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HIGH ORDER THEORIES FOR INVESTIGATION OF LAMINATED STRUCTURES WITH CLAMP CONDITION

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ABSTRACT

This paper extends the applicability of a new stress analysis method to the accurate determination of the detailed stress distributions in laminated plates in clamp region. The theoretical model presented here incorporates laminate deformations which account for the effects of transverse shear deformation, transverse normal strain-stress and a nonlinear variation of in-plane displacements with respect to the thickness coordinate. Thus, this model includes the warping of transverse cross-sections more accurately and eliminates the need of shear correction coefficients. Two numerical examples are discussed: the bending of elastically clamped beam and dynamic bending of centrally supported beam.

Key words: composite beams, clamped plates, elastic constants, identification

INTRODUCTION

The deformation of composite beams and plates is often described by means of some simplified models. A variational technique is commonly used to derive the basic equations governing the vibrations of the sandwich structures. Reference is often made to the Timoshenko [6,9,15] models.

A more general description of the bending of composite beams is given in [10]. In this model, the laminates are again described as thin plates. However, the general wave equation is used to describe the displacement in the core. The influence of boundary conditions is not discussed. A considerable computational effort is required. In [12] simple differential equations governing the apparent bending of sandwich panels are derived. The model includes shear effects. Compared with measured results, this model overestimates the shear effects. In particular, in the high frequency region, the predicted bending stiffness is too low. A modified Mindlin plate theory was suggested in [7]. The influence of some boundary conditions is also discussed in a subsequent paper [11]. Again a considerable computational effort is required. In [8] the Timoshenko beam equations are used to describe flexural vibrations of sandwich structures. In particular, variations of the shear coefficient are discussed for obtaining satisfactory results.

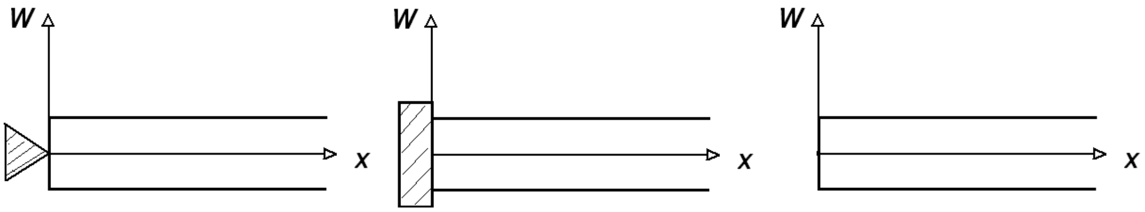
For many of those references, there is common that the derived governing differential equations are of the 4th order. Due to the frequency dependence of sandwich, the solutions with four unknowns will agree for low frequency very well. With increasing frequency, the result normally strongly disagrees with measured vibrations.

The main work on sandwich structures has been made on conventional foam-core structures with various face sheets. Little work has been done on the dynamics of honeycomb panels. In [13], there is presented an article, treating how to identify the dynamic parameters for aluminum honeycomb panels using orthotropic Timoshenko beam theory. The Authors had used a 4th order differential equation and determined the dynamic parameters comparing their theories to frequency response measurements. A 3-D theory presented in [1] is proposed for the calculation of the natural frequencies of laminated rectangular plates. The paper also includes a long reference list, each reference being classified according to the method being used.

Certainly, there is available a large number of methods which describe the vibration of composite panels. However, the aim of this work is the formulation of simple but sufficiently accurate equations governing the apparent boundary conditions for deformation of sandwich beams and plates. Boundary conditions and apparent discrete scheme for clamp constructions should also be formulated for the calculation of eigen-frequencies. The models should allow simple parameter studies for the optimization of the structures with respect to their vibration performance. The aim is also to describe a simple measurement technique for determining some of the material parameters of composite beams.

