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FREE VIBRATIONS OF TRANSVERSALLY GRADED PLATE BANDS

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ABSTRACT

In this note free vibrations of a plate band with a smooth and a slow gradation of macroscopic properties called a transversally graded plate band have been analysed. In this contribution the tolerance and the asymptotic models of these bands have been presented. Then, these models have been used to calculate fundamental free vibrations frequencies of the plate band, by means of the Ritz method. Moreover, these results have been compared to results obtained by a computer programm of the finite element method (FEM).

Key words: thin plate band, transversally graded structure, free vibrations

INTRODUCTION

Free vibrations of a thin plate band with a span L are investigated in this note. The plate band has a functionally graded material macrostructure (on the macrolevel) along its span, cf. the book by Suresh and Mortensen [14]. On the other hand, on the microlevel it has tolerance-periodic microstructure, cf. Jędrysiak [4]. It is also assumed that the material properties of the plate are independent from x_2-coordinate. The fragment of the plate band is shown in Fig. 1. One assumes that the microstructure size is described by the length l of "the cell" and is very small in comparison to the span L of the plate.

Plates of this kind are described by the partial differential equations with highly oscillating, tolerance-periodic, non-continuous coefficients. These equations are not a proper tool to investigate special problems. Thus, various averaged models, describing these plates by equations with smooth, slowly-varying coefficients, have been formulated . Because these plates are treated as made of *a functionally graded material*, cf. [14], they are called *transversally graded plates*.

Approaches used to analyse macroscopically homogeneous media, e.g. periodic, are usually applied also to describe functionally graded structures. Some of these techniques are discussed in the book [14]. One has to mention these models, which are based on the asymptotic homogenization, cf. the book [8]. Unfortunately, the governing equations of these models neglect the effect of the microstructure size on the behaviour of these structures. Some extensions in the asymptotic modelling applied to functionally graded laminates are shown by Woźniak [18].

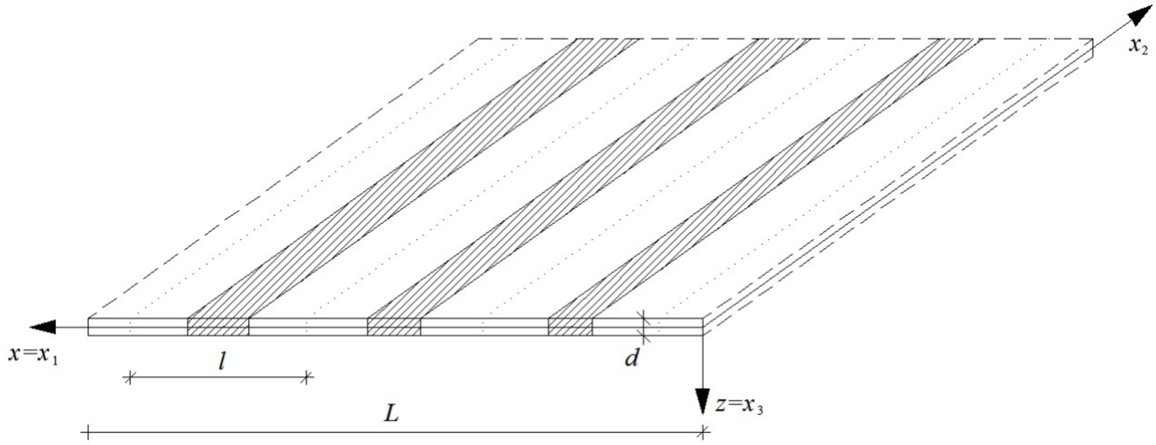

Fig. 1. The fragment of a thin transversally graded plate band

To take into account this effect *the tolerance modelling* (cf. the books [19, 20]) may be applied. The applications of this method to various periodic structures are shown in a series of papers, e.g. [1,2,6,10,11,15,16,17]. The tolerance modelling is also adopted for dynamic problems of functionally graded structures, e.g. [7,12,13]. Some applications to dynamic and stability problems for thin tolerance-periodic plates are shown by: Jędrysiak [3, 4]; Jędrysiak and Michalak [5]; Kaźmierczak, Jędrysiak and Wirowski [9], where the finite differences method is used to obtain free vibrations frequencies. The extended list of papers may be found in the books edited by Woźniak, Michalak and Jędrysiak [20], by Woźniak *et al*. [19].

