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# SOME ASPECTS OF CONTROLLING THE STOCK OF INPUTS IN THE CONTEXT OF EFFECTIVE USE OF FIRM'S CAPITAL

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### ABSTRACT

This article deals with the subject of optimizing the streams of input deliveries when rational, regular, cyclical deliveries synchronized with production needs are required. It examines the costs of storage as well as the costs of production stoppages associated with a depleted stock of inputs. The subject matter is discussed in terms of cybernetic modelling for optimal control that ultimately provides the basic parameters for the level of the safety stock and the cycle of deliveries. The optimization criterion is a function minimizing the total costs of storage and of production coming to a halt because of depleted stock of inputs.

Key words: production demands, controlling the stock of inputs, delivery model

#### **INTRODUCTION**

The issue of rationality of input deliveries and the keeping of stock levels proportionally to enterprises' demand is one of the key aspects of organizations' financial efficiency. The question whether keeping a reserve of inputs is purposeful left aside, attention should be paid to the amounts and scheduling of input deliveries to the manufacturers. This issue is especially important in a changeable market environment that makes stock management, as well as the financing of this aspect of business activity, risky and uncertain. The range of possible financial problems arising in this area may include, *inter alia*, frozen capital (sometimes of substantial value), financial burden and its impact on firm's financial results, or the bearing of the procurement processes on firm's profits.

This article intends to draw attention to some aspects of stock management, as well as presenting a method for analysing the control processes in an uncertain environment.

## VARYING PRODUCTION DEMANDS AND CONTROLLING THE STOCK OF INPUTS

Generally, demand is characterised by a temporal variability of the forecasted values concerning, for instance, input deliveries. This can be a spontaneous variability and seasonal changes, as well as short and long-term trends. Random incidents are natural determinants of firms' operation, because inputs come from different suppliers and have their specific schedules. Therefore, the stock levels tend to be self-creating. Consequently, the regularity and thus continuity of the production process are rendered uncertain when the firm runs out of stock or, alternatively, the possible overstocking gives rise to additional, unnecessary costs related to frozen capital, deteriorating inputs' quality when an extended period of storing affects their physico-chemical properties, accidents during which inputs are damaged, etc. On the other hand, synchronizing the stock of inputs with their amounts needed in the production process reduces costs and improves flow efficiency, thus making the enterprise more profitable. Further, synchronisation decreases firm's demand for capital necessary to fund its operations by reducing expenses, especially the costs of financing, etc. [Nowak 1994]. In the simplest terms, the effects of non-optimal stock management come down to wasted opportunities that capital offers. A relevant illustration is the typical approach to stock management in the enterprise. Real-life data show that incomplete information (on the volume of demand, market absorption, product life cycle, customers' expectations, etc.) results in overstocking, whose value is estimated at approximately 10% of the total stock value. This produces measureable financial consequences [Duckworth 1960, Gattorna 1994, Goldberg 1990]. For instance, assuming that in the company ALFA:

- an annual average level of stock is estimated at 35 million PLN;
- the level of overstocking invariably stands at 10% of that amount, i.e. PLN 3.5m (10% x PLN 35m = PLN 3.5m);
- opportunity costs of capital = 12% p.a.
- the financial loss due to overstocking is PLN 0.4m (PLN 3.5m x 12%)
- when the probability of occurrence of the analysed situation is 80%, the estimated weighted value of financial losses is PLN 0.4m x 80% = PLN 0.32m.

An optimal cycle of successive input deliveries is associated with the problem of optimal stock management in the enterprise. An optimisation procedure starts with an optimisation model having a bicriterion goal function [Tymiń-ska 2007, p. 153]. The proposed concept consists in:

- finding the optimal level of the safety stock,
- determining the optimal cycle of deliveries,

for a goal criterion represented by a function minimizing the costs of storage and a depleted stock. Optimality attributes are not only the sizes of successive deliveries, but also their optimal schedules, full and regular utilisation of enterprise's production capacity, and then minimisation of the stock and production stoppage costs. The formal analysis of the problem starts with the knowledge of the initial level of inputs in stock denoted as ,,Z''. The stock ,,Z'' is used with a steady intensity ,,r'' until the level ,,z'' is reached (the safety stock). The latter level must be precisely determined, as it shall send out a ,,request'' for replenishing the stock to the original level of ,,Z''. The ordered inputs are delivered in the  $\tau$ -ith time, with  $,,\tau''$  being a random variable with the cumulative distribution function of  $F(\tau)$ . The elements subject to control are simultaneously  $,,\tau''$  (the cycle of order execution) and ,,z'' (the safety stock) (Fig. 1).

