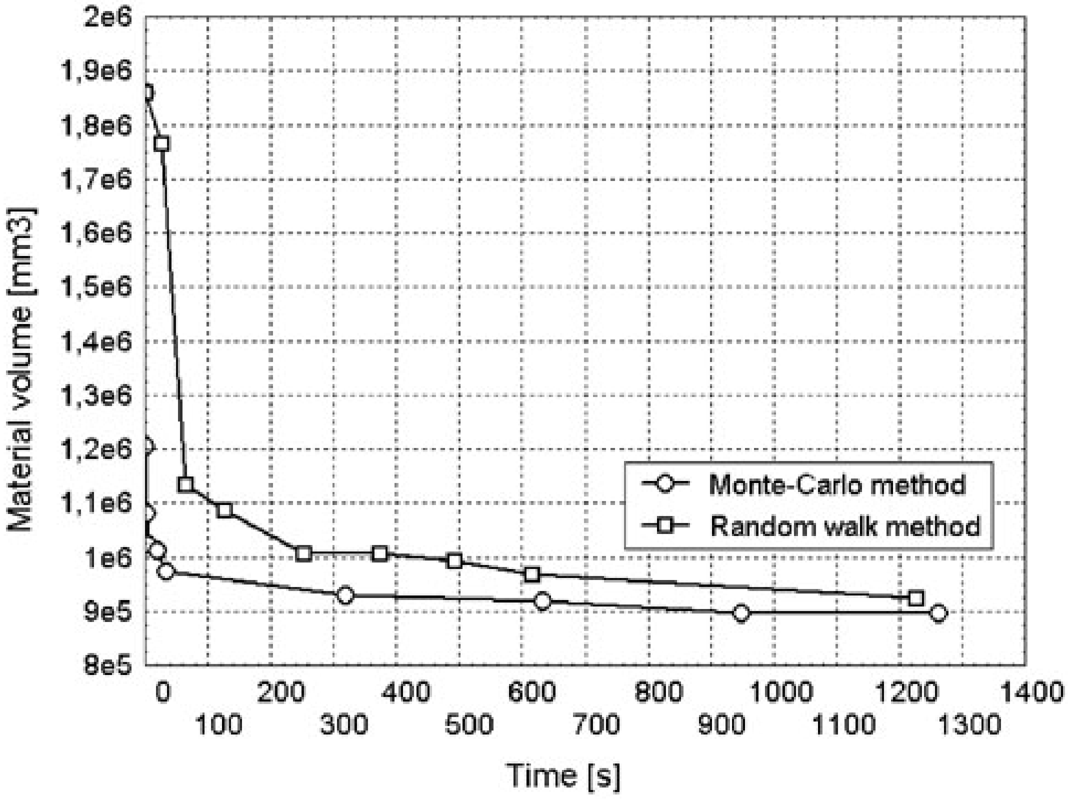
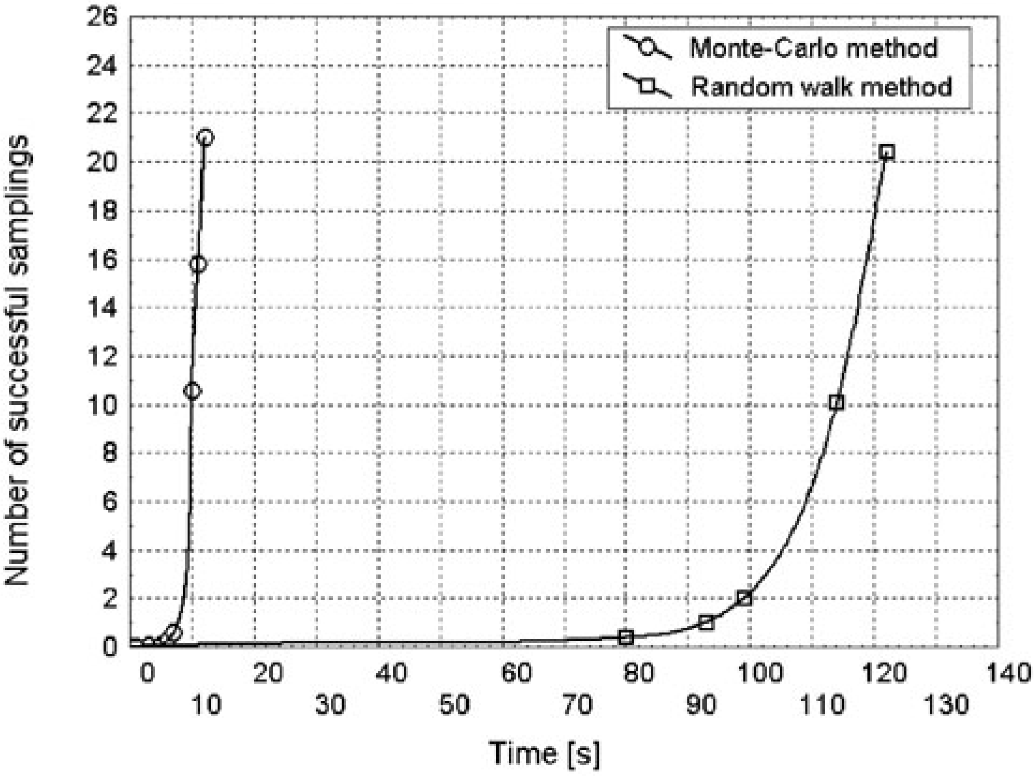