There are three aims of the paper. The first one is to present the tolerance and the asymptotic models of vibrations for thin transversally graded plate bands. The second aim is to calculate the free vibration frequencies of a simply supported plate band in the framework of the tolerance and asymptotic models using the Ritz method. The third is to compare obtained results of fundamental, lower frequencies to results calculated by the finite element method (FEM).

FORMULATION OF THE PROBLEM

It is assumed that presented considerations are treated as independent of x_2-coordinate. Let one denote $x = x_1$, $z = x_3$, $x \in [0,L]$, $z \in [-d/2, d/2]$, where d is a constant plate thickness. It means that in this case one assumes that the plate band is described in the interval $\Lambda = (0,L)$, with the basic cell $\Omega \equiv [-l/2, l/2]$ in the interval $\overline{\Lambda}$, where l, $l << L$, is the length of the basic cell. Moreover, it is assumed that $d << l$. One recalls that a cell with a centre at $x \in \Lambda$ is denoted by $\Omega(x) \equiv (x - l/2, x + l/2)$. Let the plate band be made of two elastic isotropic materials, perfectly bonded across interfaces, and characterised by Young's moduli E', E'', Poisson's ratios ν', ν'' and mass densities ρ', ρ'', respectively. It may be assumed that $E(x), \rho(x), x \in \Lambda,$ are tolerance-periodic, highly oscillating functions in x, but Poisson's ratio $\nu \equiv \nu' = \nu''$ is constant. Hence, under condition $E' \neq E''$ and/or $\rho' \neq \rho''$ the plate material structure may be treated as transversally functionally graded in the x-axis direction. By ∂ denote a derivative of x. Let $w(x,t)$ ($x \in \overline{\Lambda}$, $t \in (t_0, t_1)$) be a plate band deflection.

Let one introduce tolerance-periodic functions in x, describing plate band properties: the mass density per unit area of the midplane μ and the bending stiffness B, which may be defined by:

$$\mu(x) \equiv d\rho(x), \qquad B \equiv \tfrac{d^3}{12(1-\nu^2)} E(x). \tag{1}$$

From the well-known assumptions of the Kirchhoff-type plate theory it may be obtained for transversally graded plate bands the partial differential equation of the fourth order for deflection $w(x,t)$

$$\partial\partial[B(x)\partial\partial w(x,t)] + \mu(x)\ddot{w}(x,t) = 0, \tag{2}$$

with coefficients being highly oscillating, non-continuous, tolerance-periodic functions in x. Equation (2) describes free vibrations of the plate bands under consideration.

BASIC CONCEPTS

Following the book [19] some of basic concepts of the tolerance modelling are reminded below. For tolerance-periodic plates some of them were also presented in [4].

A cell at $x \in \Lambda_\Omega$ is denoted by $\Omega(x) \equiv x + \Omega$, $\Lambda_\Omega = \{x \in \Lambda : \ \Omega(x) \subset \Lambda\}$. The known *averaging operator* for an integrable function *f* is defined by

$$< f > (x) = \frac{1}{l} \int_{\Omega(x)} f(y) dy, \quad x \in \Lambda_\Omega. \tag{3}$$

For a tolerance-periodic function *f* in *x* its averaged value calculated from (3) is a slowly-varying function in *x*.

Let one denote the *k*–th gradient of function $f = f(x), x \in \Lambda, \ k = 0,1,...,\alpha$, ($\alpha \geq 0$), by $\partial^k f$; $\partial^0 f \equiv f$. Let $\tilde{f}^{(k)}(\cdot,\cdot)$ be a function defined in $\overline{\Lambda} \times R^m$, and δ be a tolerance parameter. Moreover, let one introduce $\Lambda_x \equiv \Lambda \cap \bigcup_{z \in \Omega(x)} \Omega(z)$, $x \in \overline{\Lambda}$.