CLASSICAL BOUNDARY CONDITIONS FOR EULER BEAM

The beam must satisfy certain boundary conditions at each end. Boundary conditions for Euler beam such as simply supported, clamped and free can be defined by means of [6,9,15]. For each boundary condition, certain requirements for the displacement w and the angular displacement $\frac{\partial w}{\partial x}$ as well as forces and bending moments must be considered.



Simply supported boundary   Clamped boundary   Free boundary

Fig. 1. Boundary conditions

The natural frequencies for a clamped and a free beam are identical, according to the Euler theory. However, when shear is considered, there is a difference between the natural frequencies for these two conditions. The natural frequencies for the clamped boundaries are the lowest. This is due to the fact that shear in the beam is induced by the clamped boundaries to a larger extend as compared to the case with free boundaries. The apparent bending stiffness for the clamped beam is therefore somewhat lower than for the beam with free ends.

REFINED CLAMP CONDITIONS. BENDING OF PLATE WITH ELASTIC CLAMPED BOUNDARY CONDITIONS

Let us consider the cylindrical bending of a plate in elastic joint [2,4,5]. The material of the plate is assumed to be more rigid than that of the interlayer (Fig. 2). The material of the plate is assumed to be anisotropic, with elasticity modulus C_{xx}, C_{xz}, C_{zz} and G_{xz}. The material of the interlayer is assumed to be isotropic and incompressible (rubber-like material). Such a joint is widely used in industry and in laboratory experiments. The equilibrium equations are derived on the basis of the kinematics hypothesis:

$$u = u_{ij} \cdot x^i \cdot z^{j-1}, \text{ and } w = w_{ij} \cdot x^i \cdot z^{j-1}, \tag{1}$$

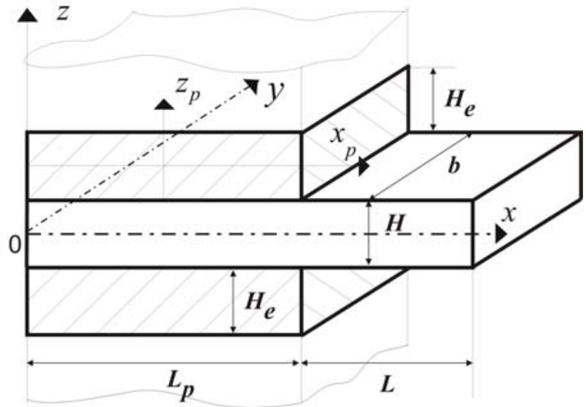


Fig. 2. Clamped plate scheme

By substitution of Eq.(1) into the Eq.(2)

$$\int_{V_p} (\sigma_{xx}\delta\varepsilon_{xx} + \sigma_{zz}\delta\varepsilon_{zz} + \tau_{xz}\delta\varepsilon_{xz})dV + \int_0^L (K^+(x)w^+ + K^-(x)w^-\delta w^-)dx +$$
$$+ \int_0^L (K_u^+(x)u^+\delta u^+ + K_u^-(x)u^- \cdot \delta u^-)dx - \int_{-H_p}^{H_p} (N(z)\delta u + T(z)\delta w)\cdot dz = 0 \qquad (2)$$

we obtain the set of the linear algebraic equations on u_{ij} and w_{ij}.Here $K^{\pm}$, $K_u^{\pm}$ are the normal and tangential elastic coefficients of the elastic interlayer, respectively – such as these used in the Winkler foundation (in the most simple case $K^{\pm} = E^{\pm}/H^{\pm}$, $K_u^{\pm} = G^{\pm}/H^{\pm}$, $E^{\pm}, G^{\pm}$ are interlayer's Young and shear modules, $H^{\pm}$ are its depths. *N(z), T(z)* are external tangential and normal forces on the right end of the beam. In more detailed analysis $K^{\pm}$, $K_u^{\pm}$ may be found by the same method [11,12,13].

In Fig. 3, 4 some numerical results are presented for a plate elastically clamped to point $L_T = 5.0$.