Fig. 6. Number of successful samplings in the time function of optimisation



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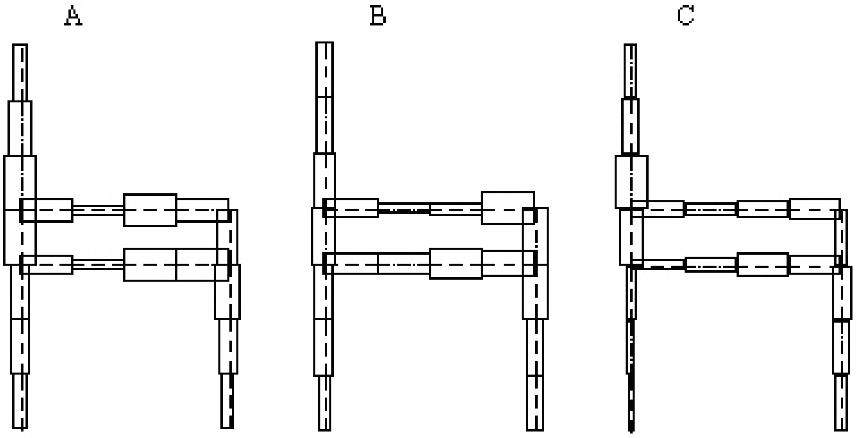
Solution A, despite a reduction in the volume of material needed to manufacture the article, cannot be accepted as a correctly optimised construction. The rapid change of cross-sectional dimensions of the underframe and the crosspiece is not only undesirable for both technological and aesthetic reasons, but is equally inappropriate from the point of view of the strength of the construction. It can, therefore, be concluded that the number of samplings after which result A was arrived at was too small to achieve the assumed optimisation task. On the other hand, solution C was reached after too long working time of the application, which can be troublesome in practical applications. Solution B was accepted as the sub-optimal result both from the point of view of construction dimensions and the volume of material required to make it. Figure 8 shows the distribution of internal forces in this solution. The comparison of solutions B and C, both constructionally acceptable, indicates that increased number of samplings had a positive influence on optimisation effect. However, in practice, an excessive prolongation of application work can be troublesome for the system user. Result C was obtained after 21 minutes of work of the application, which should be considered as quite long in situations in which many alternative constructions or loads are to be analysed. Frequently, the aim of optimisation in industrial conditions is to find a sub-optimal solution acceptable from the point of view of construction and which is also economically advantageous. From among the above-presented solutions, solution B fulfils these requirements.