Function $f \in H^\alpha(\Lambda)$ is *the tolerance-periodic function*, $f \in TP^\alpha_\delta(\Lambda,\Omega)$, if for $k = 0,1,...,\alpha$, the following conditions are satisfied

(1°) $(\forall x \in \Lambda)\, (\exists \tilde{f}^{(k)}(x,\cdot) \in H^0(\Omega))\ \ [\| \partial^k f\big|_{\Lambda_x}(\cdot) - \tilde{f}^{(k)}(x,\cdot) \|_{H^0(\Lambda_x)} \leq \delta]$,

(2°) $\int_{\Omega(\cdot)} \tilde{f}^{(k)}(\cdot,z) dz \in C^0(\overline{\Lambda})$.

Function $\tilde{f}^{(k)}(x,\cdot)$ is called *the periodic approximation of* $\partial^k f$ *in* $\Omega(x), x \in \Lambda, \ k = 0,1,...,\alpha$.
Function $F \in H^\alpha(\Lambda)$ is *the slowly-varying function*, $F \in SV^\alpha_\delta(\Lambda,\Omega)$, if

(1°) $F \in TP^\alpha_\delta(\Lambda,\Omega)$,

(2°) $(\forall x \in \Lambda)\, [\tilde{F}^{(k)}(x,\cdot)|_{\Omega(x)} = \partial^k F(x), \quad k = 0,\ldots,\alpha]$.

Function $\phi \in H^\alpha(\Lambda)$ is *the highly oscillating function*, $\phi \in HO^\alpha_\delta(\Lambda,\Omega)$, if

(1°) $\phi \in TP^\alpha_\delta(\Lambda,\Omega)$,

(2°) $(\forall x \in \Lambda)\, [\tilde{\phi}^{(k)}(x,\cdot)|_{\Omega(x)} = \partial^k \tilde{\phi}(x), \quad k = 0,1,...,\alpha]$,

(3°) $\forall F \in SV^\alpha_\delta(\Lambda,\Omega)\ \ \exists f \equiv \phi F \in TP^\alpha_\delta(\Lambda,\Omega)\ \ \tilde{f}^{(k)}(x,\cdot)|_{\Omega(x)} = F(x)\partial^k \tilde{\phi}(x)|_{\Omega(x)}, \quad k = 1,\ldots,\alpha$.

For α=0 let one denote $\tilde{f} \equiv \tilde{f}^{(0)}$.

Let *h*(·) be defined on $\overline{\Lambda}$ a highly oscillating function, $h \in HO^2_\delta(\Lambda,\Omega)$, continuous together with gradient $\partial^1 h$. However, gradient $\partial^2 h$ is a piecewise continuous and bounded. Function *h*(·) is *the fluctuation shape function* of the 2-nd kind, $FS^2_\delta(\Lambda,\Omega)$, if it depends on *l* as a parameter and conditions hold:

(1°) $\partial^k h \in O(l^{\alpha-k})$ for k=0,1,…,α, α=2, $\partial^0 h \equiv h$,

(2°) $<\mu h>(x) \approx 0$ for every $x \in \Lambda_\Omega$,

where μ>0 is a certain tolerance-periodic function; *l* is the microstructure parameter.

MODELLING ASSUMPTIONS

Following the books [19, 4] and using the basic concepts, the fundamental modelling assumptions may be formulated. *The micro-macro decomposition* of the plate band deflection w is the first assumption:

$$w(x,t)=W(x,t)+h^A(x)V^A(x,t), \quad A=1,\dots,N, \quad x\in\Lambda, \tag{4}$$

with $W(\cdot,t), V^A(\cdot,t)\in SV_\delta^2(\Lambda,\Omega)$ (for every t) as basic kinematic unknowns, and $h^A(\cdot)\in FS_\delta^2(\Lambda,\Omega)$. Function $W(\cdot,t)$ is called *the macrodeflection*; $V^A(\cdot,t)$ are called *the fluctuation amplitudes*; and $h^A(\cdot)$ are the known fluctuation shape functions.