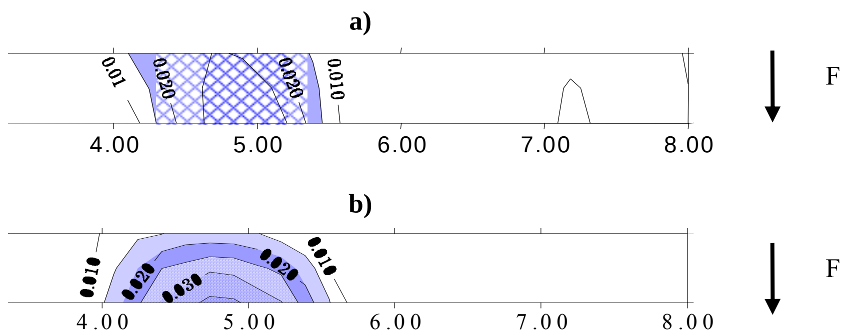


Fig. 3. Normal stress σ_{zz}/E for a distinct anisotropy level ((a) – $E/G = 0.4$; (b) – $E/G = 0.1$)

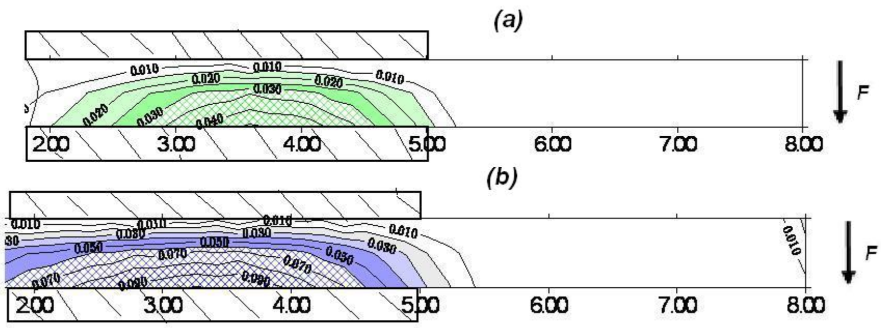


Fig. 4. Tangential stress τ_{xz}/E for a distinct anisotropy level ((a) – $E/G = 0.4$; (b) – $E/G = 0.1$)

For a good result 4 terms are needed for convergence in the normal direction and more than 13 in the tangential ($i > 13,\ j \geq 4$ in Eq.(1)). The result is incorrect, if we reduce the value j of the term in the normal direction to 1 or 2 (see [3]). The error is about 50%.

In Figs. 5a and 5b the slopes of beam near to clamp are shown. The Fig. 5a concerns an anisotropic beam and varying relationships, C_z / C_x. The Fig. 5b concerns a three-layer beam with the inner layer of thickness H_2 and face layers of the thickness H_1, each layer has various mechanical properties.