Table 1. Optimisation times and construction volumes for three selected optimisation results

Point	Number of samplings	Time [min]	Volume $[cm^3]$	Share of initial volume
A	$10 \cdot 10^3$	0.02	1252	67%
B	$500 \cdot 10^3$	0.28	991	53%
C	$40 \cdot 10^6$	21.00	877	47%

Fig.7. Dimension change of construction elements in the course of optimisation process



In order to carry out an analysis of distribution of internal forces in the optimised construction B, values of shearing forces and bending moments were determined which are shown in Figure 8. The comparison of Figures 4 and 8 revealed that it was possible to obtain a reduction in the value of the bending moment in the side underframe of the chair at the

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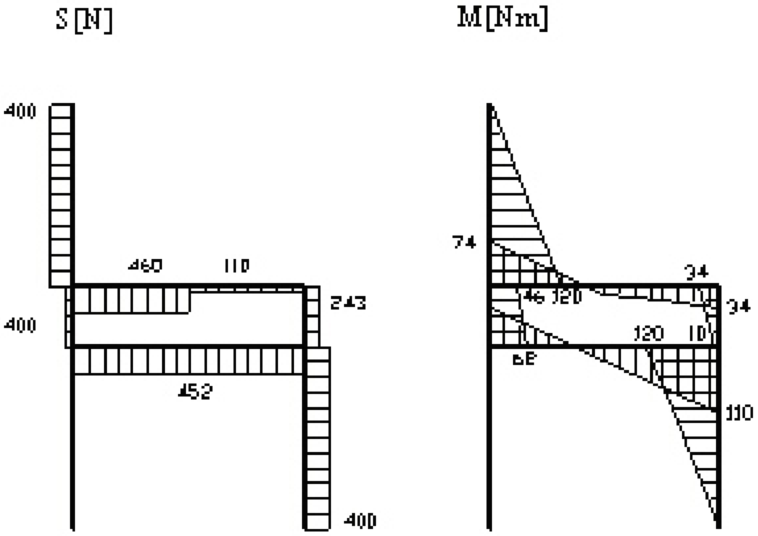
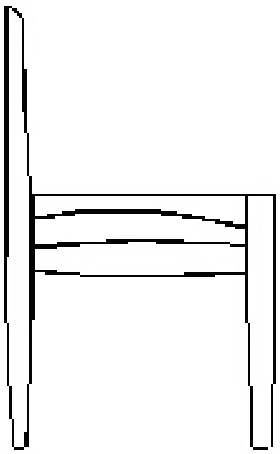
expense of an increase in the value of the bending moment in the crosspiece. It can, therefore, be concluded that the optimisation of the presented side frame of the chair resulted in the improvement of the strengthening function of the crosspiece.

Fig.8. Distribution of shearing forces and bending moments for construction B



The proposed design still needs some additional work, which will give the optimised construction appropriate aesthetic form by replacing the discreted system by curving edges of its elements (Fig. 9). This can be done automatically using algorithms of a numerical adjustment of multinomial functions. However, in order to keep the aesthetic function of the construction, this job should be left for the designer.

Fig. 9. Final design of the optimised construction



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CONCLUSIONS

Summing up, it can be concludes that:

1. Static optimisation of the construction of a chair by the Monte-Carlo method integrated with the FEM environment allowed to reduce material consumption to 53% of its initial volume within 17 seconds of work of the application.
2. Optimisation algorithm used in the Monte-Carlo method aims at an optimal solution faster. A sub-optimal solution, acceptable for engineering practice, is reached, on average, in time 18 times shorter than that reached using the random walk method.
3. High effectiveness of the application employing the Monte-Carlo algorithm is attributed to a sporadic activation of the FEM processor. The time consumption of the optimisation process is the result of numerical calculation realised in matrix operations, which takes up much of the processor's time.

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Submited:

Jerzy Smardzewski, Tomasz Gawroński
Department of Furniture Design
August Cieszkowski Agricultural University of Poznań
Wojska Polskiego 38/42, 60-637 Poznań, Poland
e-mail: JSmardzewski@woodcock.au.poznan.pl

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