The second modelling assumption is *the tolerance averaging approximation*, in which terms $O(\delta)$ are assumed to be negligibly small in the course of modelling, e.g. in formulas:

$$\begin{aligned}
&<\phi>(x)=<\bar{\phi}>(x)+O(\delta),\\
&<\phi F>(x)=<\phi>(x)F(x)+O(\delta),\\
&<\phi\partial_\alpha(h^A F)>(x)=<\phi\partial_\alpha h^A>(x)F(x)+O(\delta),\\
&x\in\Lambda;\ \alpha=1,2;\ A=1,\dots,N;\ 0<\delta<<1;\\
&\phi\in TP_\delta^2(\Lambda,,\ ),\ F\in SV_\delta^2(\Lambda,,\ ),\ h^A\in FS_\delta^2(\Lambda,,\ ),
\end{aligned}$$

where δ is a tolerance parameter.

TOLERANCE MODELLING PROCEDURE

Following the monograph [19] the modelling procedure may be outlined in the form.

The first step is the formulation of the action functional

$$\mathrm{A}(w(\cdot))=\int_\Lambda\int_{t_0}^{t_1}\Lambda(y,\partial\partial w(y,t),\dot{w}(y,t))dtdy, \tag{5}$$

with the lagrangean Λ given by

$$\Lambda=\tfrac{1}{2}(\mu\dot{w}\dot{w}-B\partial\partial w\partial\partial w). \tag{6}$$

From the principle stationary action applied to A, after some manipulations, one may obtain the known equation (2) of free vibrations for thin transversally graded plate bands.

In the next step of the tolerance modelling we substitute micro-macro decomposition (4) to action functional (5). In the third step, applying averaging operator (3) to the action functional one obtains the tolerance averaging of functional $\mathrm{A}(w(\cdot))$ in the form

$$\mathrm{A}_h(W(\cdot),V^A(\cdot))=\int_\Lambda\int_{t_0}^{t_1}<\Lambda_h>(y,\partial\partial W,\dot{W},\dot{V}^A,W,V^A)dtdy, \tag{7}$$

with the averaged form $<\Lambda_h>$ of lagrangean (6)

$$\begin{aligned}
<\Lambda_h>=-\tfrac{1}{2}\{(&<B>\partial\partial W+2<B\partial\partial h^B>V^B)\partial\partial W-\\
&-<\mu>\dot{W}\dot{W}+<B\partial\partial h^A\partial\partial h^B>V^AV^B-<\mu h^Ah^B>\dot{V}^A\dot{V}^B\}.
\end{aligned} \tag{8}$$

The principle stationary action applied to A_h leads to the system of Euler-Lagrange equations with coefficients being slowly-varying functions in x.

TOLERANCE MODEL EQUATIONS

From the system of Euler-Lagrange equations after some manipulations one may arrive to the following system of equations for $W(\cdot,t)$ and $V^A(\cdot,t)$:

$$\begin{aligned} &\partial\partial(<B>(x)\partial\partial W+<B\partial\partial h^B>(x)V^B)+<\mu>(x)\ddot{W}=0,\\ &<B\partial\partial h^A>(x)\partial\partial W+<B\partial\partial h^A\partial\partial h^B>(x)V^B+\underline{<\mu h^A h^B>(x)}\ddot{V}^B=0. \end{aligned} \tag{9}$$

The above equations involve the underlined term with the microstructure parameter l. The coefficients of equations (9) are slowly-varying functions in x. The equations (9) together with micro-macro decomposition one (4) constitute *the tolerance model of thin transversally graded plate bands*. This model makes possible to take into account the effect of the microstructure size on free vibrations of these plates. For the plate band described in $\Lambda=(0,L)$ one has to formulate boundary conditions only for *the macrodeflection W* (on the edges $x=0$, L), but not for *the fluctuation amplitudes* V^A, $A=1,\ldots,N$.

ASYMPTOTIC MODEL EQUATIONS

It may be shown that $<\mu h^A h^B>\in O(l^4)$. Neglecting the term with l in equation $(9)_2$ one arrives to the algebraic equations for the fluctuation amplitudes V^A:

$$V^A=-(<B\partial\partial h^A\partial\partial h^B>)^{-1}<B\partial\partial h^B>\partial\partial W. \tag{10}$$

After substituting the right-hand side of equation (10) into $(9)_1$ the following equation for $W(\cdot,t)$ has been obtained:

$$\partial\partial((<B>(x)-<B\partial\partial h^A>(x)(<B\partial\partial h^A\partial\partial h^B>(x))^{-1}<B\partial\partial h^B>(x))\partial\partial W)+<\mu>(x)\ddot{W}=0. \tag{11}$$

The above equation and micro-macro decomposition (4) represent *the asymptotic model of thin transversally graded plate bands*. This model may be obtained in the framework of the formal asymptotic modelling procedure, cf. the books [19, 4]. In equation (11) the effect of the microstructure size on free vibrations of the transversally graded plates is neglected. The asymptotic model describes the macrobehaviour of the plate bands under consideration.