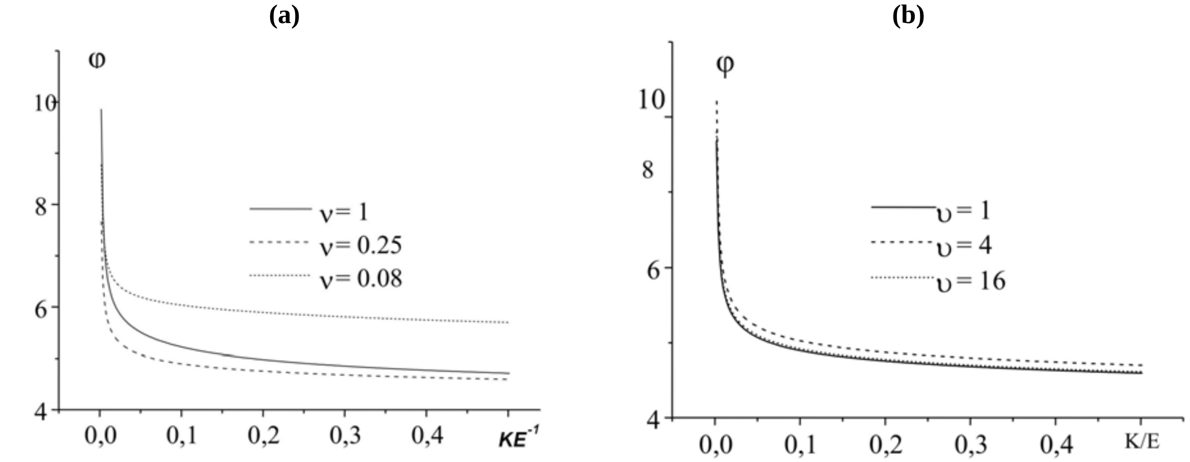


Fig. 5. (a) – the angles f of beam deflection near to clamp (K / E – clamp rigidity to Young modulus ratio, $\nu = C_{xx} / C_{zz}$); (b) – the angles of three-layer beam deflection near to clamp ($\upsilon = C_{xx}^1 / C_{xx}^2$ – outer layer to inner layer rigidity ratio for beam with layers thickness ratio, $H_1 / H_2 = 0.2$)

In Fig. 5, there is visible that by absolutely rigid clamp the slope of beam is not equal to zero.
For Euler beam clamp we can write

$$K_\Sigma = \frac{K_e K_c}{K_e + K_c}, \quad K_e = \frac{L_s^3}{3EI},$$

where K_Σ – integral rigidity, K_e – rigidity of rigidly clamped Euler beam (L_s – beam length, EI – beam bending rigidity), K_c – clamp rigidity.

DISCRETE-CONTINUUM SCHEME OF BEAM CLAMP

In Fig. 6 the discrete-continuum scheme of beam – shaker system is shown

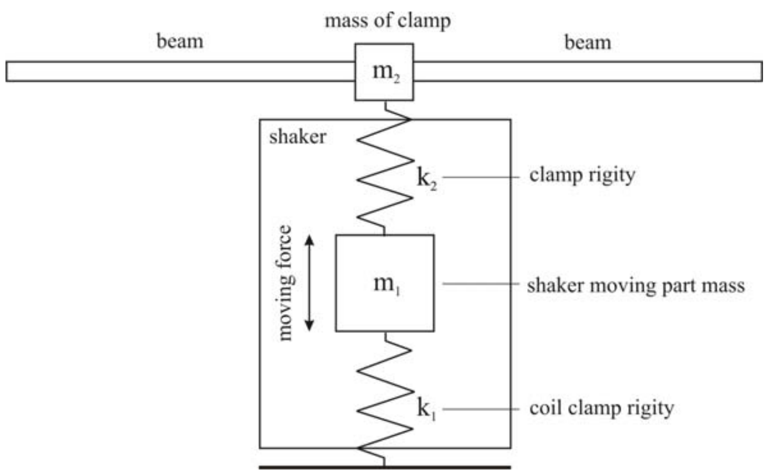


Fig. 6. Mechanical scheme for vibration measurements of sandwich beam

In the experiment the beam was excited with white noise by a shaker mounted at the middle. The experimental Bruel&Kjaer PULSE system analyzes the signals using dual FFT.

THEORETICAL-EXPERIMENTAL COMPARISON

The range of numerical experiments must be done to be sure that our theoretical approach is correct. Let us consider now the vibration testing of a beam made of a foam material. Geometrical parameters are: length of beam $L = 0.60 - 0.20$ M; Width of beam $H = 0.0127$ M. Elastic moduli are: $C_{xx} / C_{zz} = 250$ MPa, $G = 58$ MPa, $C_{xz} = 40$ MPa. The FRF for this beams are presented in Figs. 7, 8. The discrete-continual numerical scheme based on Fig. 2 is applied. In [3], elastic constants of laminates were determined by using an identification procedure based on experiment design, and multi-level theoretical approach. Results of identification for beam made of foam material are presented in Fig. 7. The FRF for this beams in the middle frequency range are presented in Fig. 8 .