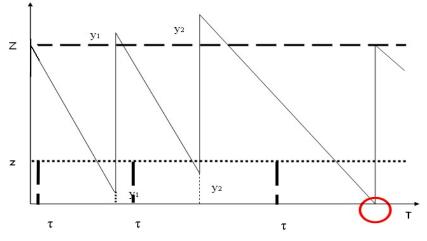


Fig. 1. Evolution of the stock of inputs as a function of time

Source: developed by the author based on [Tymińska 2007, p. 156]

In the successive replenishment cycles, the level of stock will equal, respectively:  $z, z + y_1, z + y_2$  ... etc., however, it will not exceed (z + Z). The variable z' is a parameter indicating the time when the stock has to be immediately replenished. The level of z' is an optimal stock at the point, where the probability of running out of the stock z' during order execution takes a predefined value  $\varepsilon$ . Hence, the probability of depletion of the safety stock (z) is  $P(\tau \ge z/r)$ , since  $F(\tau \le z/r)$ . Because  $\varepsilon$  is, in fact, the control operator, the sought-after value of z' will be given

by the formula:  $\varepsilon = 1 - F(\tau \le \frac{z}{r})$ .

The above gives rise to the following questions:

- what are the costs of stoppages caused by stock depletion for the predefined probability  $\varepsilon$ , and

- what is the optimal value of ", $z^*$ " for which the stock  $y(z^*)$  has to be replenished immediately;

- what is the optimal cycle of successive deliveries ( $\tau^*$ ) that prevents additional costs of the exhausted stock,

- what are the total storage costs ( $C_i$ ) in the *i*-th period separating successive deliveries,

- what is the relative, additional cost of preventing stock depletion, taking into account the probability of stock depletion ( $\varepsilon$ ). Without going into the details of mathematical transformations, the total storage cost in the *i*-th period can be given by the formula:

$$C_{i} = \frac{c}{2r} \left( Z + y(z)_{i-1} + y_{i} \right) \left( Z + y(z)_{i-1} - y(z)_{i} \right) = \frac{c}{2r} \left( Z^{2} + 2Zy(z)_{i-1} + y_{i-1}^{2} - y_{i}^{2} \right)$$

where (c) is the cost of storing a unit of stock in a unit of time,

After an infinite number of periods  $\left(\lim_{i\to\infty} \frac{Z+y(z)_{i-1}-y(z)_i}{r}\right)$ , an average storage cost is the expected value

 $E(C_1)$ , i.e.  $E(C_1) = \frac{c}{2r} \left[ Z^2 + 2ZE(y) \right]$ , where E(y) – the expected level of stock immediately preceding a replenishing delivery, with y'' allowed taking values:

$$y = \begin{cases} z - r\tau & \text{for } \tau \le z/r \\ 0 & \text{for } \tau > z/r \end{cases}$$

Given that  $F(z/r) = 1 - \varepsilon$  and  $\int_{0}^{z/r} \tau dF(\tau) = E(\tau) - \int_{z/r}^{\infty} \tau dF(\tau)$ 

Ultimately, we will have1:

$$E(y) = z - rE(\tau) + \left(r\int_{z/r}^{\infty} \tau dF(z) - z\varepsilon\right)$$

For  $\varepsilon < 1$  there will be  $z/r \ge 1$ . Because the expression in the brackets takes very small values  $E(y) \approx z - rE(\tau)$ The economic reality makes the firm accept the possible fluctuations in the cycle of replenishments. Managerial decisions influencing the stock-control processes should tend to secure the continuity of the production process. In the business practice, providing a rationale for keeping a larger safety stock is not a problem. The problem lies in answering the following question: what additional cost can be expected when a higher, safer level of the stock of inputs is kept and the assumed probability of stock depletion is ( $\varepsilon$ ).

The predefined value of  $\varepsilon$  shapes the additional (relative) costs of running out of stock. This relationship is rather obvious – the higher probability of stock depletion, the lower storage costs, but the higher risk of disturbed continuity of production.

To identify an increase in the costs it is proposed [Wolski 1998] to compare the storage costs when the time ",  $\tau$ " is a constant value and the estimate storage costs when ",  $\tau$ " is a random variable. This actually comes down to the formula [Goddard 1966, p. 228]:

<sup>1</sup> 
$$E(y) = \int_{0}^{z/r} y dF(\tau) = \int_{0}^{z/r} (z - r\tau) dF(\tau) = zF(z/r) - r \int_{0}^{z/r} \tau dF(\tau)$$

$$R = \frac{Z^2 + 2ZE(y)}{Z^2} = 1 + \frac{2}{Z}E(y) \approx 1 + \frac{2}{Z}[z - rE(\tau)].$$

The expression  $2[z - rE(\tau)]$  can be taken as a measure of the additional costs of preventing the stock from depletion (at the level of  $\varepsilon$ ). This means that the relative additional costs for the probability of stock depletion  $\varepsilon$  are  $Kz(d) = 2[z - rE(\tau)]/Z$ .