EXAMPLE – FREE VIBRATIONS OF A PLATE BAND

Introduction

Let us consider free vibrations of a simply supported thin plate band with span L along the x-axis. The properties of the plate band are assumed to be described by the following functions:

$$\rho(\cdot,z)=\begin{cases}\rho', & \text{for} \quad z\in((1-\gamma(x))l/2,(1+\gamma(x))l/2),\\ \rho'', & \text{for} \quad z\in[0,(1-\gamma(x))l/2]\cup[(1+\gamma(x))l/2,l],\end{cases} \tag{12}$$

$$E(\cdot,z)=\begin{cases}E', & \text{for} \quad z\in((1-\gamma(x))l/2,(1+\gamma(x))l/2),\\ E'', & \text{for} \quad z\in[0,(1-\gamma(x))l/2]\cup[(1+\gamma(x))l/2,\lambda],\end{cases} \tag{13}$$

where $\gamma(x)$ is a distribution function of material properties, cf. Fig. 2.

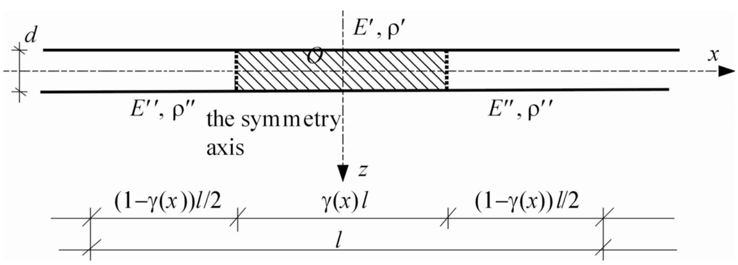

Fig. 2. A cell of the transversally graded plate band

Let one restrict the considerations assuming only one fluctuation shape function, i.e. $A=N=1$. Denoting $h\equiv h^1$, $V\equiv V^1$, micro-macro decomposition (4) of field $w(x,t)$ may be written as:

$$w(x,t) = W(x,t) + h(x)V(x,t),$$

where $W(\cdot,t), V(\cdot,t) \in SV_\delta^2(\Lambda,\Omega)$ for every $t \in (t_0,t_1)$, $h(\cdot) \in FS_\delta^2(\Lambda,\Omega)$.

Since the cell has a structure shown in Fig. 2 the periodic approximation of the fluctuation shape function $h(x)$ is assumed in the form

$$\tilde{h}(x,z) = \lambda^2[\cos(2\pi z/l) + c(x)], \quad z \in \Omega(x), \quad x \in \Lambda,$$

where parameter $c(x)$ is determined by $<\tilde{\mu}\tilde{h}>=0$ and is a slowly-varying function in x. From the aforementioned condition it takes the form

$$c = c(x) = \frac{\sin[\pi\tilde{\gamma}(x)](\rho' - \rho'')}{\pi\{\rho'\tilde{\gamma}(x) + \rho''[1 - \tilde{\gamma}(x)]\}},$$

where $\tilde{\gamma}(x)$ is the periodic approximation of the distribution function of material properties $\gamma(x)$. In calculations of derivatives $\partial\tilde{h}, \partial\partial\tilde{h}$ parameter $c(x)$ is treated as constant.