Here $f_1 = \frac{1}{2\pi}\sqrt{\frac{k_2}{m}}$ – the eigen-frequency of mass m on spring k_2 . In Fig. 8 only small influence of this value on high frequency vibration can be seen. By the dot vertical lines the Euler beam eigen-frequency are presented.

DISCUSSION

The order of approximation must be higher than 3 – for the normal displacement w and for the plane displacement u. If the frequency is higher, or the plate is not simply supported but clamped (as usually is in practice), or it has stiffeners or holes, then more convenient theories must be developed. The simple examples given in this paper show that the theories based on assumptions of a lower order approximation are unsatisfactory also in the case of a simply-supported plate. For the each case under investigation it must be decided which theory is most satisfactory. Each theory has its own limitations. It is well known that the exact solutions of elasticity theories are singular near the corners of a plate. Thus, the oscillation solutions found, for example in [14] by ABAQUS, are obviously not physically convenient to use in the cases studied herein.

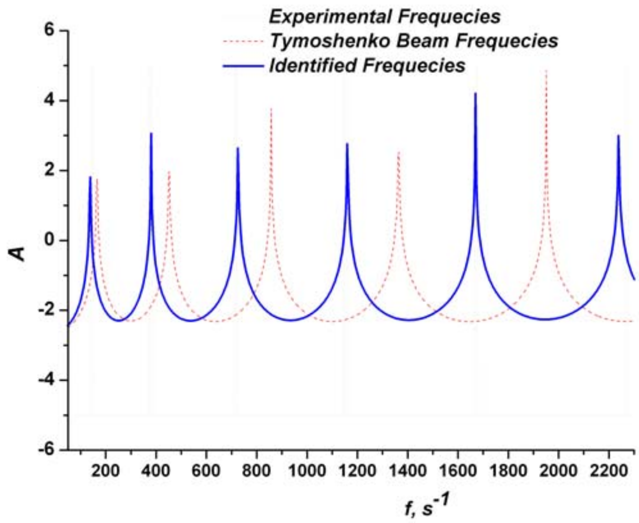


Fig. 7. Results of identification for beam made of foam material

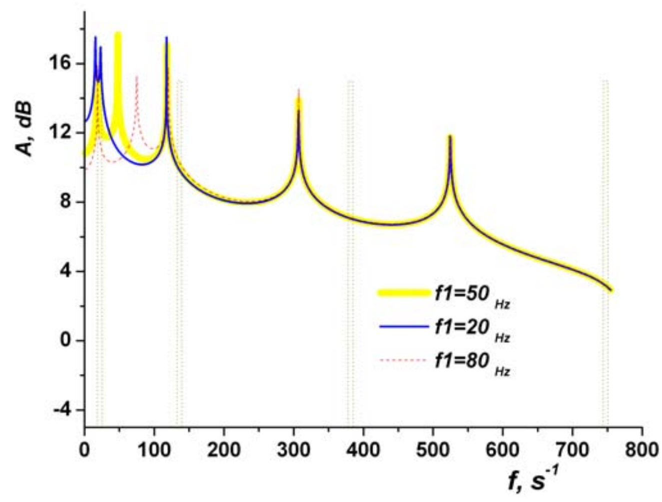


Fig. 8. Resulting FRF for beam in the middle frequency range

CONCLUSIONS

In this paper various displacement models have been developed by considering combinations of displacement fields for in-plane and transverse displacements inside a mathematical sub-layer. The developed numerical method follows a semi-analytical approach with the analytical field assumed in the longitudinal direction and a layer-wise displacement field assumed in the transverse direction. The present work aims at developing a simple numerical technique, which can produce very accurate results in comparison with the available analytical solutions and also [can help?] to decide about the level of refinement in higher order theory that is needed for accurate and efficient analysis of clamped plates.

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