Because of the character of the analysed question, we need to consider a cost model of production stopped by the depleted stock (E(C2)). This model is based on a unit cost of stock depletion (cp):

$$cp = (C_2) = Kbr \left\langle \left\{ \left( \frac{1}{\lambda} \cdot r \cdot p \right) / \varepsilon / (R-1) / Z / 12 \cdot Q \right\} \right\rangle$$

where:

cp – a unit cost of depleted stock Kbr – an annual cost of depleted stock Q – annual output in natural units

Assuming that the exponential distribution [Wolski 1998] of the form  $e^{-\lambda t}$  is typical of the probability of occurrence of stock depletion, calculations of the input control parameters are as follows:  $\lambda t = p$  (because of the equality  $e^{-\lambda t} = e^{-p}$ ), hence  $\varepsilon = e^{-p}$ .

The model of total production stoppage costs has the form:

$$E(Cx_2) = \frac{cp}{2r} \Big[ Z^2 + 2Z(z - rE(\tau)) \Big]$$

Ultimately, the difficult choice between higher storage costs due to keeping a larger stock, on one hand, and higher production costs when the firm runs out of inputs, on the other, can be resolved by applying a "combined" criterion for minimising the total costs E(C). Then the optimisation model for the total costs of stock is represented by the sum:

$$E(Cx_{1}) = \frac{c}{2r} \left[ Z^{2} + 2Z(z - rE(\tau)) \right]$$

$$E(Cx_{2}) = \frac{cp}{2r} \left[ Z^{2} + 2Z(z - rE(\tau)) \right]$$

$$E(Cx_{2}) = \frac{cp}{2r} \left[ Z^{2} + 2Z(z - rE(\tau)) \right]$$

This optimisation model allows finding a sensible solution fulfilling the assumed criterion function. The control actions are necessary for this purpose. An effective tool of control is a cybernetic model for the dynamic regulation of input deliveries.

### **CONSTRACTING A DELIVERY CONTROL MODEL**

A delivery control model is an instrument supporting the optimal choice of the safety stock ( $z^*$ ) and of the schedule of deliveries ( $\tau^*$ ). Designed according to a feedback principle, the model comes down to a series of relations reflecting the mathematical sequences within the examined problem. This can be graphically presented in the following manner (Fig. 2):

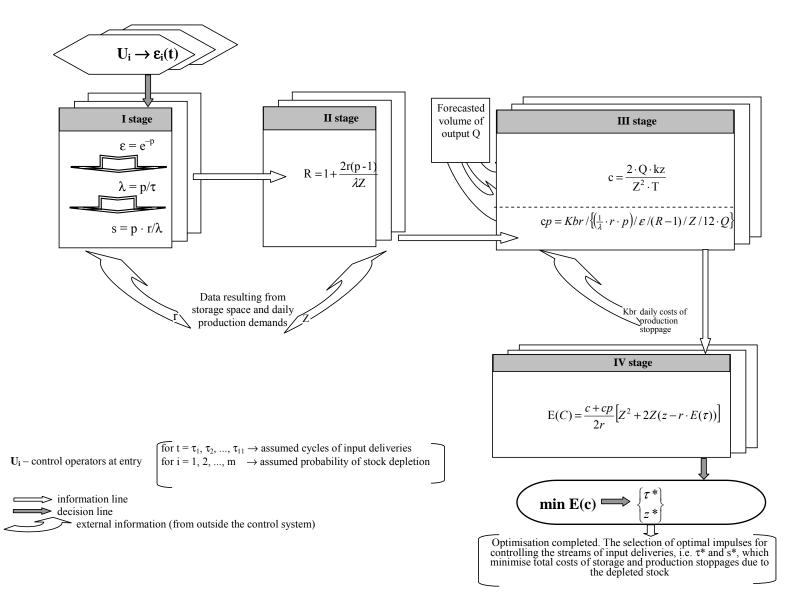


Fig. 2. The stock control and regulation system

The illustrated control model for a stock of inputs is a feedback-based sequential model, i.e. one using successive approximations. The optimisation criterion tends to minimize the total of storage costs and stoppage costs generated by depleted stock. The control procedures are applied to the successive probabilities of stock depletion ( $\varepsilon$ ) that range from 0.01 to 0.37; the optimisation process ends when the goal function takes the smallest value, i.e. min E(C). The completed optimisation means that the optimal impulses for managing the streams of deliveries, i.e.  $\tau^*$  and  $z^*$ , have been selected. These impulses ensure the maximal reduction in the total storage and stock depletion costs.

### CONCLUSIONS

- 1. Because of the variability of demand and the need to comply with the accepted customer service standards a relevant stock keeping and controlling strategy has to be in place. This problem can be solved using a rather sophisticated mathematical structure the stochastic models. The models assist in making optimal decisions within the stock management area.
- 2. The stream management process in the enterprise includes an element of fluctuations in demand. This is a source of "a conflict" that appears where the two subsystems storage and production converge. It is particularly important to synchronize the physical streams of materials in this area, because of the high costs of depleted inputs and of storage.
- 3. At the same time, a systemic approach to the optimal coordination of supply/production calls for smooth flows of information and effective management of the whole system. Implementation of modern information technologies is one of the factors supporting effective control.

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