Denote:

$$\begin{aligned} &\hat{B} \equiv < B >, \quad \tilde{B} \equiv < B\partial\partial h >, \quad \bar{B} = < B\partial\partial h \partial\partial h >, \\ &\hat{\mu} = < \mu >, \quad \bar{\mu} = l^{-4} < \mu hh >. \end{aligned} \tag{14}$$

Hence, tolerance model equations (9) for free vibrations of the transversally graded plate bands under consideration may be written as:

$$\begin{aligned} &\partial\partial(\hat{B}\partial\partial W + \tilde{B}V) + \hat{\mu}\ddot{W} = 0, \\ &\tilde{B}\partial\partial W + \bar{B}V + l^4\bar{\mu}\ddot{V} = 0. \end{aligned} \tag{15}$$

Moreover, using denotations (14), for the plate band equation (11) takes the form:

$$\partial\partial[(\hat{B} - \tilde{B}^2/\bar{B})\partial\partial W] + \hat{\mu}\ddot{W} = 0. \tag{16}$$

Equation (16) describes free vibrations of this plate band within the asymptotic model. It may be observed that all coefficients of equations (15) and (16) are slowly-varying functions in x.

The Ritz method applied to the asymptotic model equation

Because it is too difficult to find analytical solutions of equations (15) or (16), which have slowly-varying, functional coefficients, approximate formula of free vibrations frequencies may be obtained by means of the known Ritz method, cf. Jędrysiak [4]. In this method relations of the maximal strain energy Y_{max} and the maximal kinetic energy K_{max} are determined.

Because the plate band under consideration is simply supported, the solution to equation (16) and equations (15) are assumed in the form:

$$W(x,t) = A_W \sin(\alpha x)\cos(\omega t), \qquad V(x,t) = A_V \sin(\alpha x)\cos(\omega t), \tag{17}$$

with a wave number α and a free vibrations frequency ω. Introducing denotations:

$$
\begin{aligned}
\hat{B} &= \tfrac{d^3}{12(1-\nu^2)} \int_0^L \{E''[1-\tilde{\gamma}(x)] + \tilde{\gamma}(x)E'\} \sin^2(\alpha x)dx, \\
\overline{B} &= \tfrac{(\pi d)^3}{3(1-\nu^2)} \int_0^L \{(E'-E'')[2\pi\tilde{\gamma}(x) + \sin(2\pi\tilde{\gamma}(x))] + 2\pi E''\} \sin^2(\alpha x)dx, \\
\widetilde{B} &= \tfrac{\pi d^3}{3(1-\nu^2)} (E'-E'') \int_0^L \sin(\pi\tilde{\gamma}(x)) \sin^2(\alpha x)dx, \\
\breve{\mu} &= d\int_0^L \{[1-\tilde{\gamma}(x)]\rho'' + \tilde{\gamma}(x)\rho'\} \sin^2(\alpha x)dx, \\
\bar{\mu} &= \tfrac{d}{4\pi} \int_0^L \{(\rho'-\rho'')[2\pi\tilde{\gamma}(x) + \sin(2\pi\tilde{\gamma}(x))] + 2\pi\rho''\} \sin^2(\alpha x)dx + \\
&\quad + \tfrac{d}{\pi}(\rho'-\rho'') \int_0^L c(x)[\pi c(x)\tilde{\gamma}(x) - 2\sin(\pi\tilde{\gamma}(x))] \sin^2(\alpha x)dx + \\
&\quad + d\rho'' \int_0^L [c(x)]^2 \sin^2(\alpha x)dx,
\end{aligned}
\tag{18}
$$

and using (17) formulas of the maximal energies – strain Y_{max} and kinetic K_{max} for the tolerance model take the form:

$$
\begin{aligned}
\mathrm{Y}^{TM}_{max} &= \tfrac{1}{2}[(\hat{B}A_W{}^2\alpha^2 - 2\widetilde{B}A_W A_V)\alpha^2 + \overline{B}A_V{}^2], \\
\mathrm{K}^{TM}_{max} &= \tfrac{1}{2}(A_W{}^2\breve{\mu} + A_V{}^2 l^4 \bar{\mu})\omega^2,
\end{aligned}
\tag{19}
$$

However for the asymptotic model they have the form:

$$
\begin{aligned}
\mathrm{Y}^{AM}_{max} &= \tfrac{1}{2}[(\hat{B}A_W{}^2\alpha^2 - 2\widetilde{B}A_W A_V)\alpha^2 + \overline{B}A_V{}^2], \\
\mathrm{K}^{AM}_{max} &= \tfrac{1}{2}A_W{}^2\breve{\mu}\omega^2.
\end{aligned}
\tag{20}
$$

Using the conditions of the Ritz method:

$$
\frac{\partial(\mathrm{Y}_{max} - \mathrm{K}_{max})}{\partial A_W} = 0, \qquad \frac{\partial(\mathrm{Y}_{max} - \mathrm{K}_{max})}{\partial A_V} = 0,
\tag{21}
$$

from relations (19) after some manipulations we obtain the following formulas:

$$
(\omega_{-,+})^2 \equiv \frac{(\alpha l)^4 \bar{\mu}\hat{B} + \breve{\mu}\overline{B} \mp \sqrt{[\hat{B}(\alpha l)^4\bar{\mu} - \breve{\mu}\overline{B}]^2 + 4(\alpha l)^4 \breve{\mu}\bar{\mu}\widetilde{B}^2}}{2\breve{\mu}\bar{\mu} l^4},
\tag{22}
$$

of *the lower* ω_- and *the higher* ω_+ *free vibrations frequencies*, respectively, for *the tolerance model*.

In the framework of *the asymptotic model* conditions (21) are applied to equations (20) and after manipulations one arrives in the following formula:

$$
\omega^2 \equiv \alpha^4 \frac{\hat{B}\overline{B} - \widetilde{B}^2}{\breve{\mu}\overline{B}},
\tag{23}
$$

of the lower free vibrations frequency ω.

Results

In order to compare obtained results in the numerical example the considerations are restricted only to the lower free vibrations frequency ω_- (or ω) calculated by the tolerance (or the asymptotic) model.

Let us consider the distribution function of material properties $\gamma(x)$ in the following form:

$$\tilde{\gamma}(x) = \sin^2(\pi x / L)\,. \tag{24}$$

Moreover, we introduce dimensionless frequency parameters given by:

$$\Omega_-^{\;2} \equiv \tfrac{12(1-\nu^2)\rho'}{E'} L^2 \omega_-^{\;2}, \qquad \Omega^2 \equiv \tfrac{12(1-\nu^2)\rho'}{E'} L^2 \omega^2, \tag{25}$$

with the free vibrations frequency ω_- and ω determined by equations (22) and (23), respectively.

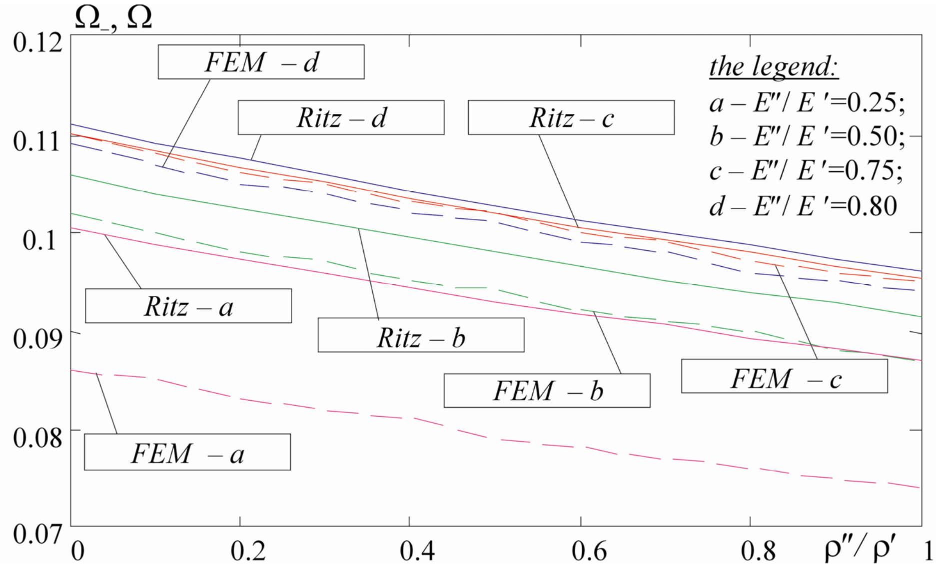

Fig. 3. The plots of the lower frequency parameters Ω_-, Ω versus the ratio of mass densities ρ''/ρ' (ν=0.3, ratio l/L=0.1, ratio d/l=0.1)

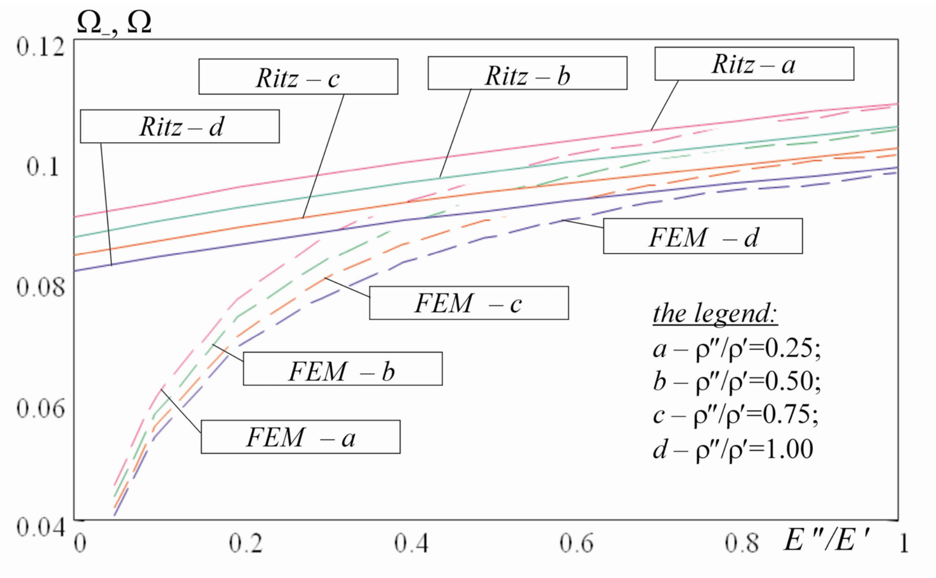

Fig. 4. The plots of the lower frequency parameters Ω_-, Ω versus the ratio of Young's moduli E''/E' (ν=0.3, ratio l/L=0.1, ratio d/l=0.1)

Calculational results are shown in Figs. 3-4. These figures present the results obtained in the framework of the tolerance (or the asymptotic) model (using the Ritz method) in comparison with those calculated by the FEM method. Fig. 3 shows plots of the lower frequency parameters versus ratio ρ''/ρ' (for E''/E'=0.25, 0.5, 0.75, 0.8), but Fig. 4 shows curves of these parameters versus ratio E''/E' (for ρ''/ρ'=0.25, 0.5, 0.75, 1). These calculations are made for the Poisson's ratio ν=0.3, the wave number $\alpha=\pi/L$, ratio l/L=0.1 and ratio d/l=0.1.

From results presented in Figs. 3-4 the following remarks may be formulated:

1. values of the lower free vibrations frequencies depend on ratios E''/E' and ρ''/ρ', i.e.:
   - they increase with the increasing of ratio E''/E' (cf. Fig. 4),
   - they decrease with the increasing of ratio ρ''/ρ' (cf. Fig. 3);
2. differences between frequencies calculated by the tolerance (or the asymptotic) model and those by the FEM method are small for ratios $E''/E'>¼$, cf. Fig. 4.

REMARKS

The tolerance modelling applied to the known differential equation of Kirchhoff-type plater, having a transversally graded macrostructure, leads to the tolerance model equations. This modelling method makes possible to replace the governing differential equation with non-continuous, tolerance-periodic coefficients by the system of differential equations with slowly-varying coefficients. The derived tolerance model equations describe the effect of the microstructure size on the overall behaviour of transversally graded plates under consideration. However, in the framework of the asymptotic model this effect is omitted.

In order to compare results obtained in the framework of the proposed models with results by the FEM method, the example is restricted to investigate only the lower free vibrations frequency. From this example it may be observed that these frequencies decrease with the increasing of the ratio of the mass densities ρ''/ρ' and increase with the increasing of the ratio of the Young's moduli E''/E'. Moreover, it may be observed that differences between frequencies calculated by the tolerance and the asymptotic models and those by the finite element method are small for ratios $E''/E'>¼$.

Other special problems of vibrations for the transversally graded plates and some evaluations of obtained results will be shown in forthcoming papers.

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